

Lecture 5

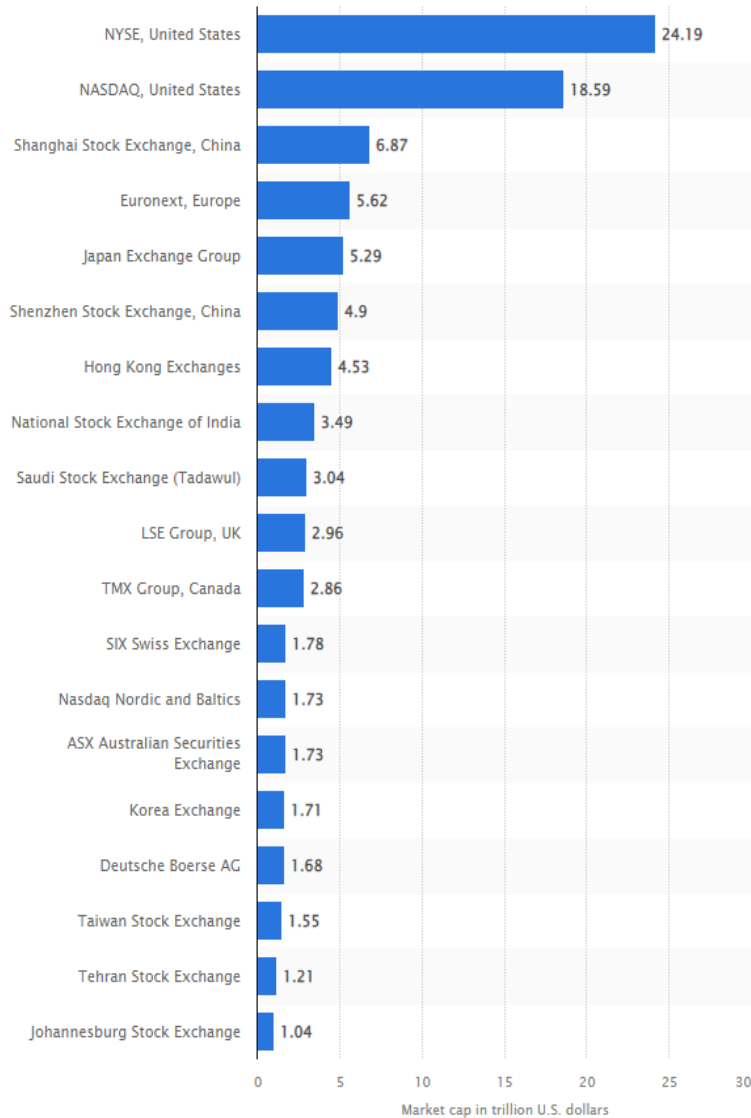
Limit Order Markets

A limit order market (LOM) is a real world institution for characterizing many of the trading platforms in the financial sector. It is also a paradigm for describing almost all trading mechanisms. This lecture defines how a LOM works, and gives you some experience trading on one.

Financial Markets

- ◆ **Electronic limit order markets** are amongst the fastest growing markets within the financial and retail sectors.
- ◆ Whether market makers set the spread (NASDAQ), specialists oversee transactions between investors (as the NYSE used to), or the market admits anyone in good standing to submit buy and sell orders (EBAY), these exchanges have a common structure.

Electronic LOMs are the biggest markets



◆ The NYSE and NASDAQ are the two largest stock exchanges.

◆ NYSE (\$24 trillion cap)

- ❑ founded 1792.
- ❑ was previously run by specialists.
- ❑ 2,600 listings.
- ❑ went fully electronic on 23 March 2020.

◆ NASDAQ (\$19 trillion cap)

- ❑ founded 1971.
- ❑ world's first electronic stock exchange.
- ❑ 3,700 listings.
- ❑ highest trading volume in the world.

◆ Establishing NASDAQ:

- ❑ probably reduced the bid-ask spread.
- ❑ was unpopular with brokers.

Trading in a generic limit order market

- ◆ Traders submit a **market order** or a **limit order**.
- ◆ Each order is for a given quantity, positive (negative) quantities standing for units demanded to buy (for sale).
- ◆ Limit orders specify a **transaction price**, market orders a **reservation price**.
- ◆ Market transactions match market orders with limit orders, and take place at the limit order price(s).
- ◆ Thus market orders are executed instantaneously, but a limit order might never be executed.

Price-time precedence

- ◆ **Market buy orders** fill (are matched against) the lowest price limit order(s) to sell, the **ask**.
- ◆ **Market sell orders** fill the highest price limit order(s) to buy, called the **bid**.
- ◆ If two limit orders to buy are submitted at the same price, the order submitted first is filled by a market sell order before the more recently submitted buy order.
- ◆ Similarly lower priced limit sell orders have a higher priority than higher priced limit sell orders, and if two bidders seeking to sell a unit at the same price the person who bid first will be filled before his rival seller.

