

# Lecture 3

## Revenue Equivalence

The previous lecture introduced several auction formats, emphasized the difference between **private** and **common** values, and defined the concept of **strategic equivalence**. This lecture analyzes a weaker relation, called **revenue equivalence**. It is based on the notion of equal expected net benefit to each bidder. Revenue equivalence only applies to private value auctions. Our discussions of pure auctions conclude with an example of what happens when there is **differential information** in a common value setting.

# Ascending and second price auctions

- ◆ To illustrate concepts introduced in the previous lecture we experimented with Dutch (descending) and first price auctions.
- ◆ We begin this lecture with an experiment involving English (ascending) and second price auctions.
- ◆ Recall that:
  - in an English auction, the auctioneer continues to raise the price until only one bidder is willing to pay it. (See some rules.)
  - in a second price (sealed bid) auction the highest bidder pays the second highest bid.

# A procurement auction

- ◆ In a procurement auction, the auctioneer seeks to buy some goods and services.
- ◆ Thus in a second price procurement auction bidders submit prices, the lowest price wins, and the winning bidder pays the second lowest price.
- ◆ In an English procurement auction, bidders have repeated opportunities to lower the price until only one remains, the winner, who pays the lowest price s/he bid.

# A second price procurement auction bidding window and instruction window

1. Instruction and auction window appears on your screen.
2. To close the instruction window click on "x". To retrieve it click on "Description".

The image displays two screenshots of the ComLabGames client interface. The left screenshot shows the 'Description' window, which is titled 'ComLabGames - Client' and has a 'Description' button circled in red. The window content includes the title 'Second Price Sealed Bid Auction', the number of invited bidders (4), and the cost of developing software (229.12). It also features a bid input field highlighted in yellow and a 'Continue' button. The right screenshot shows the 'Instructions' window, also titled 'ComLabGames - Client', with an 'x' button circled in green. The window content includes the title 'Second Price Sealed Bid and English auction', the word 'Instructions', and detailed instructions for the auction process, including the profit formula:  $\text{PROFIT} = (\text{THE SECOND LOWEST BID}) - (\text{ACTUAL COST OF VPN development})$ .

# Write a bid in a second price auction

1. Write a number and click Enter (you can enter decimal number).
2. After submitting the bid, your bid should be colored in red.
3. Wait for all players in your session to submit bids.

ComLabGames - Client

Description Username: Francesca Id: 18 Identity: 1

**Second Price Sealed Bid Auction**

Number of invited bidders: 4

Your cost of developing software is: 229.12 . It is drawn from the uniform distribution with minimum \$200 and maximum \$400.

Place your bid:  and click Enter.

The bidder with the lowest bid is awarded a project and pays the **SECOND lowest** bid.

If you win the auction, your profit is:

**PROFIT = (THE SECOND LOWEST BID) - (ACTUAL COST OF VPN development)**

Stage time limit: unlimited Round: 1 Continue

Player type: software supplier (1) Please make a move now!

Waiting 4 of 4 subject(s) to proceed the session!

ComLabGames - Client

Description Username: Francesca Id: 18 Identity: 1

**Second Price Sealed Bid Auction**

Number of invited bidders: 4

Your cost of developing software is: 229.12 . It is drawn from the uniform distribution with minimum \$200 and maximum \$400.

Place your bid: 220.17 and click Enter.

The bidder with the lowest bid is awarded a project and pays the **SECOND lowest** bid.

If you win the auction, your profit is:

**PROFIT = (THE SECOND LOWEST BID) - (ACTUAL COST OF VPN development)**

Stage time limit: unlimited Round: 1 Continue

Player type: software supplier (1) Please wait for other players to make their choices!

Waiting 3 of 4 subject(s) to proceed the session!

# Summary page for sealed bid auction

1. Click "Continue" to move to English auction instructions.
2. Wait for all players in your session to click "Continue".

ComLabGames - Client

Description Username: Francesca Id: 18 Identity: 1

## Brief summary of the second price sealed bid auction

Your cost of developing software was **229.12**, and your bid in this auction was **220.17**. Please click "Continue" and proceed to the English auction.

