

Lecture 2

Backwards Induction: Bargaining and Contracts

This lecture introduces a second principle for solving games with only singleton information sets, perfect information games. Examples of these games are found in bargaining and contracting. We analyze the solutions to several simple bargaining and contracting games. This leads us to a theory about the firm size.

Perfect information games

- ◆ In games of perfect information players take turns making their moves, knowing what the players who have moved before them have chosen.
- ◆ Every perfect information game has an extensive form in which all the information sets are singletons.
- ◆ There are no dotted lines joining nodes. Thus all the strategic interactions are sequential.

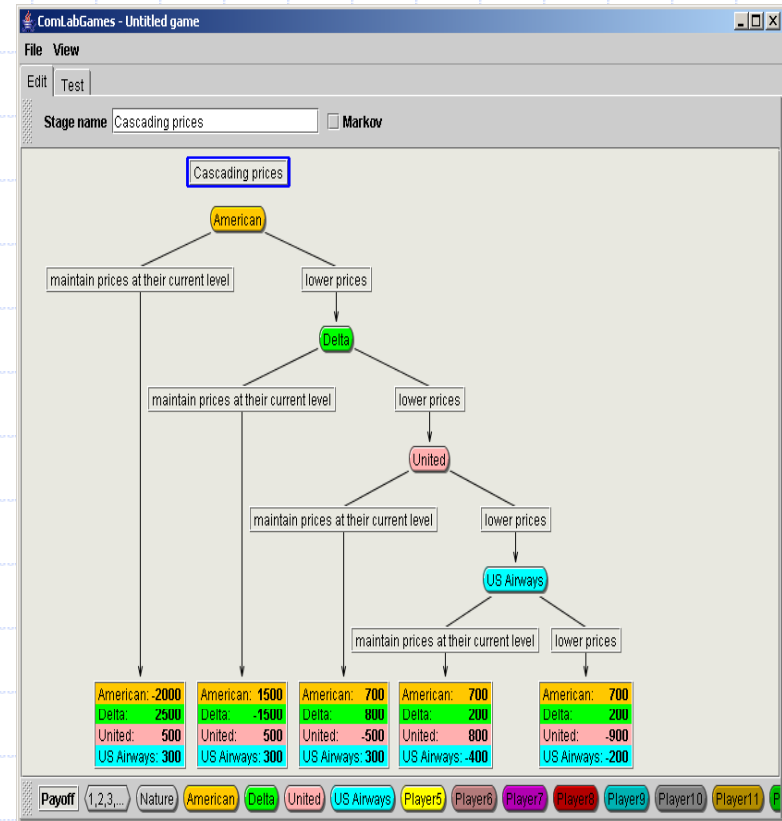
Regional markets

- ◆ A prominent feature of geographically based markets, such as personal services, retailing, distribution, and travel is that the regional markets overlap.
- ◆ Thus competition in one market can spill over into the next, creating a cascading effect.
- ◆ Perhaps nowhere is this more evident than in the airline industry.

Airline pricing cascades

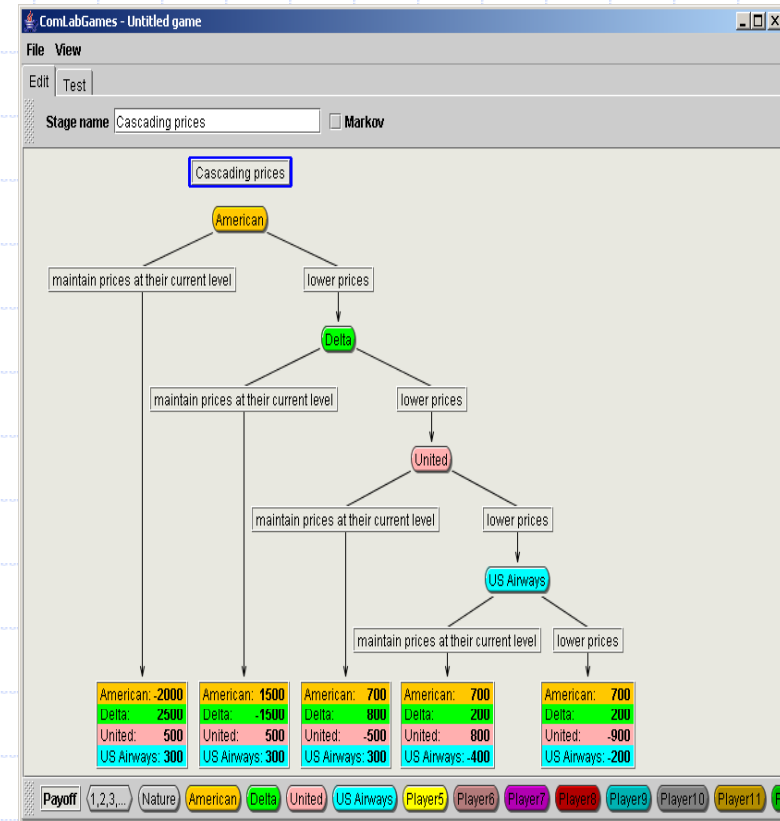
◆ In this example, American competes with Delta, Delta also competes with United, United competes with Delta, and US Airways, while US Airways only competes with United.

◆ Thus the payoff to US Airways is unaffected by rivalry between American and Delta.