Trading window

Sell	Price: 5800.00	Quantity: 9	Duration: 60000	
	Price(Markup)	Quantity (Cum.)	Revenue\Cost (Cum.)	Duration
Buy	6000.00 (-291.24)	4 (4)	24000.00 (24000.00)	59603
	5800.00 (-491.24)	9 (13)	52200.00 (76200.00)	59634
Delete	3800.00 (2491.24)	2 (2)	7600.00 (7600.00)	59301
Center	200.00 (6091.24)	4 (6)	800.00 (8400.00)	59197

- ◆ The difference between the highest priced limit buy order price (the **bid**), and the lowest priced limit sell order (the **ask**) is called the **spread**, here 2,000.
- ◆ The trader just placed a sell order for 9 units at price 5,800, with an expiry time of 60,000 seconds, reducing the spread from 2,200 by placing an order inside the previous bid ask quotes.

Many auctions are examples of simple limit order markets

- ◆ In a **first price** sealed bid auction, bidders place limit buy orders without seeing the book, and the auctioneer fills one of the limit orders with a market sell.
- ◆ In an **English** auction bidders place limit orders and the auctioneer fills one order with a market sell.
- ◆ In a **Dutch** auction, the auctioneer places limit sell orders until a bidder fills one of his orders with a market buy.
- ◆ Since an auction is the simplest form of a limit order market, studying behavior in auctions is also a useful way to learn how investors trade in limit order markets.

Not all auctions are limit order markets

- ◆ In a **second priced** sealed bid auction, players simultaneously submit their bids, the highest bidder wins the auction, and pays the second highest bid.
- ◆ **Ebay** is not technically a second price sealed bid auction, but in practice most Ebay bids are sold to sniping bidders, who bid at the close out time so that no one can respond.
- ◆ Note also that the “Buy it now” part of Ebay is just a limit sell order, in this case the ask price until the bidding closes.

Auctions versus Limit Order Markets

◆ There are some obvious similarities and differences:

- Auctions are **one sided** markets.
- Limit order have important **demand and supply** side considerations.
- Both are used in trading **financial securities**.

◆ Auctions are also used for:

- procurement.
- timber, real estate and used car fleets.
- perishable items. (For example, *Feeding America*, a large national charity for distributing food to local banks and soup kitchens).

◆ Limit Order Markets are/could also be used for:

- distributing (new or used) car inventory amongst dealers, or rental trucks across pickup outlets.
- allocating gates at airports (easier than in a mall).
- mining and drilling rights.

Introducing LOMs to Commodity Markets

- ◆ LOMs and electronic trading have revolutionized financial markets and retail finance by connecting investors in a more tangible way to facilitate transactions.
- ◆ There is scope for replacing cumbersome and costly administrative procedures and other bureaucratic mechanisms for allocating resources with LOMs.
- ◆ The key here is to define the object being traded in precise transparent way . . . as a **commodity**.
- ◆ By definition units of such **goods and services** are more or less **interchangeable**, and therefore amenable to being listed on the **same market**.

Trading Landing Gates

- ◆ For example, imagine different airlines trading landing gates within an airport as they compete for routes.
- ◆ Landing gates are a candidate for an internal LOM because there is little to differentiate a gate aside from:
 - ❑ the size of the aircraft it takes.
 - ❑ its location relative to the airport exits and baggage claim areas.
 - ❑ its location relative to the gates owned by the same airline.
- ◆ Presumably the LOM would be supplemented by a user fee paid to the airport authority, reflecting the cost of processing travelers, to account for the frequency with which any given landing gate is used.
- ◆ Introducing LOMs for landing gates would also streamline the competition for routes, that could adjust more quickly to air travel demand and supply considerations.

Landing Gates

- ◆ There are 6 airline companies. They can all buy and sell their landing gate rights.
- ◆ Each airline values owning **up to two gates** at some positive amount, but place **no value at all** on owning any more than two gates.
- ◆ An airline's valuation of its first two landing gates is drawn from a **uniform distribution** with minimum 10 and maximum 20.
- ◆ Every airline knows its own valuation, but not the valuations of its rivals.

Initial Endowment

- ◆ An airline is initially endowed with 0, 1 or 2 landing gates, with probability $1/3$ attached to each outcome.
- ◆ Every airline has an initial cash endowment of \$100 (also measured in hundreds of thousands of dollars).
- ◆ Your **initial wealth** is the sum of initial cash and valuation times number of landing gates assigned at the beginning of the game.
- ◆ The **rate of return** is calculated as a current wealth minus initial wealth divided by initial wealth, and reported as a percentage.

