Lecture 2
An Introduction to Contracts

This lecture completes our discussion of bargaining by showing what happens when there is incomplete information when one party has more information than another. Then we study how those who create and administer organizations design the incentives and institutional rules that best serve their ends. To demonstrate how to extract the most rent from a transaction, we analyze upstream contracts with suppliers, employment contracts for workers, and service contracts for consumers.

Bargaining with full information

- Two striking features characterize all the solutions of the bargaining games that we have played so far:
  1. An agreement is always reached.
  2. Negotiations end after one round.
- This occurs because nothing is learned from continuing negotiations, yet a cost is sustained because the opportunity to reach an agreement is put at risk from delaying it.

Reaching agreement may be costly

- Yet there are many situations where conflict is not instantaneously resolved, and where negotiations break down:
    1. In industrial relations, negotiations can be drawn out, and sometimes lead to strikes.
    2. Plans for construction projects are discussed, contracts are written up, but left unsigned, so the projects are cancelled.
    3. Weddings are postponed and called off.

The blame game

- Consider the following experiment in a multi-round bargaining game called BLAME. There are two players, called BBC and a GOVT.
- At the beginning of the game BBC makes a statement, which is a number between zero and one, denoted N. (Interpret N as a proportion of blame BBC is prepared to accept.)
- The GOVT can agree with the BBC statement N or refute it. If the GOVT agrees with the statement then the BBC forfeits £N billion funding, and the GOVT loses 1 - N proportion of the vote next election.

Counter proposal

- If the GOVT refutes the statement, there is a 20 percent chance that no one at all will be blamed, because a more newsworthy issue drowns out the conflict between BBC and GOVT.
- If the GOVT refutes the statement, and the issue remains newsworthy (this happens with probability 0.8), the GOVT issues its own statement P, also a number between zero and one. (Interpret P as a proportion of blame the GOVT offers to accept.)
- Should the BBC agree with the statement issued by the GOVT, the GOVT loses P proportion of the vote in the next election, and the BBC loses £5(1-P) billion in funds.

Endgame

- Otherwise the BBC refutes the statement of the GOVT, an arbitrator called HUTTON draws a random variable from a uniform distribution with support [0,2] denoted H, the BBC is fined £H billion, and the GOVT loses H/5 proportion of the vote next election.
- What will happen?
- The solution can be found using backwards induction. (See the footnotes or read the press!)

Evolving payoffs and discount factors

- Suppose two (or more) parties are jointly liable for a debt that neither wishes to pay.
- The players take turns in announcing how much blame should be attributed to each player, and the game ends if a sufficient number of them agree with a tabled proposal.
- If a proposal is rejected, the total liability might increase (since the problem remains unsolved), or decline (if there is some chance the consequences are less dire than the players originally thought).
- If the players do not reach a verdict after a given number of rounds, another mechanism, such as an independent enquiry, ascribes liability to each player.

Summarizing bargaining outcomes when there is complete information

- If the value of the match is constant throughout the bargaining phase, and is known by both parties, then the preceding discussion shows that it will be formed immediately, or not at all.
- The only exception occurs if the current value of the match changes throughout the bargaining phase as the players gather new information together.

Bargaining with incomplete information

- If the value of the match is constant throughout the bargaining phase, and is known by both parties, then the preceding discussion shows that it will be formed immediately, or not at all.
- In the segment on this topic, we will relax the assumption that all the bargaining parties are fully informed.
- We now modify the original ultimatum game, between a proposer and a responder, by changing the information structure.
- Suppose the value to the responder of reaching an agreement is not known by the proposer.

An experiment

- In this game:
  1. The proposer demands s from the responder.
  2. Then the responder draws a value v from the probability distribution F(v). For convenience we normalize v so that $v_0 \leq v \leq v_1$.
  3. The responder either accepts or rejects the demand of s.
  4. If the demand is accepted the proposer receives s and the responder receives v – s, but if the demand is rejected neither party receives anything.