Best response to the empirical distribution

- ◆ In this example, American competes with Delta, Delta also competes with United, United competes with Delta, and US Airways, while US Airways only competes with United.
- ◆ Thus the payoff to US Airways is unaffected by rivalry between American and Delta.



The third rule of strategy:

Backward induction

Sometimes called a rollback equilibrium the principle of backwards induction shows how finite games of complete information can be solved.

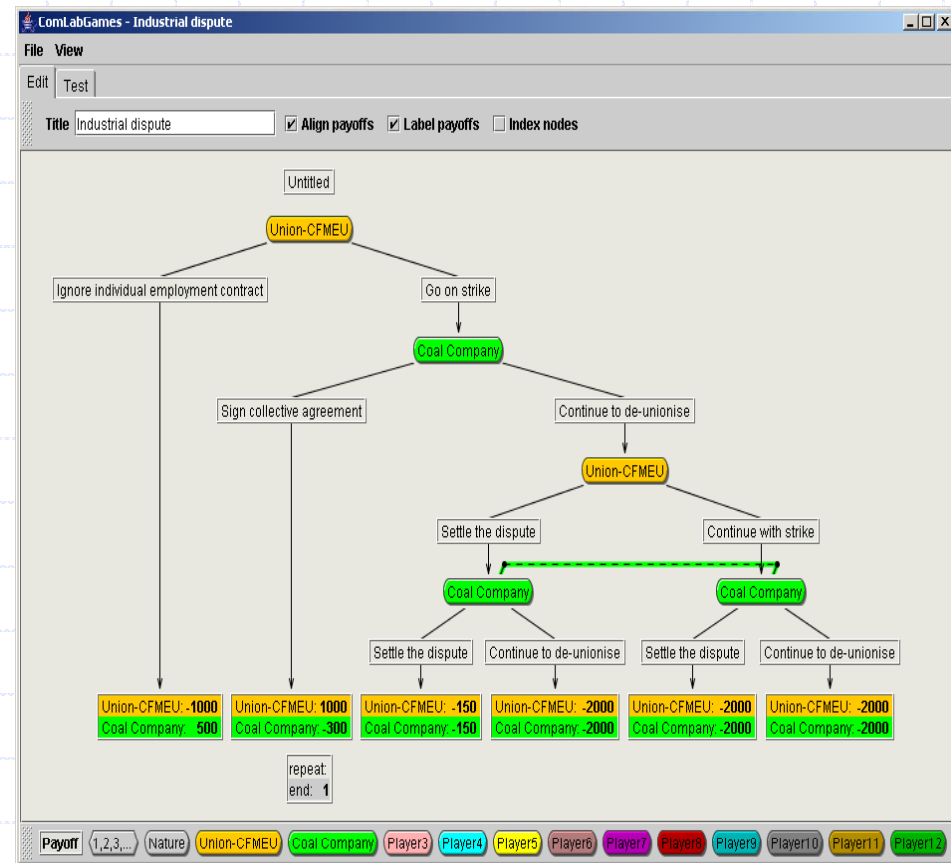
Rule 3: Look ahead and reason back.

Unrest down under

- ◆ Coal miners rank amongst the highest paid workers Australia, and the media are not shy about broadcasting this fact.
- ◆ Encouraged by a conservative government, several years ago a major coal exporter in Australia began to negotiate with its workers individually.
- ◆ In this way the exporter hoped to undermine the power of the union representing the workers, and ultimately reduce the miner's wages by encouraging entry into that profession.

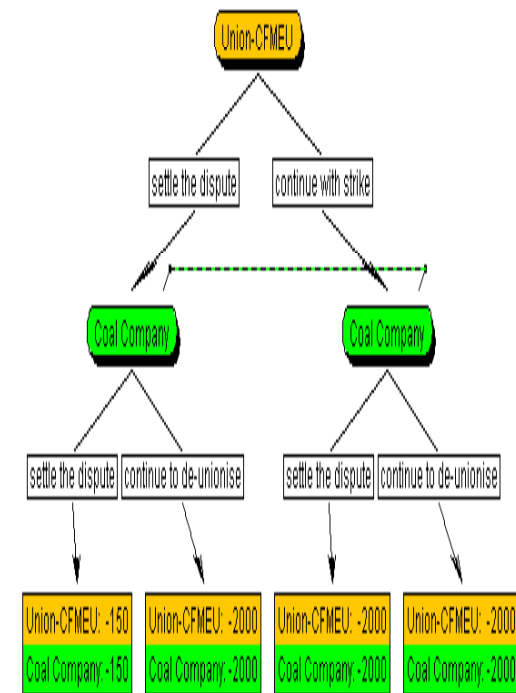
Combining a simultaneous move game with a perfect information game

This game can be solved by combining the dominance principle used in simultaneous move games and the roll back rule used in perfect information games.