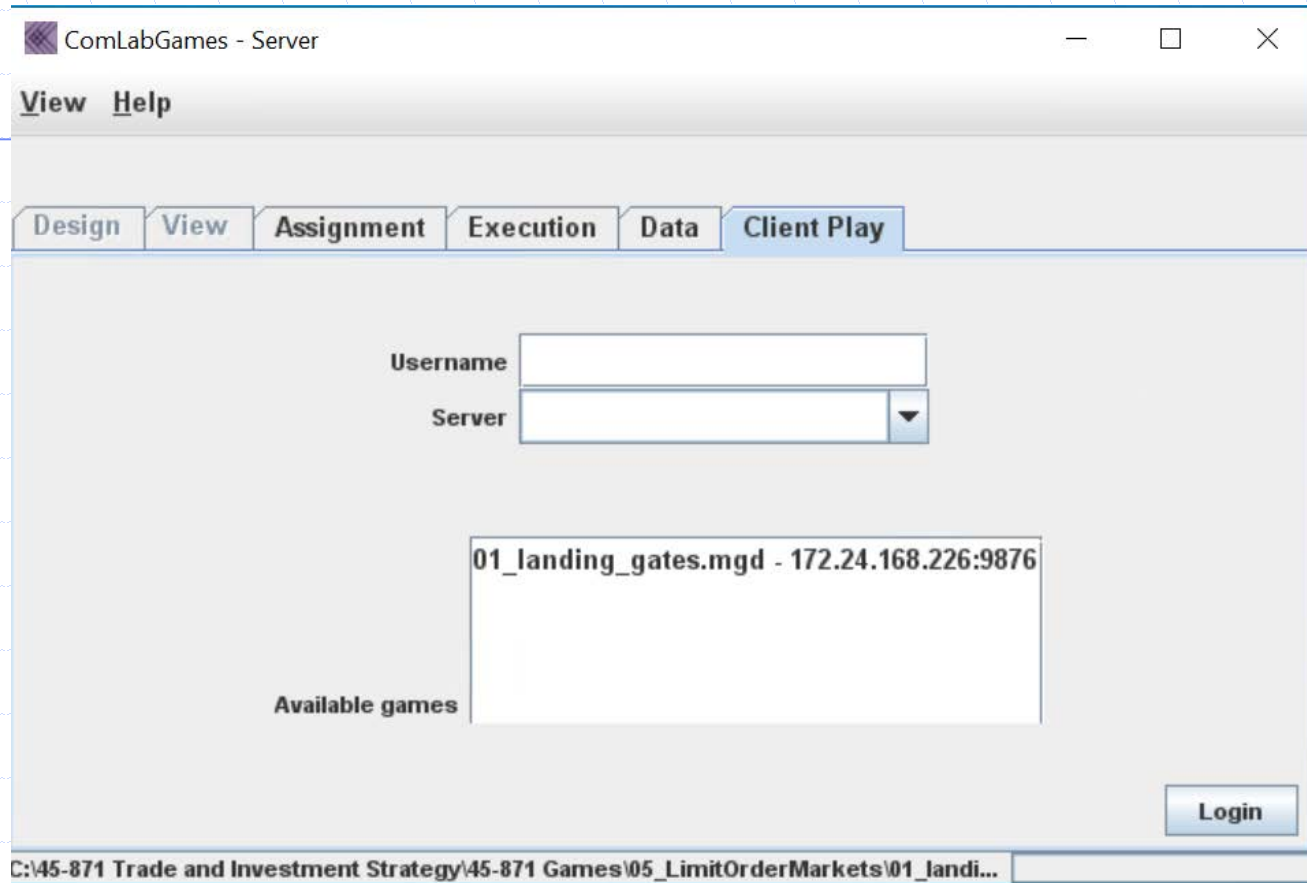
The trading mechanism

- ◆ You can:
 - ❑ place **limit** orders or **market** orders to sell or buy.
 - ❑ **cancel a limit order** any time unless another trader fills it beforehand (by placing a market order).
 - ❑ see the book of limit orders (all buy and sell limit orders) on your screen.

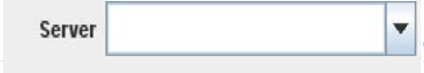
- ◆ You cannot:
 - ❑ **short sell**. The number of limit orders to sell is bounded by the number of units you own.
 - ❑ **borrow**. The number of limit orders to buy is bounded by your cash endowment.

