Dynamic Considerations

Robert A. Miller

Trade and Investment Strategy

November 2023

Introduction

What does valuation mean?

- The preceding lectures have summarized what matters about an asset by its *valuation*.
- The remaining lectures address the questions:
  - How is a valuation calculated?
  - Why do consumers, investors and workers hold assets?
- How an asset is priced depends on:
  - the trading mechanism
  - the asset's intrinsic value or value-in-use
- Intrinsic value should account for:
  1. *uncertainty of* benefits (and costs).
  2. benefits accruing at *different points in time*.

Expected Value

A summary measure

- The *expected net present value* (EPV) of an asset captures the first two features within one summary measure. Benefits received:
  - at different times are perfect substitutes priced using the *interest rate*.
  - for different uncertain outcomes are priced by their *probabilities*.
- Intuitively the asset sheds payouts, over time and with designated probabilities that translate to *generalized purchasing power* consumers intrinsically value.
- Measuring value by EPV is *simple*, *internally consistent* and therefore.
- But EPV is essentially a price aggregator, not an intrinsic value.
- Given a trading mechanism, how is intrinsic values related to price?
- These next three lectures analyze:
  1. *intertemporal substitution* (timing of consumption benefits)
  2. *risk premiums* (cost of uncertainty)
  3. *modern portfolio theory* (which synthesizes these two factors).

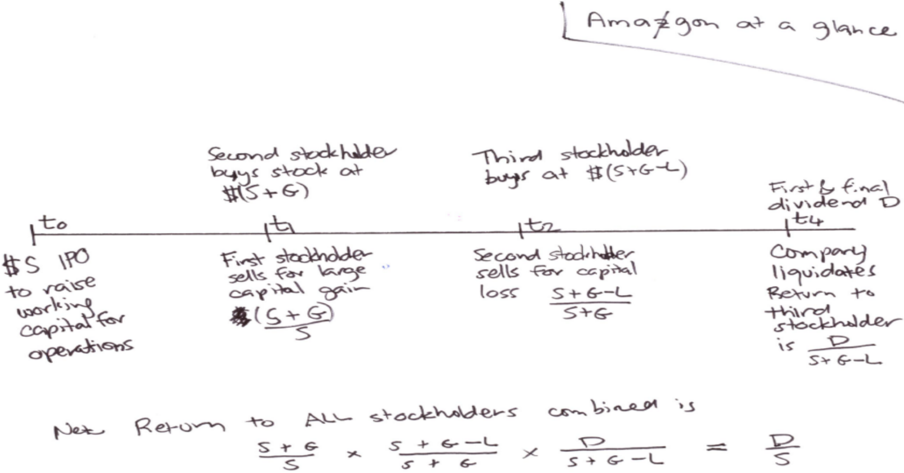
Investment in Firms

Corporations as an intermediary between current and future consumption

- The demand for shares in firm ownership comes from those wishing to defer consumption.
- The primary reason for stock issues is to finance the firm's operations.
- A compact between investor (or shareholder) and asset manager (or executive officer) embodies the trade off between present and future.
- The stock's dividend stream ultimately determines its value.
- Not all financial securities pay dividends in aggregate:
    - Many financial derivatives do not pay dividends directly.
    - Ponzi schemes swindle investors by paying some but ultimately sacrificing others.
- However:
    - derivatives are worthless unless the underlying stocks pay dividends.
    - Even Ponzi schemes exist only because some buyers believe the so-called securities they buy will pay dividends.

Investment in Firms

Why share value is (roughly) the discounted sum of the expected dividend stream



Allocating Resources over Time

Financing lifecycle consumption and investment (including human capital)

- Demand for investment vehicles and banking services arise in response to the lifetime goals of individuals and households who:
  1. first educate themselves rather than working.
  2. then form households, working less while raising young children.
  3. purchase first home when family and employment situations stabilize.
  4. work harder in prime age years.
  5. ramp down work schedule with declining mental and physical capacity.
- Their financial decisions mirror these lifecycle patterns, as they:
  1. borrow to pay for education and current consumption.
  2. repay loans, more slowly when children are young.
  3. take on homeownership, a risky asset.
  4. increase home equity, wealth and social security benefits.
  5. live rent free off assets upon retirement.