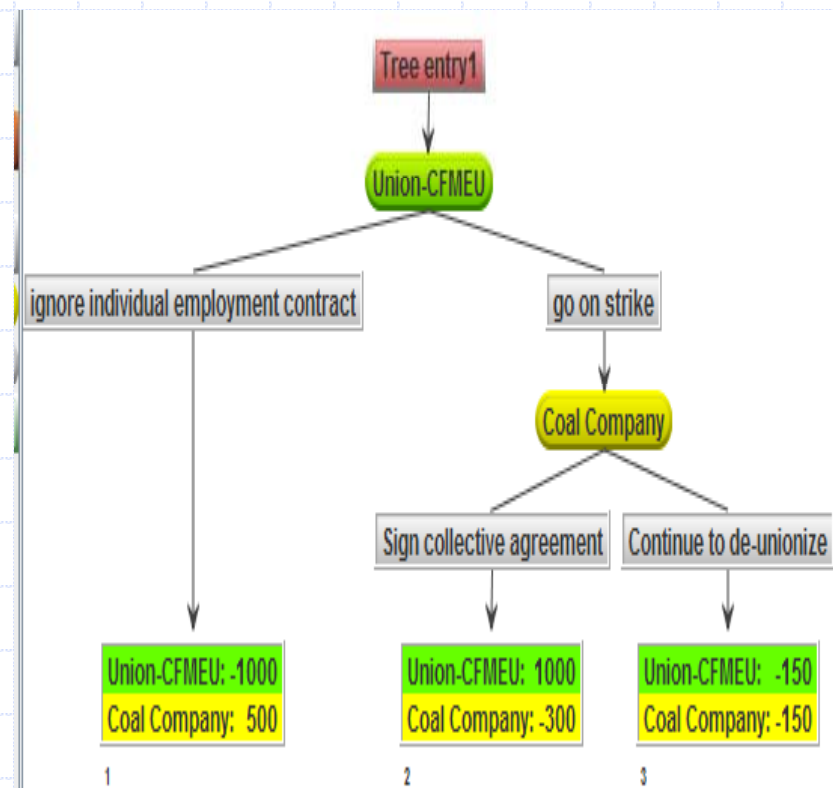
Subgame for industrial dispute

- ◆ Consider what would happen if management chose “Continue to de-unionize”.
- ◆ Having reached the second choice node of the union, the remainder of the game could be considered a game in its own right. This is why we call it a **subgame**.
- ◆ In this case the subgame is itself a simultaneous move game.