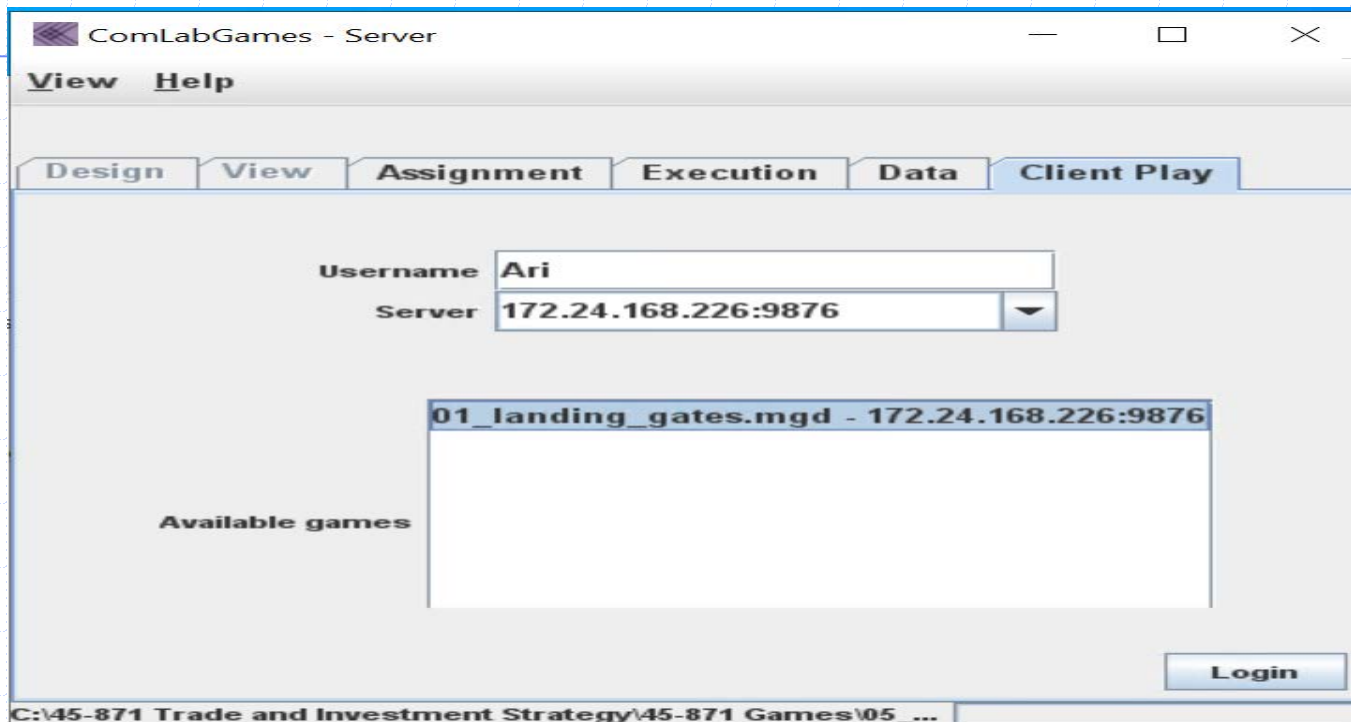
Login Instructions

1. Click on Client Play
2. Click on Available games: "01_landing_gates.mgd"



Continuing Login Instructions

1. Server address: 172.24.168.226 will appear automatically after selecting "01_landing gates". If not type 172.24.168.226 in .
2. Write a username.
3. Click on "Login".



Trading window before all subjects login

1. In each session six subjects must login before any of them see the instructions and a trading window.
2. Information about the number of subjects waiting to start the game is shown at the bottom left corner.

ComLabGames

Username: Ari Id: 18 Identity:

History

Name

View more history

Limit order book

Price	Quantity	Cumulative	Player id	Player type
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Cash Duration: 600 Price: 0 Quantity: 1 Sell Buy

Summary data

Asset name	Valuation	Endowment	Sell orders	Buy orders	Bid (Quantity)	Ask (Quantity)	Trading price (Quantity)
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Subject's own limit orders

Price	Quantity	Revenue/Cost	Duration
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Transaction history

Price	Quantity	Buyer ID	Buyer T...	Seller ID	Seller T...	Time
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Waiting 5 of 6 subjects to start a session!

Round:

Trading window and Instruction window

1. Instruction and trading window appears on your screen.
2. To close the instruction window click on "x". To retrieve it click on "Description".

ComLabGames - Client
Username: Jamie Id: 20 Identity: 3

Gate valuation

Name	20:46:32 (3...)	20:46:31 (3...)
Gate valuation	13.91	13.91
Number of gates	1	0
Price	0	0
Cash	100	100
Initial wealth	113.91	0
Current wealth	113.91	0
Rate of return (...)	0	0

Limit order book

Price	Quantity	Cumulative	Player id	Player type
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Subject's own limit orders

Asset name	Valuation	Endowment
Gate valuation	13.91	1

Transaction history

Price	Quantity	Buyer ID	Buyer Ty...	Seller ID	Seller Ty...	Time
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Player type: Airline (3) Round: 1

History shows all the updates of variables.

The last update is shown in the first left column.

Price represents a last trading price in the market.

Trading Landing Gates

There are 6 airline companies. They can all buy and sell their landing gate rights. An airline's valuation of a landing gate is drawn from a uniform distribution with minimum 10 and maximum 20. Every airline knows its own valuation, but not the valuations of its rivals. Your valuation for owning a landing gate is 13.91 measured in hundreds of thousands of dollars. You value the two gates at 13.91 each, but place no value on owning any more than two.

An airline is initially endowed with 0, 1 or 2 landing gates, with probability 1/3 attached to each outcome. Your endowment of landing gates is 1. Every airline has an initial cash endowment of \$ 100 (also measured in hundreds of thousands of dollars).

You will be able to place limit orders or market orders (sell or buy). Your initial wealth is the sum of initial cash and valuation times number of landing gates assigned at the beginning of the game. Your current wealth is the sum of current cash and valuation times number of landing gates up to two gates. (There is no additional intrinsic value from owning more than two landing gates.) The rate of return is calculated as a current wealth minus initial wealth divided by initial wealth, and reported as a percentage.