The proposer's objective

- The responder accepts the offer if $v > s$ and rejects the offer otherwise.
- Now suppose the proposer maximizes his expected wealth, which can be expressed as:

$$\Pr\{v > s\}s = [1 - F(s)]s$$

- Notice the term in the square brackets $[1 - F(s)]$ is the quantity sold, which declines in price, while s is the price itself.

Solution to the game

- Let s^o denote the optimal choice of s for the proposer. Clearly $v_0 \leq s^o < v_1$.
- If $v_0 < s^o < v_1$, then s^o satisfies a first order condition for this problem:

$$1 - F'(s^o)\, s^o - F(s^o) = 0$$

- Otherwise $s^o = v_0$ and the proposer receives:

$$[1 - F(v_0)]v_0 = v_0$$

- The revenue generated by solving the first order condition is compared with v_0 to obtain the solution to the proposer's problem.

F(s) is a uniform distribution

- Suppose:

  $F(v) = (v - v_0)/ (v_1 - v_0)$ for all $v_0 \leq v \leq v_1$

  which implies

  $F'(v) = 1 /(v_1 - v_0)$ for all $v_0 \leq v \leq v_1$

- Thus the first order condition reduces to:

  $1 - s^o/(v_1 - v_0) - (s^o - v_0)/ (v_1 - v_0) = 0$

  $\Rightarrow \quad v_1 - v_0 - s^o - s^o + v_0 = 0$

  $\Rightarrow \quad 2\, s^o = v_1$

Solving the uniform distribution case

- In the interior case $s^o = v_1/2$. It clearly applies when $v_0 = 0$, but that is not the only case.
- We compare v_0 with the expected revenue from the interior solution $v_1(v_1 - 2v_0)/(v_1 - v_0)$.
- If $v_0 > 0$ define $v_1 = kv_0$ for some $k > 1$.
- Then we obtain an interior solution if $k > 1$ and:

$$k(k - 2) > k - 1 \quad => \quad k^2 - 3k + 1 > 0$$

- So an interior solution holds if and only if k exceeds the larger of the two roots to this equation, that is

$$k > (3 + 5^{1/2})/2 \ .$$

F(s) is [0,1] uniform

- More specifically let:

  $F(v) = v$ for all $0 \leq v \leq 1$

- Then the interior solution applies so $s^o = ½$, and $F(v) = ½$. Thus exchange only occurs half the time it there are gains from trade.

- The trading surplus is:

$$\int_{s_o}^{v_1} v dF(v)$$

- Given our assumption about F(v) it follows that ¼ of the trading surplus is realized, which is ½ of the potential surplus .

Counteroffers

- Since there is only one offer, there is no opportunity for learning to take place during the bargaining process.
- We now extend the bargaining phase by allowing the player with private information to make an initial offer. If rejected, the bargaining continues for one final round.
- For convenience we assume throughout this discussion that F(v) is uniform [0,1].

Solution when there are counteroffers

- The textbook analyzes solutions of the following type:
  1. There is a threshold valuation v* such that in the first found every manager with valuation v > v* offers the same wage w*, and every manager with valuation v < v* offers lower wages.
  2. In the first round the union rejects every offer below w*, and accepts all other offers.
  3. If the bargaining continues to the final round the union solves the first order condition for the one round problem using the valuations of the manager as truncated at v*.

Outcomes of two round bargaining game

- Note that if the probability of continuation is too high, management will not offer anything in the first round, because it would reveal too much about its own private value v.
- In this case the bargaining process stalls because management find it strategically beneficial to withhold information that can be used against them.

Designing the bargaining rules

- An implication of our studies on bargaining is the manifest value from setting the rules and conventions that determine how bargaining proceeds.
- Almost by definition managers are placed in a strong position to set the rules of bargaining games they play.
- In the remaining parts of this lecture we focus focus on upstream supply contracts, downstream consumer agreements, and employment contracts with labor.