- In this country people save through their:
  1. contributions to social security
  2. investment in mutual funds
  3. home purchase and mortgage repayments
- One way of gauging the savings propensity for households is to survey their wealth:

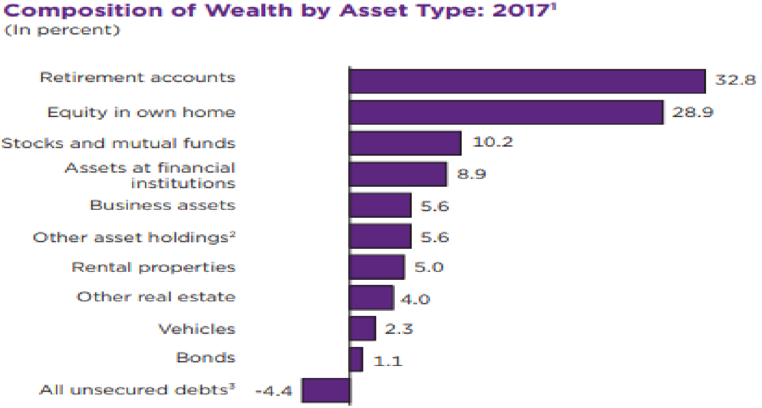
The Value of Household Wealth by Percentile: 2017

Percentile	2017 dollars
10th	-5,724
25th	5,608
50th	104,000
75th	427,700
90th	1,212,000

Source: U.S. Census Bureau, Survey of Income and Program Participation, Survey Year 2018.

Allocating Resources over Time

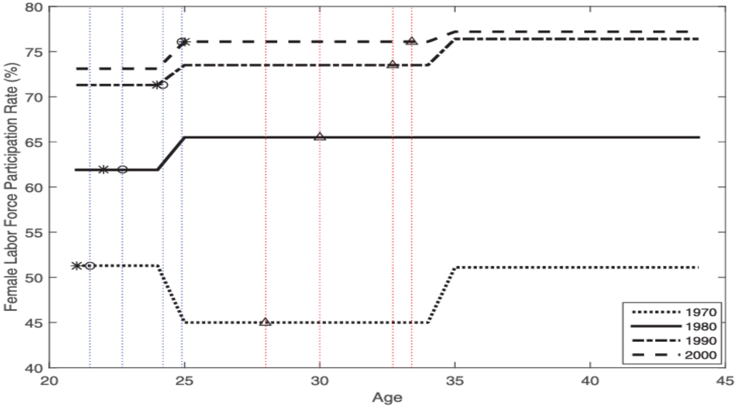
The composition of wealth



[1] Excludes households in the top 1 percent of wealth.
[2] Includes annuities, trusts, cash life insurance policies, educational savings accounts, mortgages held for sale of real estate, amount due from sale of business property, and other financial assets.
[3] Because wealth is assets minus debts, unsecured debts are subtracted from the distribution of wealth and are shown as negative.
Source: U.S. Census Bureau, Survey of Income and Program Participation, Survey Year 2018.