Summary results will be given at the end of the experiment.

Stage time limit: unlimited Round: 1 **Continue**

**Player type: software supplier (1) Please make a move now!**

**Waiting 4 of 4 subject(s) to proceed the session!**

# English auction instructions

ComLabGames - Client

Description Username: Francesca Id: 10 Identity: 1

## English Auction

### Instructions

You are about to participate in CISCO's online bidding auction for supplying VPN software. There are 4 suppliers/bidders who are participating in this event. Each of you has a different cost of developing VPN software. Each of you knows your own costs but does not know the costs of the other bidders. However, you know that the costs are drawn from uniform distribution with minimum \$200 and maximum \$400. Operationally, a computer will generate a random number between \$200 and \$400, so that any number in this range is equally likely.

You will be competing against the other 3 bidders by successively lowering the price at which you are willing to develop the software. The price will be lowered automatically by 10 by the computer each time one of the bidders click on the yellow bid within 5 seconds. The bidding will stop when nobody is willing to submit a bid any further in the allocated time of 5 seconds, and CISCO will award the project to the supplier who was the last to submit the bid.

If you click on your yellow bid that looks like this 380 within 5 seconds it means that you are willing to provide the software for that price. If you click on 380 the computer automatically lowers the price by 10 dollars. The current price will show the lowest bid submitted so far in the auction. If nobody submits a bid (clicks on the bid) within the allocated 5 seconds the

Stage time limit: 46 Round: 1

Continue

Player type: software supplier (1) Please make a move now!

Waiting 4 of 4 subject(s) to proceed the session!

# English auction page

1. The price will be lowered automatically by \$10 each time one of the bidders click on the **yellow bid** within 5 seconds.
2. The bidding will stop when nobody is willing to submit a bid any further in the allocated time of 5 seconds.
3. If you click on your **yellow bid** within 5 seconds it means that you are willing to provide the software for that price.

ComLabGames - Client

Description Username: Francesca Id: 18 Identity: 1

Number of bidders participating: 4

English Auction

Your cost of developing software is 229.12 . It is drawn from the uniform distribution with minimum \$200 and maximum \$400. You have 50 seconds to click on the yellow offer below. If nobody submits the bid within this period the auction is closed. The project is awarded to the last bidder who submitted the bid and it is shown in current price.

Current price: 400

Click on your bid 390 to submit it.