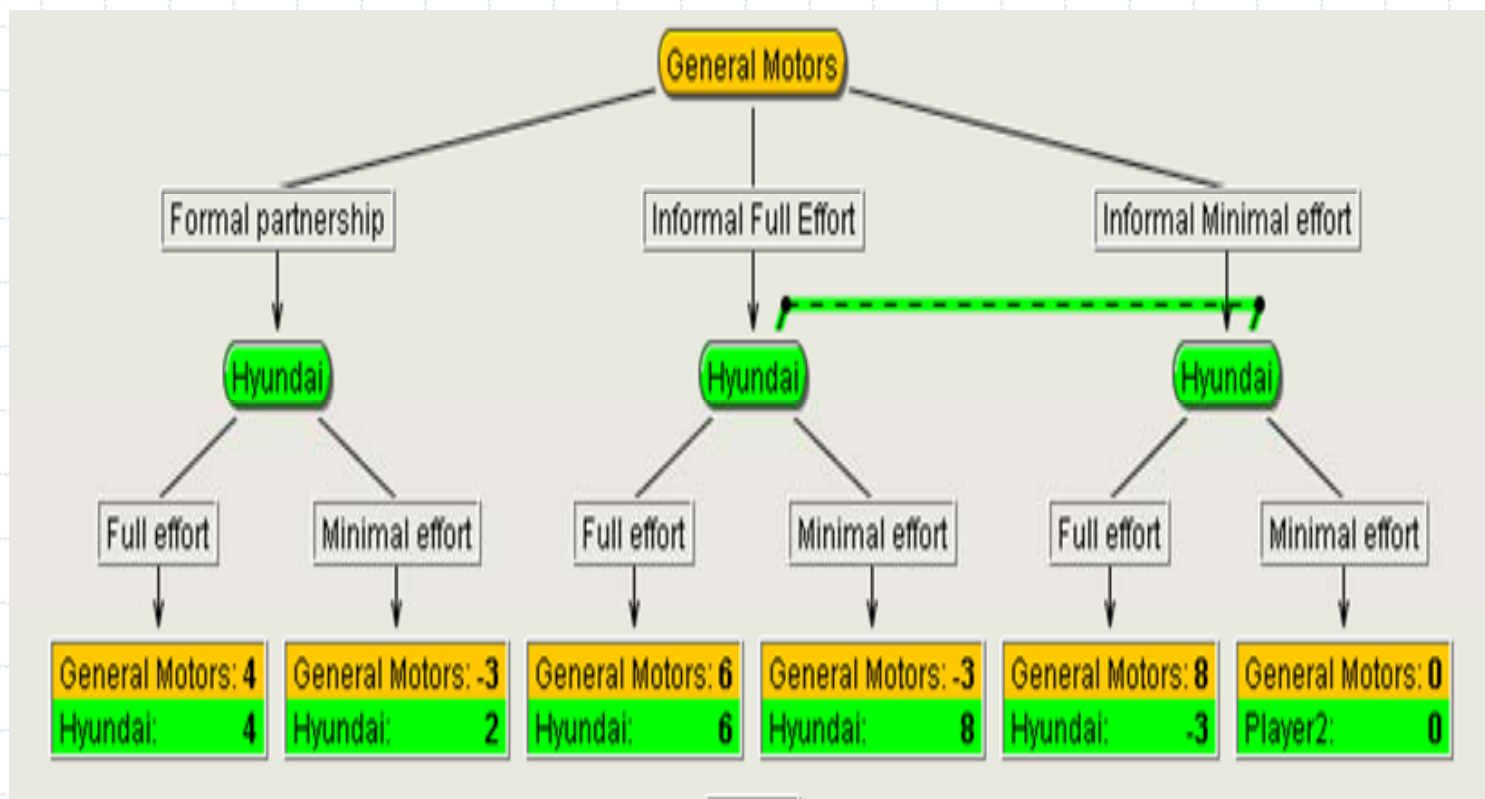


The reduced game

- ◆ Having solved the subgame, we now substitute its solution into the main game, to form a reduced game
- ◆ Notice that this procedure mimics the roll back method we used to solve perfect information games.



Alliance



Strategic form of a subgame for Alliance

- ◆ If GM does pursue a formal relationship with Hyundai, then it would enter a simultaneous move subgame in which each company determined its effort level informally.

		Hyundai	
		Full effort	Minimal effort
General Motors	Informal full effort	6, 6	8, -3
	Informal minimal effort	-3, 8	0, 0

- ◆ In the subgame modeling an informal alliance, both players have a dominant strategy of exerting “minimal effort”.
- ◆ The value of a formal relationship, and incurring the costs of lawyers, becomes evident.

Walmart and Kmart

- ◆ Consider a two round bargaining game in which Walmart may buy some of Kmart's outlets to help "rationalize" geographical sales regions.
- ◆ Insider experts agree that the sum of the increase in value to both firms from transferring ownership of these stores from Kmart to Walmart is worth \$100 million.
- ◆ Also the physical assets themselves are valued at another \$50 million.
- ◆ To summarize: if the stores are sold, the two parties Kmart and Walmart split \$150 million, but otherwise they both net zero.

Time line for bargaining

- ◆ First Walmart makes an offer to Kmart.
- ◆ The very next instant there is a 50 percent chance that the FTC issues a ruling preventing the sale.
- ◆ Otherwise Kmart can accept the offer, or reject it.
- ◆ If Kmart rejects, Walmart has a 40 percent chance of making a second offer, but with chance, Kmart has a 60 percent of coming back first, with its own price offer.
- ◆ If a second offer is made by either party, there is a 50 percent chance FTC may belatedly prevent the sale.
- ◆ If the second offer is allowed by the FTC, but is rejected anyway by the counterparty, then no one gets anything, and Kmart retains ownership of its stores.
- ◆ In that last case the FTC stops negotiations for sure because the journalists have well and truly caught on.

Bargaining Rules

- ◆ As these examples suggest backwards induction can be sometimes applied in bargaining situations.
- ◆ Bargaining is one way of resolving a conflict. (Others include litigation, theft and warfare).
- ◆ For example:
 1. Unions bargain with their employers about wages and working conditions.
 2. Professionals negotiate their employment or work contracts when changing jobs.
 3. Builders and their clients bargain over the nature and extent of the work to reach a work contract.

A finite round bargaining game

- ◆ Suppose there are T rounds of bargaining between 2 players who are trying to split some gains from trade that are normalized to one unit.
- ◆ In the first round of bargaining the designated proposer offers anything between 0 and 1, and the responder either accepts or rejects the offer.
- ◆ If an agreement is reached at any time during the bargaining game, the bargaining ends immediately and the payoffs reflect the terms of the agreement.

If an agreement is not reached . . .

- ◆ If the proposal in round $t < T$ is rejected, with probability $1 - p$ the bargaining game ends, and the players walk away empty handed.
- ◆ If the proposal in round $t < T$ is rejected, the bargaining continues for another round with probability p , where $0 < p < 1$.
- ◆ In that case the player who has just rejected the most recent proposal makes a counter offer.
- ◆ If T proposals are rejected, the bargaining ends, and if no agreement is reached, both players receive nothing.

Solution to $T=1$ bargaining game

- ◆ The responder should always accept any positive amount.
- ◆ The solution is for the proposer to extract (almost) all the surplus, and for the responder to accept the proposal.
- ◆ Note that there is no scope for retribution in this game.
- ◆ Anger or other emotions at unfairness should already be incorporated into the payoffs. If you are representing a company then ask yourself whether the company wants your feelings accounted for when negotiating on its behalf.

Solution to $T=2$ bargaining game

- ◆ In the final period the second player recognizes that the first will accept any final strictly positive offer, no matter how small.
- ◆ Therefore the second player reject any offer with a share less than p in the total gains from trade.
- ◆ The first player anticipates the response of the second player to his initial proposal.
- ◆ Accordingly the first player offers the second player proportion p , which is accepted.

Subgames with K rounds and backwards induction

- ◆ If the game reaches round $T - K$ without reaching an agreement, the player proposing at that time will treat the last K rounds as a K round game in which he leads off with the first proposal.
- ◆ Therefore the amount a player would initially offer the other in a K round game, is identical to the amount he would offer if there are K rounds to go in $T > K$ round game and it was his turn.
- ◆ So if there are K rounds to go and v_K is the value to the proposer at that time, then we might think that:

$$v_K = 1 - pv_{K-1}$$

Solution to alternating offers bargaining game

- ◆ Let v_T denote the value of being the proposer at the beginning of a T period bargaining game.
- ◆ Appealing to the mathematical induction principle, v_T is a geometric series with first term 1 and ratio $-p$.
- ◆ The formula for a geometric series implies that:

$$v_T = [1 - (-p)^T] / (1 + p)$$

- ◆ Observe that as T diverges, v_T converges to:

$$v = 1 / (1 + p)$$

which is the value of the infinite horizon game.