Lifecycle Trends over the Years

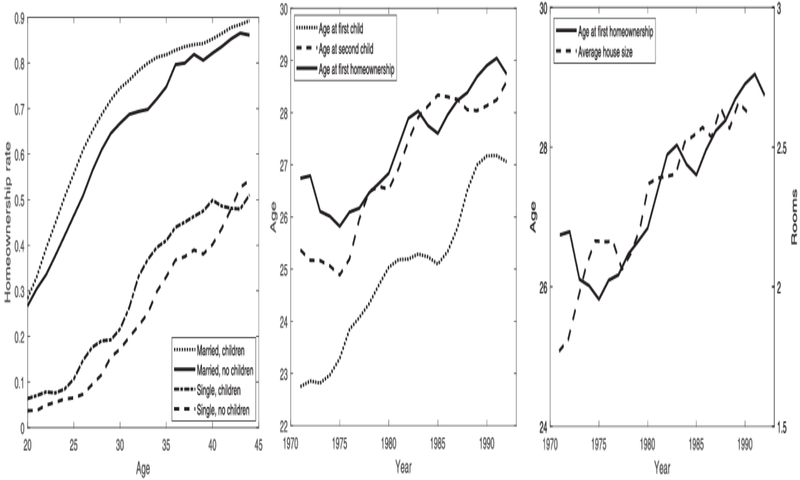
Female labor supply, marriage, first birth & homeownership (Khorunzhina and Miller, 2022)



NOTES: "star" denotes median age at first marriage, "circle" denotes average age at first birth, "triangle" denotes average age at first homeownership Age at first marriage is taken from the U.S. Census Bureau, age at first birth is taken from the National Vital Statistical Reports (Mathews and Hamilton, 2002), age at first homeownership is computed from the PSID, whereas labor force participation rates are taken from publications of the U.S. Bureau of Labor Statistics (Toossi, 2002, 2012).

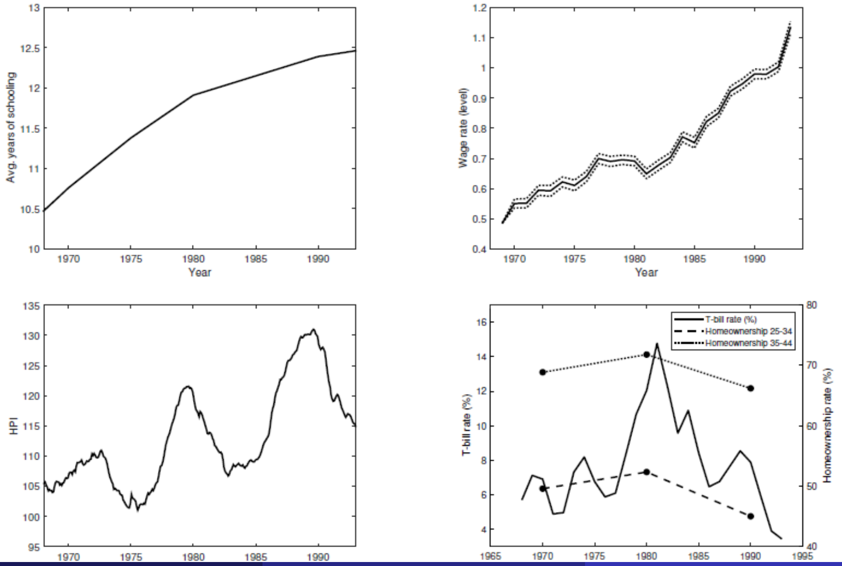
Lifecycle Trends over the Years

Timing of children, homeownership and house size (Khorunzhina and Miller, 2022)



Lifecycle Trends over the Years

The driving factors: education, female wages, housing prices and interest rates



Lifecycle Trends over the Years

The effects of the driving factors (Khorunzhina and Miller, 2022)

- Two main factors have driven the trend to the decline in homeownership:
  1. rising house prices
  2. rising female wages, medical advances and increased education, .
- The second factor is significant because higher wages:
  - encourage women to increase their labor force participation.
  - postpone and reduce childbearing
  - home ownership is more valued by families with children.
- Two factors mitigate this trend:
  - Higher educated women are more likely to be homeowners.
  - Higher real interest rates increase home ownership: households save more when the interest rate increase, and home ownership is a savings vehicle.

