Session 3.4
Differential Information in FPSB Auctions

This session derives the equilibrium behavior of two bidders competing in a FPSB auction, who have differential information about how valuable the auctioned item is. One (informed) bidder knows the true value before the auction. The other (uninformed) bidder receives a signal about the true value before the auction, only discovering its true value after the auction if she wins.

Bidding with differential information

- Differential information between bidders refers to an auction that some bidders know more about the value of the object being auctioned than the others.
- What happens if bidders are differentially informed about a common value?
- An extreme form of dependent signals occurs when one bidder know the signal and the others do not:
  - How should an informed player bid?
  - What about an uninformed player?

A theorem on first price sealed bid auctions

- The previous slide shows that the uninformed bidder plays a mixed strategy in this game.

- The slides below prove that in equilibrium, when the auctioned item is worth v:
  - the informed bidder bids:

    $$\beta(v) = E[V|V \leq v]$$

  - the uninformed bidder makes a random bid from the interval [0, E[V]] according to the probability distribution H defined by:

    $$H(b) = \text{Prob}[\beta(v) \leq b]$$

Return to the uninformed bidder

- If the uninformed player bids more than E[V], then his expected return is negative, since he would win the auction every time $v < E[V]$ but less frequently when $v > E[V]$.

- We now show that if his bid $b < E[V]$, his expected return is zero, and therefore any bid $b < E[V]$ is a best response to the informed player's bid.

- If the uniformed bids less than E[V] and loses the auction, his return is zero. If he bids less than E[V] and wins the auction, his return is

$$E[V|\ \beta(V) < b] - b = E[V|\ V < \beta^{-1}(b)\ ] - b$$
$$= \beta(\beta^{-1}(b)\ ) - b$$
$$= 0$$

Return to the informed bidder

- Since the uninformed player bids always less than E[v], so does the informed player.

- Noting that $\beta(w)$ varies from $\underline{v}$ to E[v], we prove it is better to bid $\beta(v)$ rather than $\beta(w)$. Given a valuation of v, the expected net benefit from bidding $\beta(w)$ is:

$$H(\beta(w))[v - \beta(w)] = \Pr\{V \leq w\}[v - \beta(w)] = F(w)[v - \beta(w)]$$

- Differentiating with respect to w, using derivations found on the next slide, yields $F'(w)[v - w]$ which is positive for all $v > w$ and negative for all $v < w$, and zero at $v = w$. Therefore bidding $\beta(v)$ is optimal for the informed bidder with valuation v.

The derivative

- Noting

it follows from the fundamental theorem of calculus that

and so the derivative of $F(w)[v - \beta(w)]$ with respect to w is :