Stage time limit: 4 Round: 1 Continue

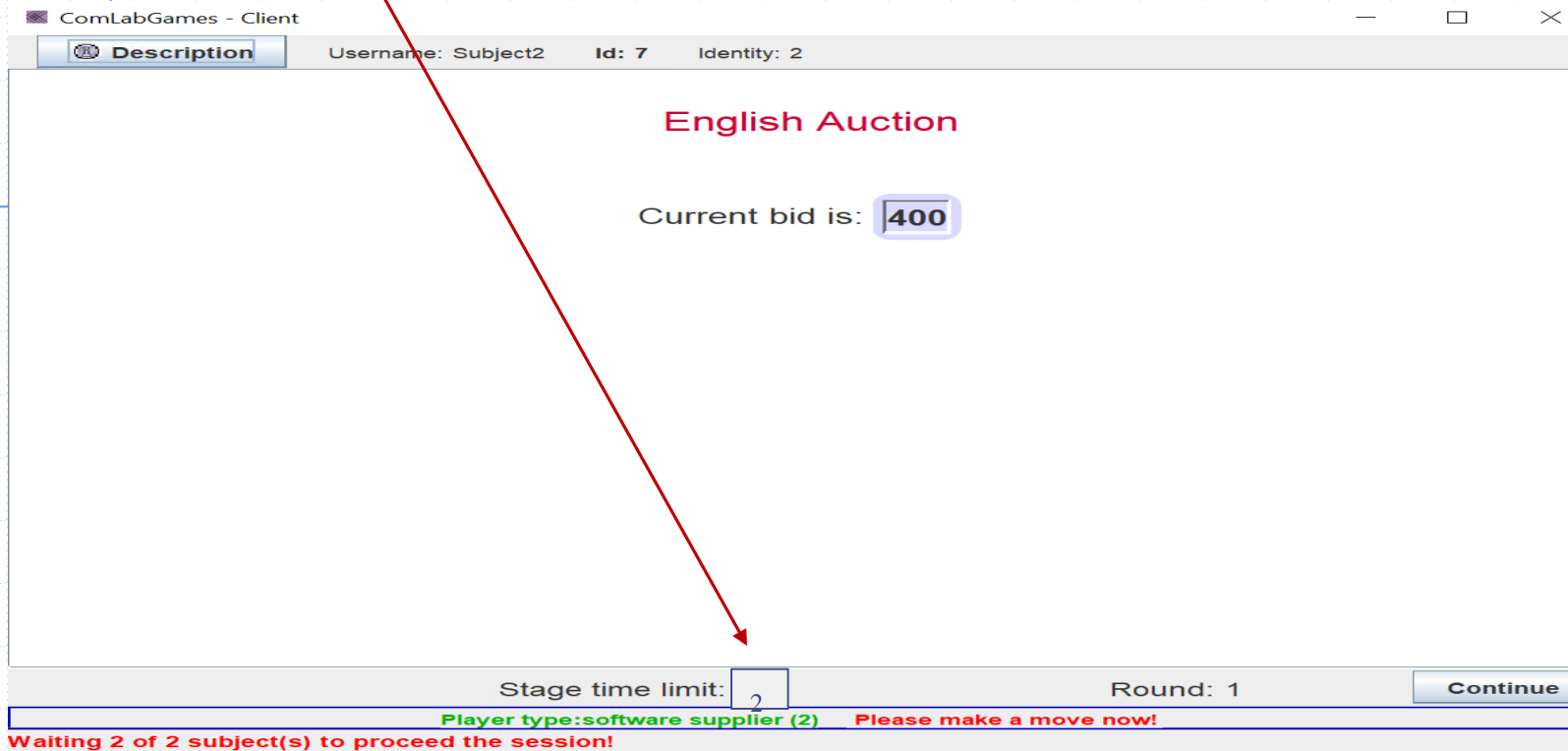
Player type: software supplier (1) Please make a move now!

Waiting 1 of 1 subject(s) to proceed the session!



# Current bid page for English auction

1. For 2 seconds a window with the current bid is shown before the next round of bidding.



# Summary page

ComLabGames - Client

— □ ×

Description

Username: Francesca

Id: 18

Identity: 1

## Summary of results for the Second Price Sealed Bid Auction

Your cost for developing the software was **229.12**, your bid was **220.17** the price was **220.17**, and your profit is **0**.

## Summary Results for the English Auction

Your cost for developing the software was **229.12**, your last bid was **320** the price was **320** and your profit was **90.88**. Below is the summary data:

Player name	Costs	Bid - secon...	Price - sec...	Profit - sec...	Your pricec...	Final price ...	Profit -Engl...
Francesca	229.12	220.17	220.17	0	320	320	90.88
Jason	377.27	300	0	0	0	320	0
Ann	221.1	211.17	0	-0.93	350	320	0
Paul	383.37	322	0	0	390	320	0

Stage time limit: unlimited

Round: 1

Continue

Player type: software supplier (1)

Game is over!

# Strategic equivalence between second-price and ascending auctions

- ◆ In private value second-price and in ascending auctions:
  - ❑ nobody learns anything about their own valuation during the auction.
  - ❑ the winner learns nothing about the highest losing bid until the auction ends.
  - ❑ the highest bidder pays (roughly) the second highest price.
- ◆ These features also hold when there are only 2 bidders, regardless of whether the valuations are private or not.
- ◆ In both cases an ascending auction is strategically equivalent to a second price sealed bid auction.
- ◆ In common value auctions these two auction mechanisms **are not strategically equivalent** if there are more than 2 players.

*Rule 2: If there are only two bidders, and/or valuations are private, choose the same reservation price in English and second price auctions.*

# Bidding in a second-price auction

- ◆ If you know your own valuation, there is a general result about how to bid in a second price sealed bid auction, or where to stop bidding in an ascending auction.
- ◆ Bidding should not depend on what you know about the valuations of the other players, nor on what they know about their own valuations.
- ◆ It is a dominant strategy to bid your own valuation.
- ◆ A corollary of this result is that if every bidder knows his own valuation, then the object will be sold for the second highest valuation.