Preferences over Time

Individuals do not maximize the present value of their consumption stream

- Clearly, household paid work (production) does not correspond to the timing of consumption.
- In other words market value and intrinsic value are not equivalent.
- Rather market value arises, or is derived from, intrinsic value.
- To explain this process, we must start with a model of consumer or household preferences over goods or commodities that accounts for:
  - the probability a commodity is consumed (uncertainty)
  - when that might occur (timing)
- Suppose an individual consuming goods and services valued at c_t in periods $t \in \{1, \ldots, T\}$ receives a *lifetime utility* of:

$$U(c_1, \ldots, c_T) = \sum_{t=1}^{T} \beta^t u(c_t)$$

where $u(c_t)$ is concave increasing. That is $u'(c) > 0$ and $u''(c) \leq 0$.

- Two key features of this formulation are:
  1. The *marginal rate of substitution* between c_t and c_{t+1} defined as:
  $$MRS\left(c_{t+1}, c_t\right) = \frac{\partial U\left(c_1, \ldots, c_T\right)/\partial c_{t+1}}{\partial U\left(c_1, \ldots, c_T\right)/\partial c_t}$$
  measures the amount of c_{t+1} a person is willing to sacrifice for an extra unit of c_t to achieve exactly the same level of utility, and comes from:
  $$\frac{\partial U}{\partial c_{t+1}} dc_{t+1} + \frac{\partial U}{\partial c_t} dc_t = 0 \Longrightarrow MRS\left(c_{t+1}, c_t\right) dc_{t+1} + dc_t = 0$$
  2. The *subjective discount factor* denoted by β:
     - Typically $0 < \beta < 1$; it measures the individual's impatience.
     - If consumption in periods t and $t+1$ is the same, then $MRS\left(c_{t+1}, c_t\right) = \beta$.
- If consumption is c^* in both periods t and $t+1$, then utility is $\beta^t u\left(c^*\right)$ utility in period t, but only $\beta^{t+1} u\left(c^*\right)$, so the ratio of period $t+1$ to period t utility is β; hence its name.

Consumption and Saving over the Lifecyle

Some examples

- For any $\alpha > 0$ consider the following commonly used parameterizations:

$$u(c_t) = \log(1 + \alpha c_t) \text{ and } MRS(c_{t+1}, c_t) = \frac{\beta(1 + \alpha c_t)}{(1 + \alpha c_{t+1})}$$

$$u(c_t) = -\exp(-\alpha c_t) \text{ and } MRS(c_{t+1}, c_t) = \exp[-\alpha(c_{t+1} - c_t)]$$

$$u(c_t) = \frac{1}{1-\alpha} c_t^{1-\alpha} \text{ and } MRS(c_{t+1}, c_t) = \left(\frac{c_t}{c_{t+1}}\right)^{\alpha}$$

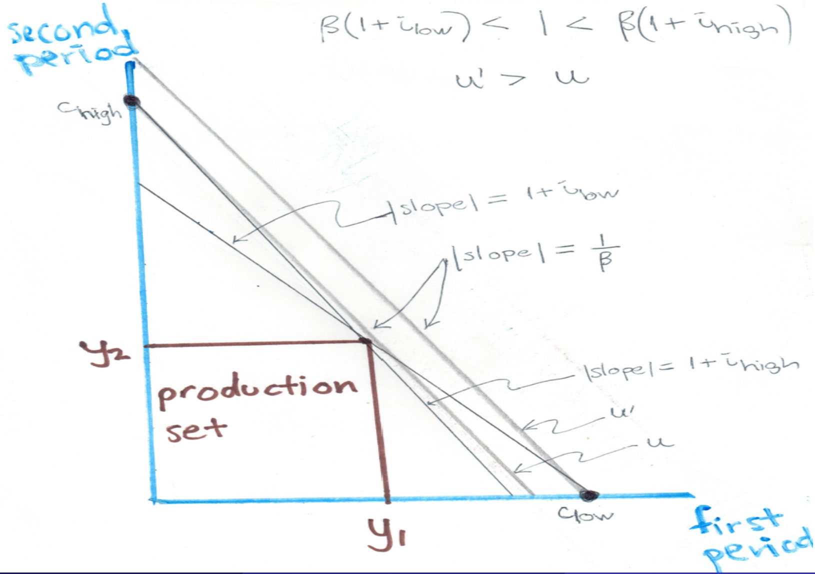
- A simpler formulation is to assume individuals have linear utility:

$$u(c_t) = c_t \text{ and } MRS(c_{t+1}, c_t) = \beta$$

- A person with linear utility would tend to consume nothing in some periods and maximal permitted amounts in others.