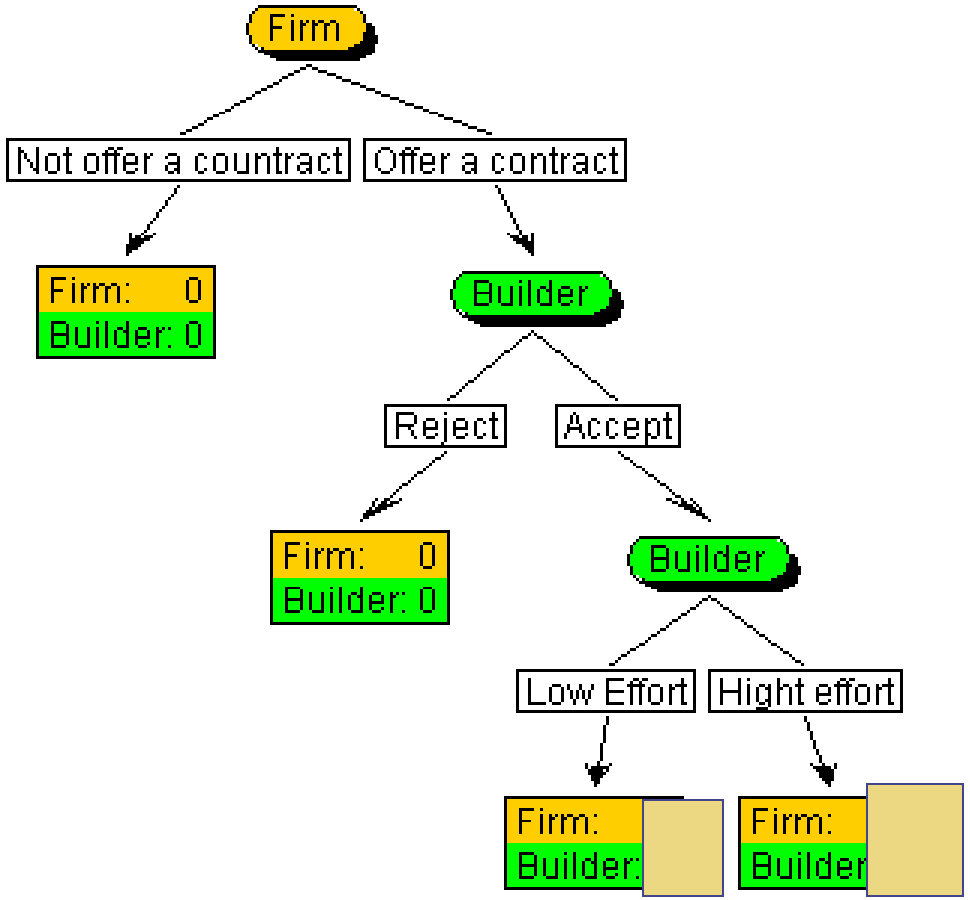
A rent extraction problem

- Employers seek to minimize their wage bill, or in the case of sole proprietors loss in expected utility, subject to two constraints:
    1. They must attract workers they wish to hire. This is called the participation constraint.
    2. The workers must perform the tasks to which they are assigned. This constraint is called incentive compatibility.

Full information principal agent problem

- A firm wishes to build a new factory, and will hire a builder.

  How should it structure the contract?