---

*Rule 3 : In a second price sealed bid auction, bid your valuation if you know it.*

# Proving the third rule

- ◆ Suppose you bid above your valuation, win the auction, and the second highest bid also exceeds your valuation. In this case you make a loss. If you had bid your valuation then you would not have won the auction in this case. In every other case your winnings would have been identical. Therefore bidding your valuation dominates bidding above it.
- ◆ Suppose you bid below your valuation, and the winning bidder places a bid between your bid and your valuation. If you had bid your valuation, you would have won the auction and profited. In every other case your winnings would have been identical. Therefore bidding your valuation dominates bidding below it.
- ◆ The proof is completed by combining the two parts.

SHOULD YOU BID  
Value or  $B_H$ ?

Bidding in an English or a  
Second Price Sealed Bid Auction

\$ price



Bid  $B_H$ , Value or  $B_L$  and  
lose auction if highest bid is  $B_1$   
and make zero.

Bid Value, lose auction to  
 $B_{first}$ , and make zero, or  
bid  $B_H$ , win auction and  
lose  $(B_{first} - Value)$ .

Bid Value, win auction,  
and make  $(Value - B_{second})$ ,  
or bid  $B_L$  and lose  
auction.

Bid  $B_H$ , Value or  $B_L$   
and win auction to  
make  $(Value - B_0)$  if next  
highest bid is  $B_0$ .

SHOULD YOU BID  
Value or  $B_L$ ?

# Revenue versus Strategic Equivalence

- ◆ In strategically equivalent auctions, the strategic form solution strategies of the bidders, and the payoffs to all them, are identical. Are bidders ever indifferent to auctions that lack strategic equivalence?
- ◆ Two auction mechanisms are revenue equivalent if, given a set of players their valuations, and their information sets, the expected surplus to each bidder and the expected revenue to the auctioneer is the same.
- ◆ Revenue equivalence is a less stringent condition than strategic equivalence. Thus two strategic equivalent auctions are invariably revenue equivalent, but not all revenue equivalent auctions are strategic equivalent.

# Revenue equivalence theorem

- ◆ The **revenue equivalence theorem** (Session 3.1) states that in private value auctions, the expected surplus to each bidder does not depend on the auction mechanism itself providing the following conditions are satisfied:
  1. Every bidder is risk-neutral.
  2. Valuations are independent and identically distributed.
  3. In equilibrium the bidder with highest valuation wins.
  4. The lowest possible valuation has zero expected value.
- ◆ Note that if all bidders obtain the same expected surplus, the auctioneer obtains the same expected revenue too.



# Intuition from revenue equivalence

- ◆ Calibrate your bid to your valuation only to the extent that it affects your beliefs about the highest valuation of the all the other bids.
- ◆ Working from the assumption that yours is the highest valuation, bid high enough to induce the next highest bidder to make a small expected loss in order to beat your bid.
- ◆ The theorem implies that the auctioneer's expected revenue is also the same (Session 3.2).

# Write a bid in First price sealed bid auction

1. Write a number and click Enter.
2. Your bid should be colored in red
3. Wait for other players in your session to submit a bid.

ComLabGames - Client

Description Username: Ari Id: 2 Identity: 1

First Price Sealed Bid Auction

Number of of invited bidders: 4

Your cost of developing software is : 367.2 . It is drawn from the uniform distribution with minimum \$200 and maximum \$400.

Place your bid:  and click Enter.

The bidder with the lowest bid is awarded a project.

If you win the auction, your profit is:  
 $\text{PROFIT} = (\text{THE LOWEST BID}) - (\text{ACTUAL COST OF VPN development})$

If there is a tie, computer will randomly select one of the lowest bids.

If the difference is negative, it represents a loss. If you loose the auciton, you will earn 0.

Stage time limit: unlimited Round: 1 Continue

Player type:bidder (1) Please make a move now!

Waiting 4 of 4 subject(s) to proceed the session!

