Session 3.2
Revenue from Private Value Auctions

This session exploits the Revenue Equivalence Theorem to derive the derive the revenue from any private value auction.

Steps for deriving expected revenue

- The expected revenue from any auction satisfying the conditions of the theorem, is the expected value of the second highest bidder.
- To obtain this quantity, we proceed in two steps:

  1. derive the probability distribution of the second highest valuation
  2. obtain its density and integrate to find the mean.

Probability distribution of the second highest valuation

- Since any auction satisfying the conditions for the theorem can be used to calculate the expected revenue, we select the second price auction.
- The probability that the second highest valuation is less than v is the sum of the the probabilities that:
  1. all the valuations are less than v, or $P(v)^N$
  2. N-1 valuations are less than v and the other one is greater than v. There are N ways of doing this so the probability is:

$$NP(v)^{N-1}[1 - P(v)] = NP(v)^{N-1} - NP(v)^N$$

- The probability distribution for the second highest valuation is therefore:

$$NP(v)^{N-1} - (N - 1) P(v)^N$$

Expected revenue from Private Value Auctions

- The probability density function for the second highest valuation v is therefore:

  $N(N-1)P(v)^{N-2}\,[1 - P(v)]P'(v)$

- Therefore the expected revenue to the auctioneer, or the expected value of the second highest valuation, denoted by $v^{(2)}$, is:

$$E\left[v^{(2)}\right] = N(N-1)\int_{v_0}^{\bar{v}} vP(v)^{N-2}\left[1-P(v)\right]P'(v)\,dv$$