Consumption and Saving over the Lifecyle

Linear utility and extremal consumption in a two period model



- Define the *risk free return*, the extra consumption available in period $t+1$ by reducing consumption in period t by one unit.
- Letting r_{t+1} denote the risk free rate of return, then $i_t \equiv r_{t+1} - 1$ is the *real interest rate* in period t.
- Let W_t denote the remaining lifetime wealth of the individual.
- It follows a law of motion given by

$$W_{t+1} = (W_t - c_t + w_t)\, r_{t+1}$$

where w_t includes income in period t, both wage earnings and nonwage income.

- Reducing c_t by dc and increasing c_{t+1} by $r_{t+1}dc$, but holding consumption in every other period constant, changes lifetime utility by:

$$\begin{aligned} d\left[\sum_{t=1}^{T}\beta^t u\left(c_t\right)\right] &= \beta^t\left[u\left(c_t\right)-u\left(c_t-dc\right)\right] \\ &\quad +\beta^{t+1}\left[u\left(c_{t+1}+r_{t+1}dc\right)-u\left(c_{t+1}\right)\right] \\ &= -\beta^t u'\left(c_t\right)dc+\beta^{t+1}u'\left(c_{t+1}\right)r_{t+1}dc \end{aligned}$$

- If this expression is:
    - *positive*, then increase c_{t+1} and reduce c_t.
    - *negative*, then increase c_t and reduce c_{t+1}.
    - *zero*, then utility can't be increased by reallocating between t and $t+1$.

- The optimal consumption sequence c_t^o uniquely satisfies this marginal condition for all $t \in \{1, \ldots, T-1\}$ if the budget constraint is satisfied with equality.
- Denoting c_t^o the optimal level of c_t, it now follows that:

$$-\beta^t u'\left(c_t^o\right) + \beta^{t+1} u'\left(c_{t+1}^o\right) r_{t+1} = 0$$

- Cancelling β^t and dividing through by $u'\left(c_t^o\right)$ yields
- We rewrite this condition as:

$$1 = (1 + i_t) \frac{\beta u'\left(c_{t+1}^o\right)}{u'\left(c_t^o\right)} \equiv r_{t+1} \frac{\beta u'\left(c_{t+1}^o\right)}{u'\left(c_t^o\right)} \equiv r_{t+1} MRS_{t+1}\left(c_{t+1}, c_t\right)$$

where as defined above:

$$MRS_{t+1}\left(c_{t+1}, c_t\right) \equiv \beta u'\left(c_{t+1}\right) / u'\left(c_t\right)$$

is the *marginal rate of substitution function* between consecutive periods of consumption.

Consumption and Saving over the Lifecyle

A parametric example

- Assume that for some $\alpha > 0$:

$$u(c) = \frac{1}{1-\alpha} c^{1-\alpha}$$

- Then:

$$u'(c) = c^{-\alpha}$$

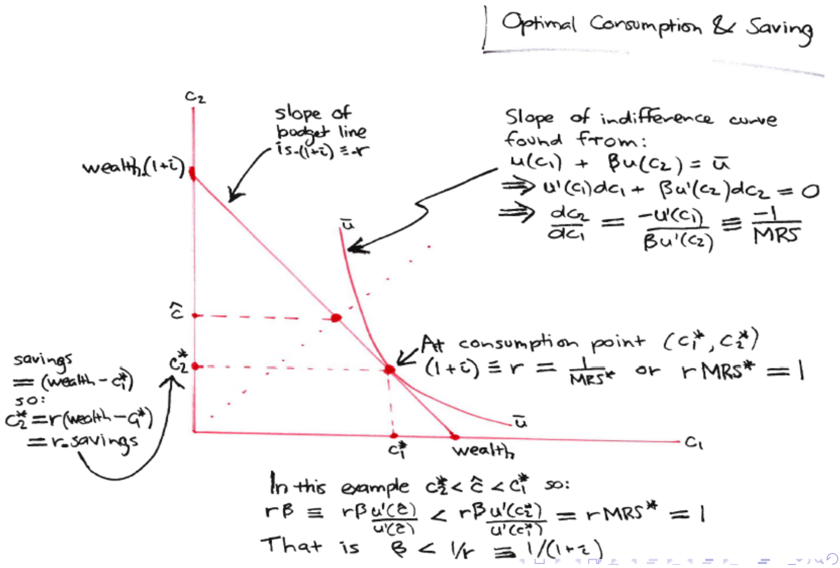
so in this example:

$$1 = r_{t+1} MRS_{t+1}(c_{t+1}, c_t) = r_{t+1} \frac{\beta u'(c^o_{t+1})}{u'(c^o_t)} = r_{t+1} \beta \left(\frac{c^o_t}{c^o_{t+1}} \right)^{\alpha}$$

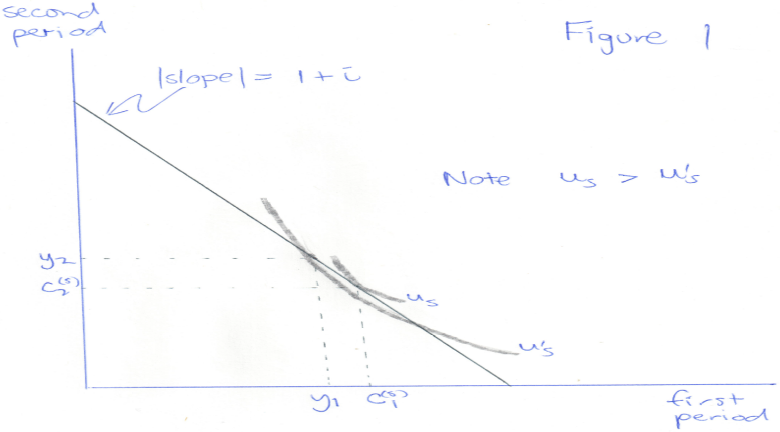
- Notice that if :
  - if $\beta \uparrow$ then $c^o_{t+1} \uparrow$ and $c^o_t \downarrow$
  - similarly $r_{t+1} \uparrow$ then $c^o_{t+1} \uparrow$ and $c^o_t \downarrow$

Consumption and Saving over the Lifecyle

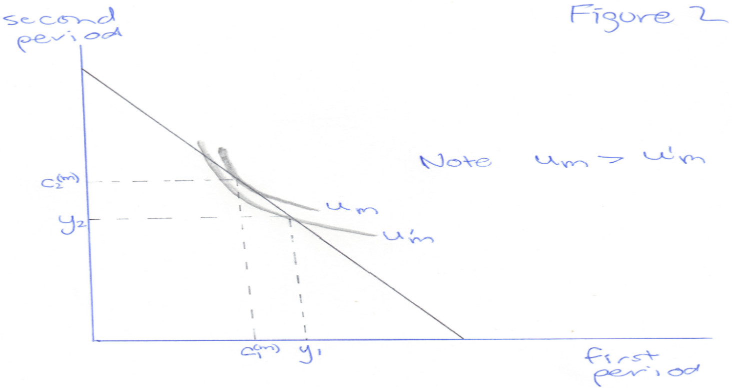
Trading off current and future consumption in a two period lifecycle model