ComLabGames - Client

Description Username: Ari Id: 2 Identity: 1

First Price Sealed Bid Auction

Number of of invited bidders: 4

Your cost of developing software is : 367.2 . It is drawn from the uniform distribution with minimum \$200 and maximum \$400.

Place your bid: 399 and click Enter.

The bidder with the lowest bid is awarded a project.

If you win the auction, your profit is:  
 $\text{PROFIT} = (\text{THE LOWEST BID}) - (\text{ACTUAL COST OF VPN development})$

If there is a tie, computer will randomly select one of the lowest bids.

If the difference is negative, it represents a loss. If you loose the auciton, you will earn 0.

Stage time limit: unlimited Round: 1 Continue

Player type:bidder (1) Please wait for other players to make their choices!

Waiting 3 of 4 subject(s) to proceed the session!

# Summary page for sealed bid auction

ComLabGames - Client



Description

Username: Ari Id: 2 Identity: 1

## Summary of results

Your costs were **367.2**, your bid was: **399**. The the lowest bid was **286** and your profit is **0**. Below are the list of all the bids.

Player ID	Costs	Bids	Lowest Bid	Profit
Ari	367.2	399	0	0
Simona	256.37	286	286	29.63
Leila	395.44	390	0	0
Marco	387.48	400	0	0

Stage time limit: unlimited

Round: 1

Continue

Player type:bidder (1)

Game is over!

# Comparison of bidding strategies

- ◆ The bidding strategies in the first and second price auctions markedly differ.
- ◆ In a second price auction bidders should submit their valuation regardless of the number of players bidding on the object.
- ◆ In the first price auction bidders should shave their valuations, by an amount depending on the number of bidders.

# Using the revenue equivalence theorem to derive optimal bidding functions

- ◆ We can also derive the solution bidding strategies for auctions that are revenue equivalent to the second price sealed bid auction.
- ◆ Consider, for example a first price sealed bid auctions with independent and identically distributed valuations (Session 3.3).
- ◆ The revenue equivalence theorem implies that each bidder will bid the expected value of the next highest bidder conditional upon his valuation being the highest.

# An example: the uniform distribution

- ◆ Suppose valuations are uniformly distributed within a closed interval, with probability distribution:

$$P(v) = (v - v_0) / (\bar{v} - v_0)$$

- ◆ Then in equilibrium, a player with valuation  $v$  bids a weighted average of the lowest possible valuation and his own, where the weights are  $1/N$  and  $(N-1)/N$ :

$$b(v_n) = v_n - P(v_n)^{1-N} \int_{v_0}^{v_n} P(v)^{N-1} dv$$

$$= v_n - (v_n - v_0)^{1-N} \int_{v_0}^{v_n} (v - v_0)^{N-1} dv$$

$$= v_0 / N + v_n (N - 1) / N$$

# Differential information in common value auctions

- ◆ In common value auctions the second lecture highlighted, everyone was **equally informed**. Each bidder knew their own signal was as good as every other.
- ◆ What happens if bidders are **differentially informed** about a common value?
- ◆ For example, one bidder might know more about the value of the object being auctioned than the others.
- ◆ An extreme form of dependent signals occurs when one bidder **knows the common value exactly** and the others do not:
  - How should the informed player bid?
  - What about an uninformed player?

# Second price sealed bid auctions

- ◆ In a SPSB auction, Rule 3 implies the informed player optimally bids his true value.
- ◆ The uninformed player bids any pure or mixed distribution:
  - If she wins the auction she pays the common value.
  - If she loses she pays nothing.
  - She neither gains or loses on any bid.
- ◆ This implies the revenue from the auction is indeterminate, that is anywhere between zero the common value.
- ◆ For example, if the uninformed player bids:
  - zero, the auction nets zero.
  - more than the item could ever be worth, the auctioneer nets the common value.
- ◆ What about a FBSB auction?



# Differential Information First price auction

- 1. Bidding screen for the informed bidder, *OpenChannel management team*: the actual value of business shown.
- 2. Bidding screen for the *uninformed bidder, Stripe*: the business value estimate shown.