To trade type a price and quantity you want to sell/buy, and click on sell/buy button. The order shows in the order book:

Limit order book

Price	Quantity	Cumulative	Player id	Player type
16.5	1	0	82	Airline (2)
15.1	1	1	81	Airline (1)
13.7	1	0	84	Airline
12.4	2	1	83	Airline

The outstanding sell orders are in red and outstanding buy orders are in green. To buy/sell you can directly select the best order by clicking on the price of the order in the order book and click on buy/sell.

Landing Gate

Limit order book

Price	Quantity	Cumulative	Player id	Player type
18.5	1	0	87	Airline (2)
15.1	1	1	81	Airline (1)
13.7	1	0	84	Airline
12.4	2	1	83	Airline

Price: 13.7 Quantity: 1 Sell Buy

Alternatively you can enter a price and quantity and select buy/sell.

The history shows the valuation of the landing gate, the transaction prices, current cash, initial wealth, current wealth and rate of return.

History

Name	11:26:42 (140)	11:18:12 (775)	11:18:11 (775)
Landing Gate	14.2	14.2	14.2
price	13.91	0	0
Cash	113.91	100	100

Submit a price/quantity in trading window

1. Type a price and quantity, and then click **Sell** to sell and **Buy** to buy.
2. The limit order book shows outstanding limit orders, and the Player id of their submitters.

ComLabGames

Description Username: Jamie Id: 20 Identity: 3

Gate valuation

History	20:46:32 (2480)	20:46:31 (2481)
Gate valuation	13.91	13.91
Number of gates	1	0
Price	0	0
Cash	100	100
Initial wealth	113.91	0
Current wealth	113.91	0
Rate of return (%)	0	0

View more history

Limit order book

Price	Quantity	Cumulative	Player id	Player type

Cash 100 Duration 600 Price 16.5 Quantity 1 **Sell** **Buy**

Asset name	Valuation	Endowment	Sell orders	Buy orders	Bid (Quantity)	Ask (Quantity)	Trading price (Quantity)
Gate valuat...	13.91	1					

Subject's own limit orders

Price	Quantity	Revenue/Cost	Duration

Transaction history

Price	Quantity	Buyer ID	Buyer Ty...	Seller ID	Seller Ty...	Time

Player type: Airline (3)

Round: 1

ComLabGames

Description Username: Jamie Id: 20 Identity: 3

Gate valuation

History	20:46:32 (2760)	20:46:31 (2761)
Gate valuation	13.91	13.91
Number of gates	1	0
Price	0	0
Cash	100	100
Initial wealth	113.91	0
Current wealth	113.91	0
Rate of return (%)	0	0

View more history

Limit order book

Price	Quantity	Cumulative	Player id	Player type
16.5	1	0	20	Airline (3)

Cash 100 Duration 600 Price 16.5 Quantity 1 **Sell** **Buy**

Asset name	Valuation	Endowment	Sell orders	Buy orders	Bid (Quantity)	Ask (Quantity)	Trading price (Quantity)
Gate valuat...	13.91	0	1 (16.5)			16.5 (1)	

Subject's own limit orders

Price	Quantity	Revenue/Cost	Duration
16.5	1	16.5	586 x

Transaction history

Price	Quantity	Buyer ID	Buyer Ty...	Seller ID	Seller Ty...	Time

Player type: Airline (3)

Round: 1

Trading window description

1. Limit order book shows all outstanding sell orders in red and all outstanding buy orders in green.
2. Summary data shows the bid (ask) in green (red).

ComLabGames

Description Username: Jamie Id: 20 Identity: 3

Gate valuation

History

Name	20:46:32 (512...)	20:46:31 (512...)	Price	Quantity	Cumulative	Player id	Player type
Gate valuation	13.91	13.91	16.5	1	0	20	Airline (3)
Number of gates	1	0	13	1	1	22	Airline (3)
Price	0	0	12.99	1	0	19	Airline
Cash	100	100	12.99	1	1	19	Airline
Initial wealth	113.91	0	12.9	1	2	19	Airline
Current wealth	113.91	0	11	1	3	20	Airline
Rate of return (%)	0	0	11	1	4	20	Airline
			10	1	5	23	Airline

View more history

Cash 78 Duration 600 Price 11 Quantity 1 Sell Buy

Summary data

Asset name	Valuation	Endowment	Sell orders	Buy orders	Bid (Quantity)	Ask (Quantity)	Trading price (Quantity)
Gate valua...	13.91	0	1 (16.5)	2 (22)	12.99 (2)	13 (1)	

Sell orders show total outstanding quantity submitted by a subject and in the brackets, total value of all subject's (Jamie) sell orders. There is only one sell order for \$16.5.
Buy orders show total value of all submitted buy orders, which is two for Jamie. Total value of all submitted buy orders is \$22 and is written in brackets.