Proving the formula for the sum of a geometric sequence

- Denote by v_k the sum of k terms where the first term is a and the ratio of two adjacent terms is r . That is:

$$v_k = a + ar + ar^2 + \dots + ar^{k-1} \quad (1)$$

- Multiply this equation by the ratio r to obtain:

$$rv_k = ra + ar^2 + ar^3 + \dots + ar^k \quad (2)$$

- Subtract Equation (2) from Equation (1), which yields:

$$v_k - rv_k = v_k(1 - r) = a - ar^k = a(1 - r^k)$$

- Divide both sides through by $(1 - r)$ to derive the formula for v_k :

$$v_k = a(1 - r^k) / (1 - r)$$

- As $k \rightarrow \infty$ then $r^k \rightarrow 0$ and $v_k \rightarrow a / (1 - r)$ if $|r| < 1$.

Randomizing the role of proposer and responder each period

- ◆ Suppose there is an equal chance of being the proposer in each period. How does the solution differ depend on the chance of being selected?
- ◆ As before p denote the probability of continuing negotiations if no agreement is reach at the end of the first round.
- ◆ We consider a 2 round game. If the first round proposal is rejected, then the expected payoff to both parties is $p/2$.

The value of being the proposer

- ◆ In the solution to a two round alternating offers bargaining game, the proposer in the first period receives a payoff of $v = 1 - p$.
- ◆ In the solution to a two round bargaining game where the roles are randomized each period, the initial proposer attains a payoff of $v = 1 - p/2$.
- ◆ This is more than in the alternating offers game, because he has a higher chance of being the proposer next period.
- ◆ In the extreme case, where the initial proposer has another opportunity to make an offer in the second round if his first offer is refused (so the responder never becomes a proposer), his solution payoff is v .

Bargaining with full information

- ◆ Two striking features characterize the solutions of both bargaining games:
 1. An agreement is always reached.
 2. Negotiations end after one round.
- ◆ This occurs because nothing is learned from continuing negotiations, yet a cost is sustained because the opportunity to reach an agreement is put at risk from delaying it.

Reaching agreement may be costly

◆ Yet there are many situations where conflict is not instantaneously resolved, and where negotiations break down:

1. In industrial relations, negotiations can be drawn out, and sometimes lead to strikes.
2. Plans for construction projects are discussed, contracts are written up, but left unsigned, so the projects are cancelled.

The role of information

- ◆ If the value of the match is constant throughout the bargaining phase, and common knowledge, then it will be formed immediately, or not at all.
- ◆ What if the proposer does not know the value of the match to the responder?
- ◆ Suppose the proposer demands s from the responder. The responder draws a value v from the probability distribution $F(v)$. We normalize v so that $v_0 \leq v \leq v_1$.
- ◆ The responder either accepts or rejects the demand of s . If the demand is accepted the proposer receives s and the responder receives $v - s$, but if the demand is rejected neither party receives anything.

The proposer's objective

- ◆ The responder accepts the offer if $v > s$ and rejects the offer otherwise.
- ◆ Now suppose the proposer maximizes his expected wealth, which can be expressed as:

$$\Pr\{v > s\}s = [1 - F(s)]s$$

- ◆ Notice the term in the square brackets $[1 - F(s)]$ is the quantity sold, which declines in price, while s is the price itself.

Solution to the game

- ◆ Let s^0 denote the optimal choice of s for the proposer. Clearly $v_0 \leq s^0 < v_1$.
- ◆ If $v_0 < s^0 < v_1$, then s^0 satisfies a first order condition for this problem:

$$1 - F'(s^0) s^0 - F(s^0) = 0$$

- ◆ Otherwise $s^0 = v_0$ and the proposer receives:

$$[1 - F(v_0)]v_0 = v_0$$

- ◆ The revenue generated by solving the first order condition is compared with v_0 to obtain the solution to the proposer's problem.

$F(s)$ is $[0,1]$ uniform

- ◆ For example suppose:

$$F(v) = v \text{ for all } 0 \leq v \leq 1$$

- ◆ If the proposer set demands an amount that every responder would accept, he nets 0.

- ◆ Noting $F'(v) = 1$ in this example, the first order condition for the interior solution is:

$$1 - s - s = 0$$

- ◆ Solving, the interior solution applies so $s^0 = 1/2$, and $F(v) = 1/2$.
- ◆ Thus exchange only occurs half the time where there are gains from trade.