Borrowing in equilibrium



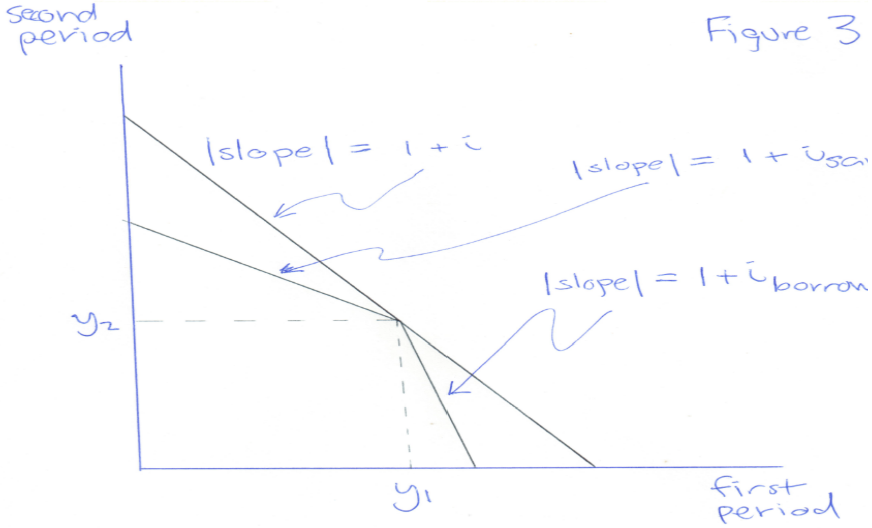
Spendthrift (s) borrows $(c_1^{(s)} - y_1)$ in first period and repays $(y_2 - c_2^{(s)}) = (c_1^{(s)} - y_1)(1+i)$ in the second.



Miser (m) saves $(y_1^{(m)} - c_1^{(m)})$ in first period and spends an extra $(c_2^m - y) = (c_1^m - y)$ in the second.

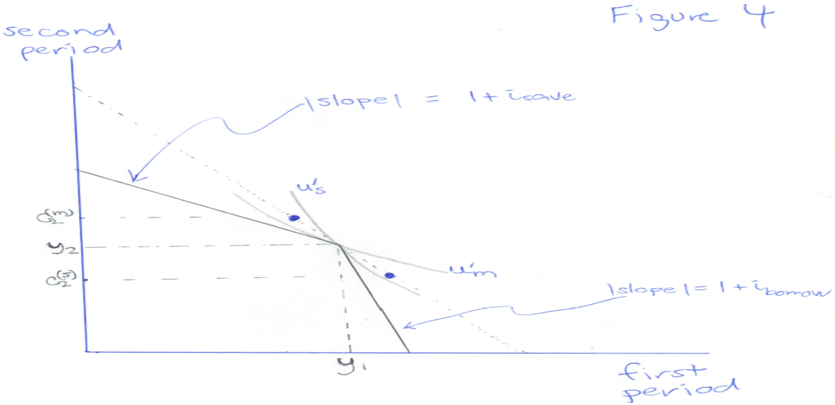
Consumption and Saving over the Lifecyle

A wedge is driven between the borrowing and lending rates



Consumption and Saving over the Lifecyle

The miser and the spendthrift neither save nor borrow



If the difference ($i_{borrow} - i_{save}$) is big enough both miser and spendthrift consume their current income each period.

Consumption and Saving over the Lifecyle

Can this wedge explain why Americans save so little?

- First of all they don't "save so little":
  - Social security is a form of forced savings.
  - Many contribute to defined benefit and defined retirement plans.
  - Many are eligible for health subsidies in their old age.
  - Households gradually acquire ownership of their own homes.
- Second of all these payments differ:
  - Social security payments and benefits differ by income.
  - Retirement plans are largely discretionary.
  - Health plans differ across households.
  - Homes differ vastly in their quality and amenities.
- Nevertheless people might spend their disposable income immediately because of the wedge between the borrowing and savings rates.
- Intertemporally sharing and managing wealth between borrower and lender is a costly activity.