Subject's own limit orders

Price	Quantity	Revenue/Cost	Duration	
16.5	1	16.5	341	x
11	1	11	565	x
11	1	11	572	x

Transaction history

Price	Quantity	Buyer ID	Buyer Ty...	Seller ID	Seller Ty...	Time
11	1					
11	1					

To delete an order click on "x". In our example sell limit order for 16.5 and q=1 was deleted

Player type: Airline (3)

Round: 1

ComLabGames

Description Username: Jamie Id: 20 Identity: 3

Gate valuation

History

Name	20:46:32 (513...)	20:46:31 (513...)	Price	Quantity	Cumulative	Player id	Player type
Gate valuation	13.91	13.91	13	1	0	22	Airline (3)
Number of gates	1	0	12.99	1	0	19	Airline
Price	0	0	12.99	1	1	19	Airline
Cash	100	100	12.9	1	2	19	Airline
Initial wealth	113.91	0	11	1	3	20	Airline
Current wealth	113.91	0	11	1	4	20	Airline
Rate of return (%)	0	0	10	1	5	23	Airline

View more history

Cash 78 Duration 600 Price 11 Quantity 1 Sell Buy

Summary data

Asset name	Valuation	Endowment	Sell orders	Buy orders	Bid (Quantity)	Ask (Quantity)	Trading price (Quantity)
Gate valua...	13.91	1	0 (0)	2 (22)	12.99 (2)	13 (1)	

Subject's own limit orders

Price	Quantity	Revenue/Cost	Duration	
11	1	11	482	x
11	1	11	489	x

Transaction history

Price	Quantity	Buyer ID	Buyer Ty...	Seller ID	Seller Ty...	Time

Player type: Airline (3)

Round: 1

Trading

- To buy/sell directly select the best order by clicking on the price of the order and click **Buy** / **Sell**.
- Transaction history shows the executed order.

ComLabGames - Description - Username: Jamie - Id: 20 - Identity: 3

Gate valuation

Name	20:46:32 (518...)	20:46:31 (518...)
Gate valuation	13.91	13.91
Number of gates	1	0
Price	0	0
Cash	100	100
Initial wealth	113.91	0
Current wealth	113.91	0
Rate of return (%)	0	0

Price	Quantity	Cumulative	Player id	Player type
13	1	0	22	Airline (5)
12.9	1	0	19	Airline
11	1	1	20	Airline
11	1	2	20	Airline
10	1	3	23	Airline

Price \$10 was selected and by clicking on the price at which sell order is executed is \$12.9 (i.e. the highest outstanding buy limit order price.)

Cash: 78, Duration: 600, Price: 10, Quantity: 1, Sell, Buy

Asset name	Valuation	Endowment	Sell orders	Buy orders	Bid (Quantity)	Ask (Quantity)	Trading price (Quantity)
Gate valua...	13.91	1	0 (0)	2 (22)	12.9 (1)	13 (1)	

Price	Quantity	Revenue/Cost	Duration	
11	1	11	529	x
11	1	11	535	x

Transaction history

Price	Quantity	Buyer ID	Buyer Ty...	Seller ID	Seller Ty...	Time
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Player type: Airline (3)

Round: 1

ComLabGames - Description - Username: Jamie - Id: 20 - Identity: 3

Gate valuation

Name	11:12:14 (18)	20:46:32 (519...)	20:4...
Gate valuation	13.91	13.91	13.91
Number of gates	0	1	0
Price	12.9	0	0
Cash	112.9	100	100
Initial wealth	113.91	113.91	0
Current wealth	112.9	113.91	0
Rate of return (%)	-0.88	0	0

Price	Quantity	Cumulative	Player id	Player type
13	1	0	22	Airline (5)
11	1	0	20	Airline
11	1	1	20	Airline
10	1	2	23	Airline

Cash: 90.9, Duration: 600, Price: 10, Quantity: 1, Sell, Buy

Asset name	Valuation	Endowment	Sell orders	Buy orders	Bid (Quantity)	Ask (Quantity)	Trading price (Quantity)
Gate valua...	13.91	0	0 (0)	2 (22)	11 (2)	13 (1)	12.9 (1)