Firm: $R_L - w_L$ $R_H - w_H$

Builder: $w_L - u_L$ $w_H - u_H$

Constraints facing the firm

- We can use backwards induction to solve the problem:

1. The incentive compatibility constraint is:

$$w_H - u_H \geq w_L - u_L \text{ if } H$$

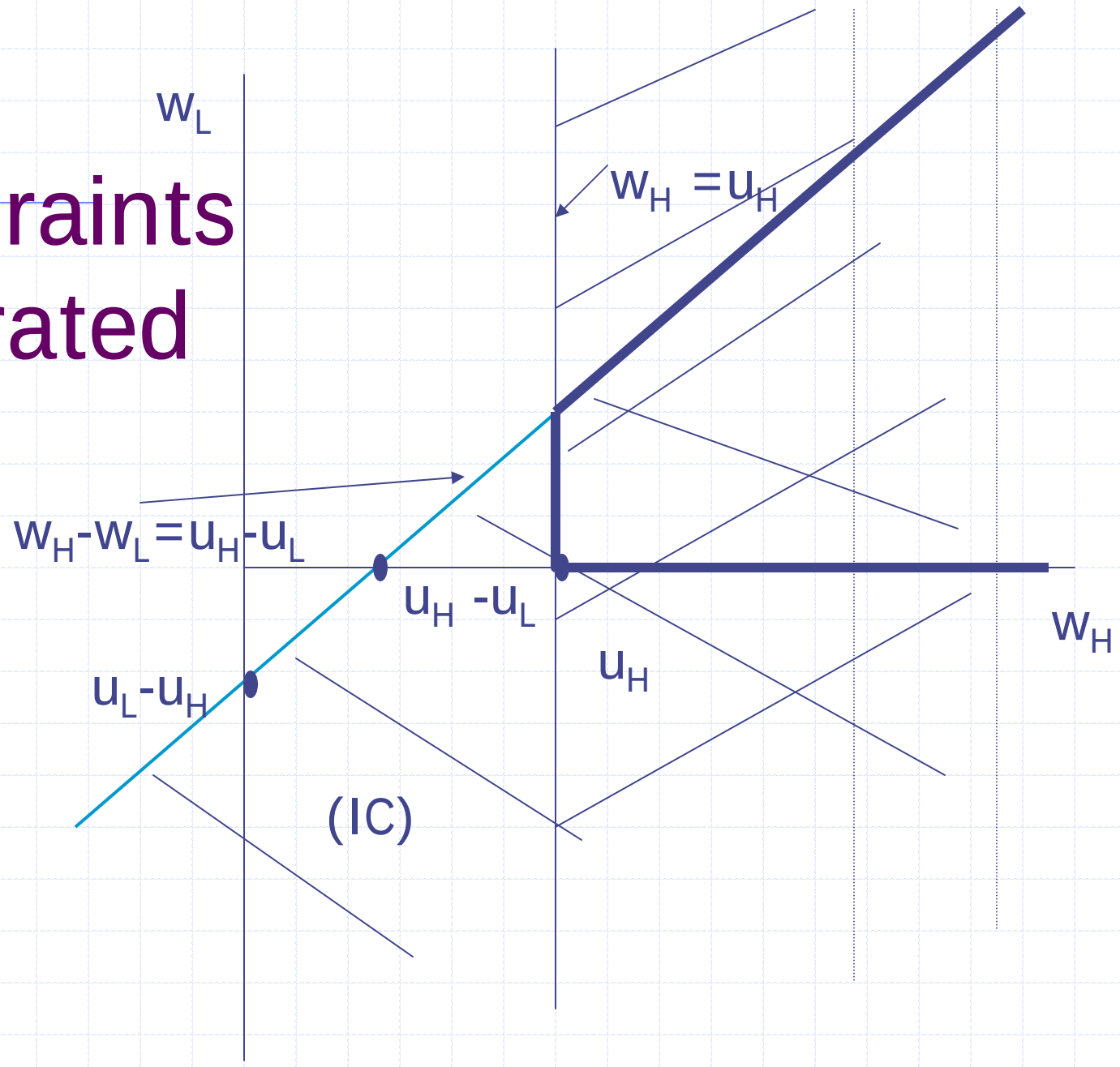
$$w_L - u_L \geq w_H - u_H \text{ if } L$$

