Session 3.2
Bidding in a First Price Sealed Bid Auction

This session derives the Nash equilibrium bidding function in a FPSB auction with private values using the Revenue Equivalence Theorem. We then apply the general formula to the special case in which valuations are uniformly distributed on a closed interval. In that case the bid function is a weighted average of the bottom valuation and the bidder's valuation, where the weights are determined by the number of rivals,

Using the revenue equivalence theorem to derive optimal bidding functions

- We can derive the solution bidding strategies for auctions that are revenue equivalent to the second price sealed bid auction.
- Consider, for example a first price sealed bid auctions with independent and identically distributed valuations.
- The revenue equivalence theorem implies that each bidder will bid the expected value of the next highest bidder conditional upon his valuation being the highest.

Bidding in a first price sealed bid auction

- In a symmetric equilibrium to first price sealed bid auction, we can show that a bidder with valuation v_n bids:

$$b(v_n) = v_n - \frac{\int_{v_0}^{v_n} P(v)^{N-1} dv}{P(v_n)^{N-1}}$$

The derivation

- The probability the remaining N - 1 valuations are less than v given the highest valuation is v_n is:

$$\Pr\left\{v^{(2)} \le v \middle| v_n\right\} = \left[P(v)/P(v_n)\right]^{N-1}$$

- Differentiating, the conditional density for the second highest valuation is then:

$$(N-1)P(v_n)^{1-N}P(v)^{N-2}P'(v)$$

- If he wins, the bidding function for n is the expected value of the second highest valuation:

$$b(v_n) = (N-1)P(v_n)^{1-N}\int_{v_0}^{\bar{v}} vP(v)^{N-2}P'(v)dv$$

- Integrating by parts, we simplify this formula to:

$$b(v_n) = P(v_n)^{1-N}\left[vP(v)^{N-1}\right]_{v=v_0}^{v=v_n} - P(v_n)^{1-N}\int_{v_0}^{\bar{v}} P(v)^{N-1}dv$$

$$= v_n - P(v_n)^{1-N}\int_{v_0}^{\bar{v}} P(v)^{N-1}dv$$

An example: the uniform distribution

- Suppose valuations are uniformly distributed within a closed interval, with probability distribution:

$$P(v) = (v - v_0)/(\bar{v} - v_0)$$

- Then in equilibrium, a player with valuation v bids a weighted average of the lowest possible valuation and his own, where the weights are 1/N and (N-1)/N:

$$b(v_n) = v_n - P(v_n)^{1-N} \int_{v_0}^{v_n} P(v)^{N-1} dv$$

$$= v_n - (v_n - v_0)^{1-N} \int_{v_0}^{v_n} (v - v_0)^{N-1} dv$$

$$= v_0 / N + v_n (N-1)/ N$$