Losses from strategic play

- ◆ How much of the potential gains from trade are realized in this bargaining game with incomplete information?
- ◆ The trading surplus is found by integrating the gains over the valuation:

$$\int_{s_o}^{v_1} vF'(v)dv = \int_{1/2}^1 vdv = \left[v^2/2 \right]_{1/2}^1 = 3/8$$

- ◆ Given our assumption about $F(v)$ it follows that 3/8 of the trading surplus is realized, which is 3/4 of the potential surplus, which we derive as:

$$\int_{v_o}^{v_1} vF'(v)dv = \int_0^1 vdv = \left[v^2/2 \right]_0^1 = 1/2$$

Contracting

- ◆ The preceding examples illustrate the value of setting the bargaining rules and conventions that determine how the potential gains from trade should be divided.
- ◆ What is the most desirable way of writing a contract?
- ◆ Almost by definition, managers have a hand in writing employment contracts for their workers, and often determine upstream supply and downstream service contracts as well.
- ◆ In these last two examples we consider key features of consumer and employment contracts.

Service provider

- ◆ Multipart pricing schemes are commonly found in the telecommunications industry, amusement parks, sport clubs, and time sharing vacation houses and small jets.
- ◆ In this example a provider incurs a fixed cost of c to connect the consumer to the facility, and a marginal cost of 1 for every unit provided.
- ◆ It follows that if the consumer purchases x units the total cost to the provider is: $c + x$.
- ◆ We assume the monetary benefit to the consumer from a service level of x is: $8x^{1/2}$.
- ◆ How should the provider contract with the consumer?

Charging a uniform price

- ◆ If the service provider charges per unit, the consumer would respond by purchasing a level of service a a function of price.
- ◆ Anticipating the consumer's demand, the provider constructs the consumer's demand curve, and sets price where marginal revenue equals marginal cost.
- ◆ The provider serves the consumer if and only if the revenue from providing the service at this price exceeds the total cost.

Solving for the optimal uniform price

- ◆ Supposing p is the price charged for a service unit, the consumer maximizes:

$$8x^{1/2} - px$$

- ◆ The first order condition yields the consumer demand

$$4x^{-1/2} = p = p(x) \quad \text{or} \quad x = 16p^{-2}$$

- ◆ The service provider maximizes:

$$p(x)x - c - x = 4x^{1/2} - c - x$$

with respect to x to obtain the interior solution

$$2x^{-1/2} = 1 \quad \text{or} \quad x = 4 \quad \text{and} \quad p = 2$$

- ◆ In this case the firm extracts a rent of $4 - c$ if $c < 4$.

Optimal contracting

- ◆ To derive the optimal contract, we proceed in two steps:
 1. derive the optimal level of service, by asking how much the consumer would use if she controlled the facility herself.
 2. calculate the equivalent monetary benefit of providing the optimal level of service to the consumer, and sell it to the consumer if this covers the total cost to the provider.
- ◆ The equivalent monetary benefit can be extracted two ways, as membership fee with rights to consume up to a maximal level, or in a two part pricing scheme, where the consumer pays for use at marginal cost, plus a joining fee.

A parameterization

- ◆ In our example we maximize

$$8x^{1/2} - c - x$$

with respect to x to obtain interior solution

$$4x^{-1/2} = 1 \text{ or } x = 16$$

- ◆ Hence the costs from an interior solution are $c + 16$, and the monetary equivalent from consuming the optimal level of service is 32.

- ◆ Therefore the provider can extract:

$$32 - (c + 16) = 16 - c \text{ if } c < 16.$$

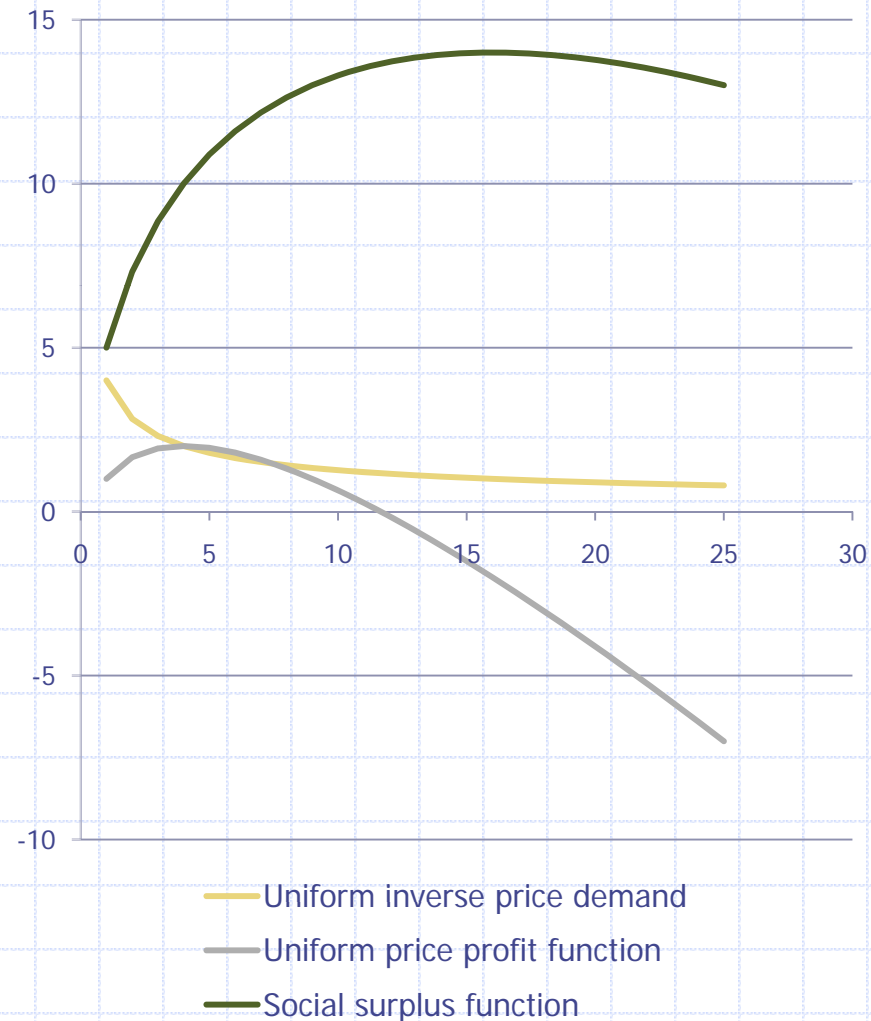
- ◆ A two part pricing scheme that achieves this goal is to charge a joining fee of 16 and a unit price of 1, achieving profits of $16 - c$.