Price	Quantity	Revenue/Cost	Duration	
11	1	11	456	x
11	1	11	461	x


Transaction history

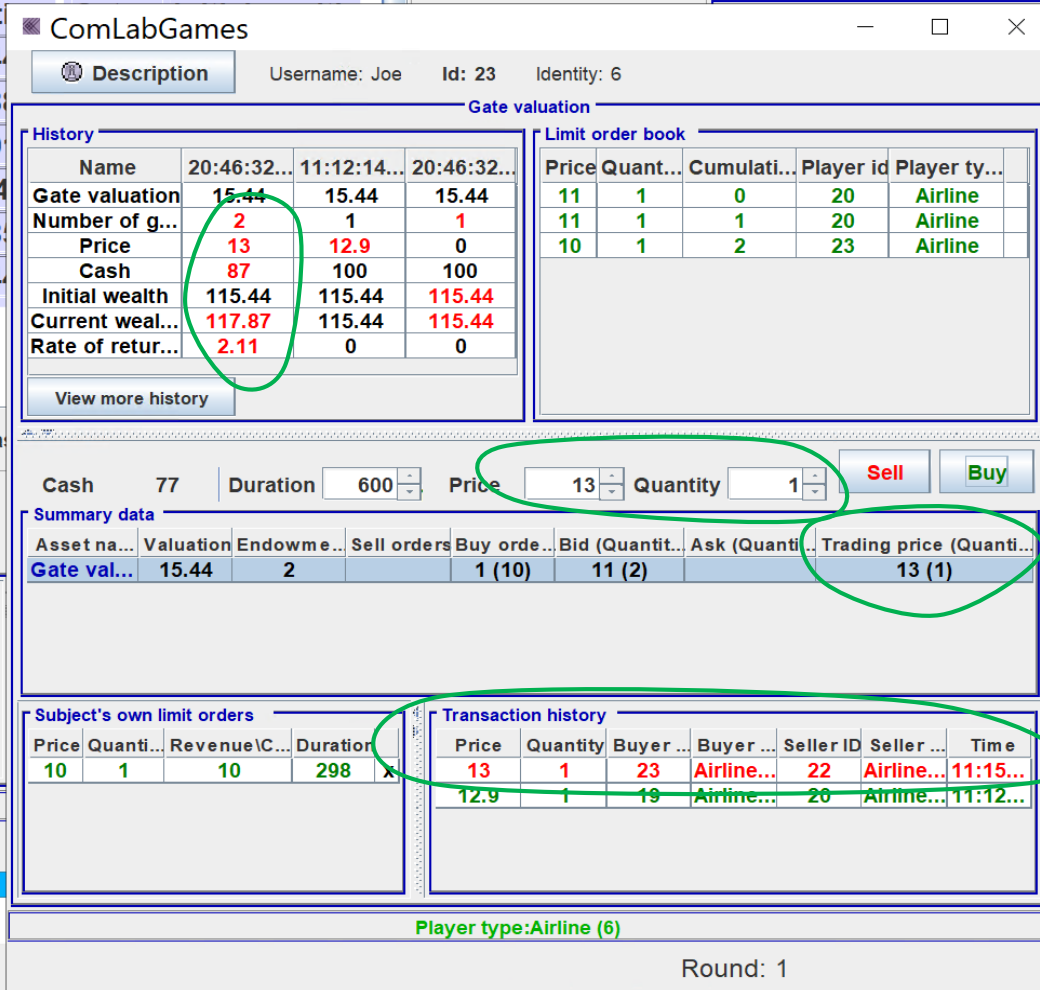
Price	Quantity	Buyer ID	Buyer Ty...	Seller ID	Seller Ty...	Time
12.9	1	19	Airline (2)	20	Airline (3)	11:12:12

Player type: Airline (3)

Round: 1

More trading

1. Price \$13 was selected and by clicking on , buyer (id = 23) bought the gate. His cash decreased to \$87 and his current wealth is $\$117.87 = (2 * 15.44 + 87)$.



The screenshot displays the 'ComLabGames' interface for a user named 'Joe' (Id: 23, Identity: 6). The window is titled 'Gate valuation' and contains several sections:

- History:** A table showing the player's performance over three time periods. The current wealth is highlighted as 117.87.
- Limit order book:** A table showing the current market orders. The selected price of 13 is circled in green.
- Trading controls:** A section with input fields for Cash (77), Duration (600), Price (13), and Quantity (1). The 'Buy' button is highlighted in green.
- Summary data:** A table showing the current state of the asset. The trading price is highlighted as 13 (1).
- Subject's own limit orders:** A table showing the player's current orders. The selected price of 13 is circled in green.
- Transaction history:** A table showing the player's recent transactions. The selected price of 13 is circled in green.

At the bottom of the window, it indicates 'Player type: Airline (6)' and 'Round: 1'.

Summary page for trading game

ComLabGames - Client

Description Username: Ari Id: 18 Identity: 1

	Valuation	Gates	Initial wealth	Current wealth	Rate of return
	19.44	0	100	100	0
	12.88	2	112.88	112.86	-0.02
	13.91	0	113.91	112.9	-0.88
Competitive equilibrium price	10.4	0	100	100	0
	10.85	0	110.85	113	1.94
	15.44	2	115.44	117.87	2.11

Market Stage time limit: unlimited Round: 1 Continue

Player type: Airline (1)

Game is over!

Your (personal) quote

- ◆ Define your highest limit order buy price as your **personal bid**:
 - If you have no limit buy orders, then your personal bid is zero.

- ◆ Define your lowest limit order sell price as your **personal ask**:
 - If you have no limit sell orders, then personal ask is infinity.

- ◆ Your **quote** is the pair:
(personal bid, personal ask)

Rule: If your quote straddles your valuation then your portfolio cannot lose value.

Exhausting the gains from trade

- ◆ There are two ways of making profitable trades in a limit order market . . .
- ◆ **Create liquidity** by placing limit orders (adding to the book):
 - with a quote that straddles your valuation.
- ◆ **Dissolve liquidity** by filling limit orders (enlarging the spread):
 - when the bid is less than your valuation (placing market buy orders)
 - or the ask is greater than your valuation (placing market sell orders).
- ◆ Suppose all bidders place a profitable:
 1. market order if that is feasible.
 2. limit order if not.

⇒ if the limit orders are placed sufficiently close to the valuation (so the personal bid and ask are nearly equal) then a **competitive equilibrium allocation** is iteratively attained.