ComLabGames - Client

Description Username: Francesca Id: 29 Identity: 1

First price sealed bid common value auction

Number of bidders participating: 2

The actual value of OpenChannel business is **133.39**. You are informed about this value. The other bidder participating in the auction does not have this information. She is given just an estimate of the actual value of OpenChannel business. For your information, this estimate of actual business value is randomly drawn from the interval (Actual business value - \$30) to (Actual business value + \$30).

Place your bid  and click return.

Remember the lowest bid wins the auction!

PROFIT = (ACTUAL BUSINESS VALUE (V)) - (THE HIGHEST BID)

If the difference is negative, it represents a loss. If you loose the auction, you will receive zero.

Stage time limit: unlimited Round: 1 Continue

Player type: Informed - OpenChannel management (1) Please make a move now!

ComLabGames - Client

Description Username: AI Id: 29 Identity: 2

First price sealed bid common value auction

Number of bidders participating: 2

Your estimate of the actual value of OpenChannel business is **148.22**. This estimate of the value of the business is randomly drawn from the interval (Actual business value value - \$30) to (Actual business value + \$30). Each estimate in this interval is equally likely. Actual business value (V) will always be between \$100 and \$300 inclusively, and each value in this range is equally likely. The other bidder participating in the auction *does have* the information about the actual business value (V) and you do not have this information.

Place your bid  and click return.

Remember the lowest bid wins the auction!

PROFIT = (ACTUAL BUSINESS VALUE (V)) - (THE HIGHEST BID)

If the difference is negative, it represents a loss. If you loose the auction, you will receive zero.

Stage time limit: unlimited Round: 1 Continue

Player type: Uninformed - Stripe (1) Please make a move now!

# Summary page for differential auction

## Informed bidder

ComLabGames - Client

Description Username: Francesca Id: 28 Identity: 1

### Summary

Your bid for acquiring OpenChannel business was **100**, the actual value of the company was **133.39**, and the winning price was **110**. Your profit is: **0**.

Data for individual session:

Player's use...	Player type	S	V	Bids	Winning bid	Profit
Francesca	Informed - ...	0	133.39	100	110	0
AI	Uninformed...	148.22	133.39	110	110	23.39

Stage time limit: unlimited Round: 1 [Continue](#)

Player type: Informed - OpenChannel management (1)

Game is over!

## Uninformed bidder

ComLabGames - Client

Description Username: AI Id: 29 Identity: 2

### Summary

Your estimate of OpenChannel business's value was **148.22**. Your bid was **110**, the actual company value was **133.39**. The winning price was **110** and profit is: **23.39**.

Data for individual session:

Player's u...	Player type	S	V	Bids	Winning bid	Profit
Francesca	Informed - ...	0	133.39	100	110	0
AI	Uninform...	148.22	133.39	110	110	23.39

Stage time limit: unlimited Round: 1 [Continue](#)

Player type: Uninformed - Stripe (1) Please make a move now!

Game is over!

# Uninformed bidders should not be predictable

- ◆ Suppose the uninformed bidder always makes the same positive bid, denoted  $b_{\text{fixed}}$ . This is an example of a **pure strategy**.
- ◆ Is this pure strategy part of a Nash equilibrium?
- ◆ The **best response of the informed bidder** is to bid a little more than  $b_{\text{fixed}}$  when the value of the object  $v$  is worth more than  $b_{\text{fixed}}$ , and less than  $b_{\text{fixed}}$  otherwise.
- ◆ Therefore the uninformed bidder **loses** or makes **zero** by playing a pure strategy in this auction. A better strategy would be to bid zero (Session 3.4).

# Principles for Bidding in Auctions

*Summarizing some general principles for bidding:*

1. Assume your own valuation is the high enough to win. Bid high enough to pay the break-even value of the highest losing bid.
2. Shave your bid to account for the winner's curse if you don't know your own valuation but only have a signal that is correlated with those of the other bidders.
3. Be wary of bidding with less knowledge than your rivals.
4. The greater the number of bidders the less opportunities there are for profit.
5. In a second price auction bid your valuation, and in an ascending auction, up to your valuation, if you know it.
6. Bid the same way in a first price sealed bid auction as a descending auction.