Comparing multipart with uniform pricing schemes

- ◆ In a two part contract rents are $16 - c$ but with a uniform price the rent is only $4 - c$ if $c < 4$.
- ◆ Furthermore if $4 < c < 16$, a uniform price scheme cannot yield a profit but a two part price scheme can.
- ◆ Since lower levels of service are provided in the uniform price case, and since the consumer achieves a greater level of utility than in the two part contract, the provider charging a unit price realizes less rent than in the two part contract.

Profits as a function of units provided

Plotting the inverse demand, profits from charging a uniform price, and the surplus the service provides, for different values of x , we see that at high values x , the surplus is not very sensitive to changes in x , but the uniform profit function is negative.



Terms of employment

- ◆ The same principles apply to hiring a worker. For example let y denote the income the worker receives for her labor, in other words her wage earnings.
- ◆ Let h denote her hours of labor supplies to the firm if she is employed by the firm.
- ◆ Assume the worker's utility function takes the form

$$y + 9\log(16 - h)$$

where 16 is the maximum number of hours she would consider working, and $9/(16 - h)$ is the marginal rate of substitution between income and leisure time.

- ◆ We also assume that if she is not employed with the firm her income equivalent is v .

Firm value

- ◆ Suppose firm profits are:

$$3h - y$$

where 3 is the output price (or value of the worker's product in terms of hourly units).

- ◆ The firm chooses h and y to maximize profits subject to the participation constraint that the worker chooses to be employed:

$$y + 9\log(16 - h) \geq v$$

Optimization

- ◆ If the firm offered more than v , then it could always reduce y by so that hours remains unchanged.
- ◆ Therefore the participation constraint is met with equality and we set:

$$y = v - 9\log(16 - h)$$

- ◆ The firm maximizes:

$$3h + 9\log(16 - h) - v$$

- ◆ Dividing through by 3, and differentiating we obtain the first order condition:

$$1 = 3/(16 - h) \quad \text{or} \quad h = 13$$

Solution

- ◆ Substituting this equation for hours into the profit function we obtain:

$$39 + 9\log(3) - v$$

- ◆ This is the interior optimum, which only applies if and only if profits are strictly positive.
- ◆ Therefore the firm sets

$$h = 13$$

if profits are positive, meaning

$$39 + 9\log(3) > v$$

and otherwise $h = 0$.

Outsource

- ◆ A second type of work contract is for the worker to approach the firm, and propose an arrangement to the firm, which the firm can either accept or reject.
- ◆ This is quite close to outsourcing tasks that might have been undertaken within the firm.
- ◆ In this case the worker chooses both the payment y and hours or output h to maximize her utility

$$y + 9\log(16 - h)$$

subject to the constraint that the firm accepts her proposal (does not make losses): $y \leq 3h$

Solution to Outsourcing

- ◆ The solution is almost identical to the employment contract problem, except that all the rent accrues to the worker instead of the firm.
- ◆ The outsourcer sets a contract so that the firm only just breaks even, meaning $y = 3h$.
- ◆ Hours are now chosen by the outsourcer to maximize

$$3h + 9\log(16 - h)$$

yielding the same choice of hours as in the original problem.

Sales commission: the worker chooses her hours

- ◆ An alternative method of payment is for the firm to pay its employee a sales commission, denoted by s , on her output.

- ◆ In this case the worker chooses h to maximize

$$sh + 9\log(16 - h).$$

- ◆ Analogous to the previous problem, the solution to this maximization problem is the labor supply function $h(s)$:

$$h = 16 - 9/s$$

$$\text{if } 16s - 9 + 9\log(9) - 9\log(s) > v$$

and $h = 0$ otherwise.

Sales commission: the firm chooses the commission

- ◆ Upon solving for $h(s)$, the worker's supply of hours as function her commission, the firm chooses s to maximize:

$$\begin{aligned}(3 - s)h(s) &= (3 - s)(16 - 9/s) \\ &= 57 - 16s - 27/s\end{aligned}$$

- ◆ This solution to this maximization problem is found by solving the first order condition to the firm's optimization problem: $16 = 27/s^2$

Solving for s gives: $s = 3(3)^{1/2}/4.$

Comparing the schemes

- ◆ Total profit under the sales commission is:

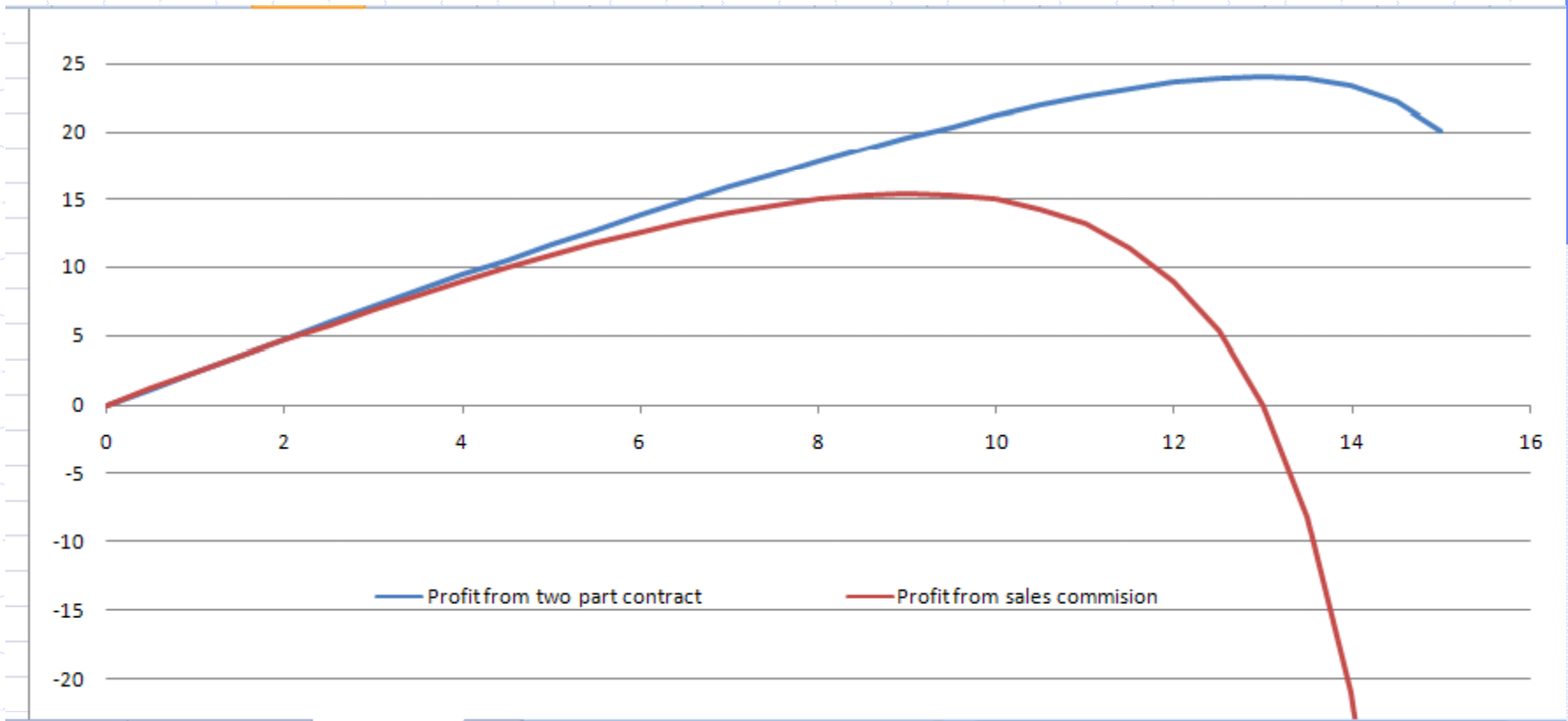
$$57 - 12 * 3^{1/2} - 36 * 3^{-1/2}$$

- ◆ Total profit under the optimal wage contract is:

$$39 + 9 \log(3) - v$$

- ◆ Since the share is less than the output price, the participation constraints imply there are values of v where participation occurs under the wage contract but not the sales commission.
- ◆ Give participation in both schemes, the worker is better off under the commission system than under the contract. Since the contract extracts all the gains from trade, it is more profitable than the commission.

Optimization



Complications and complexity

- ◆ Complicated or complex contracts are likely to bring the bargaining parties under the umbrella of the firm.
- ◆ As the complexity of the optimal contract increases, so does the propensity for error, leading to responders:
 1. rejecting the contract
 2. accepting the contract but taking an action the manager did not intend
 3. accepting the contract, taking the intended action, but leaving the manager without any rent.

Size and scope of firms

- ◆ Simpler contracts (such as uniform prices or letting another player make an offer) yield less rent than optimal contracts, but are easier to implement without unintended consequences.
- ◆ This leads us to consider how strategic interactions help shape the firm's boundaries.
- ◆ High value activities within the value chain demand the strategist's attention as the firm seeks to capture the rents. Here the firm is proactive in approaching its bargaining partners and making contract offers.
- ◆ Conversely opportunities that offer little rent are best dealt with at arm's length, for example within the market place.

Summary

- ◆ A player should use the rollback principle to solve perfect information games, if she is confident that the other players in the game are capable of doing the same.
- ◆ The rollback principle is sometimes useful in predicting the outcome of bargaining and setting contracts.
- ◆ Optimal contracting, or setting those rules provides an opportunity for the contractor to extract rents from his business partners, employees, customers and clients.
- ◆ Extracting maximal rent may require complicated contracts, which if written incorrectly, carry the prospect of loss. These factors form the basis for defining where firm boundaries should be relative to the market.