2. The participation constraint is:

$$w_H - u_H \geq 0 \text{ if } H$$

$$w_L - u_L \geq 0 \text{ if } L$$

The constraints illustrated



Minimum cost of achieving L

- The minimum cost of achieving L is found by minimizing w_L such that:
  1. $w_L \geq u_L$
  2. $w_L - u_L \geq w_H - u_H$
- The first constraint bounds w_L from below by u_L.
- Since $u_L \geq u_H$ the second constraint is satisfied by not making the wage depend on effort.
- Therefore the minimum cost of achieving L is found by setting

$$w^* = u^*_L$$

Minimum cost of achieving H

- The minimum cost of achieving H is found by minimizing w_L such that:
  1. $w_H \geq u_H$
  2. $w_H - u_H \geq w_L - u_L$
- The first constraint bounds w_H from below by u_H.
- Since $u_L \geq u_H$ we must penalize the worker to deter him from choosing L, by setting:

$$w_L < w_H - u_H + u_L$$

- Therefore the minimum cost of achieving H is:

$$w^*_H = u^*_H$$

$$w^*_L = w_H - u_H + u_L - \text{Penalty}$$

Profit maximization

- The net profits from achieving L are

  $R_L - u_L^*$

- The net profits from achieving H are

  $R_H - u_H^*$

- Therefore the firm hires a worker to achieve H if

  $R_H - u_H^* > R_L - u_L^*$

  and hires a worker to achieve only L otherwise.

Service provider

- Multipart pricing schemes are commonly found in the telecommunications industry, amusement parks. sport clubs, and time sharing vacation houses and small jets.
- In this example a provider incurs a fixed cost of c_0 to connect the consumer to the facility, and a marginal cost of c_1 for every unit provided.
- It follows that if the consumer purchases x units the total cost to the provider is: $c_0 + c_1 x$.
- We assume the monetary benefit to the consumer from a service level of x is: $x^{1/2}$.
- How should the provider contract with the consumer?

Optimal contracting

- To derive the optimal contract, we proceed in two steps:
  1. derive the optimal level of service, by asking how much the consumer would use if she controlled the facility herself.
  2. calculate the equivalent monetary benefit of providing the optimal level of service to the consumer, and sell it to the consumer if this covers the total cost to the provider.
- The equivalent monetary benefit can be extracted two ways, as membership fee with rights to consume up to a maximal level, or in a two part pricing scheme, where the consumer pays for use at marginal cost, plus a joining fee.

A parameterization

- In our example we maximize

$$x^{1/2} - c_0 - c_1 x$$

with respect to x to obtain interior solution

$$x = (2c_1)^{-2}$$

- It follows that the costs from an interior solution are:

$$c_0 + 1/4c_1$$

and the monetary equivalent from consuming the optimal level of service is $1/2c_1$.

- Therefore the provider extracts $1/2c_1$ if:

$$4c_0c_1 < 1$$

Charging a uniform price

- If the service provider charges per unit instead, the consumer would respond by purchasing a level of service a a function of price.
- Anticipating the consumer's demand, the provider constructs the consumer's demand curve, and sets price where marginal revenue equals marginal cost.
- The provider serves the consumer if and only if the revenue from providing the service at this price exceeds the total cost.
- Since lower levels of service are provided, and since the consumer achieves a greater level of utility, than in the two part contract, the provider charging a unit price realizes less rent than in the two part contract.

The parameterization revisited

- In our example the consumer demands

$$x = (2p)^{-2}$$

where p is the uniform unit price of the service.

- The service provider maximizes:

$$x^{1/2}/2 - c_0 - c_1 x$$

with respect to x to obtain the interior solution

$$x = (4c_1)^{-2}$$

which is the optimal choice if:

$$16c_0c_1 < 1$$

Comparing multipart with uniform pricing schemes

- Since lower levels of service are provided, and since the consumer achieves a greater level of utility, than in the two part contract, the provider charging a unit price realizes less rent than in the two part contract.