Competitive equilibrium defined

◆ Denote by:

d_k demand for landing gate by airline k

s_k supply of landing gates by airline k

$S = s_1 + s_2 + \dots + s_6$ total supply of gates

$v_d = \min\{v_k : d_k > 0\}$ the lowest valuation airline getting a gate

$v_s = \max\{v_k : d_k = 0\}$ the highest valuation airline not getting a gate

◆ A competitive equilibrium is a price p_c satisfying:

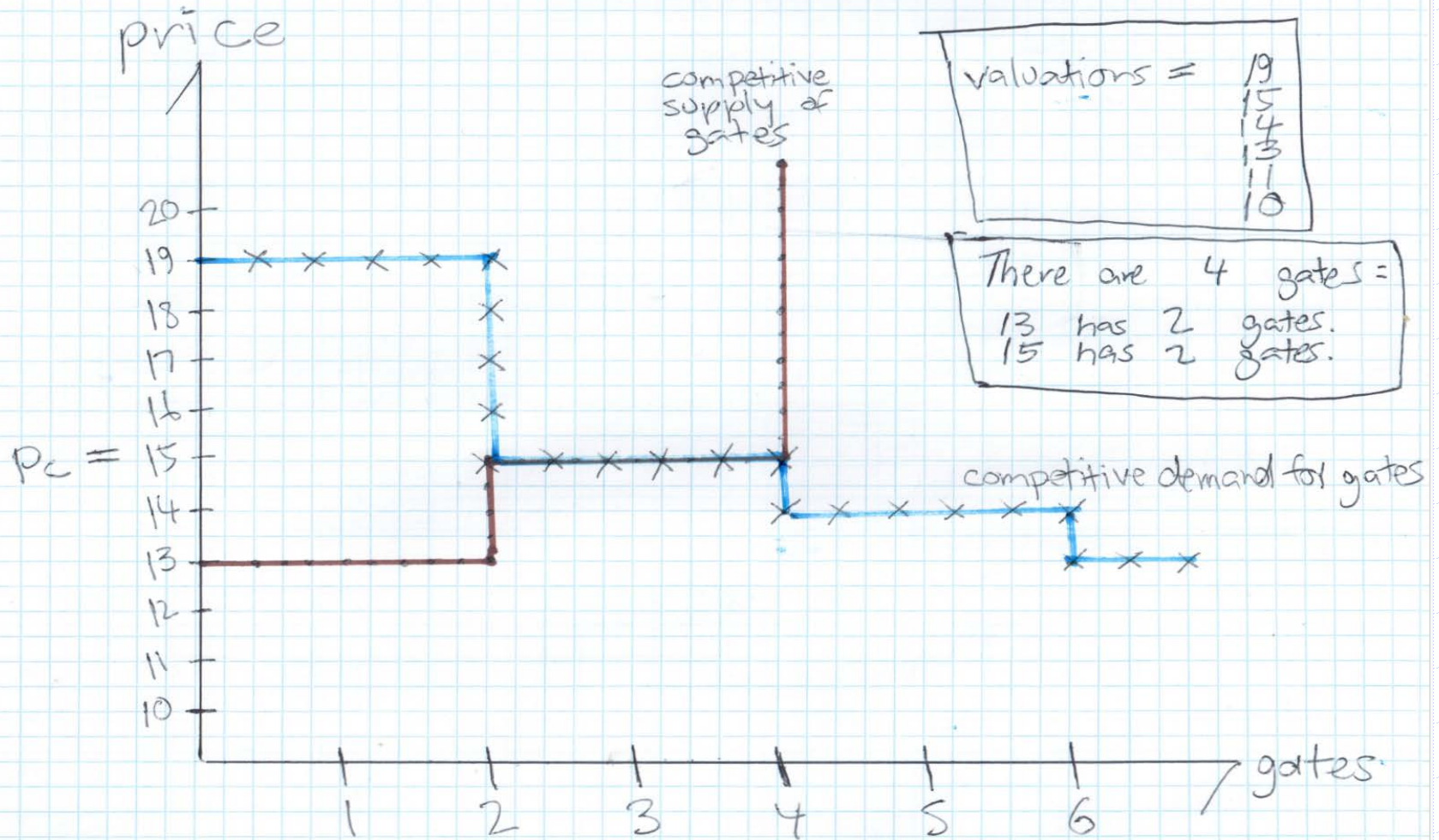
1. $v_d \geq p_c \geq v_s$

2. If $v_k > p_c$ then airline k is allocated 2 gates

3. If $v_k < p_c$ then airline k is allocated 0 gates

4. The total number of allocated gates is s .

Competitive equilibrium illustrated



A question about trading

◆ The essential features of limit order markets are described by:

1. limit orders
2. market orders
3. bid and ask prices
4. limit order book
5. the spread
6. personal quotes
7. transaction prices

◆ We defined an algorithm, whose every two-step iterate is profitable to the trader in question, that exhausts the gains from trade and attains a competitive equilibrium allocation.

◆ But each steps taken by a traders in this algorithm do not maximize his or her individual profits!

◆ To be continued . . .