Lecture 3
Employment Contracts

Last week we analyzed optimal contracting with upstream suppliers and downstream customers. Lectures 3 and 4 focus on labor contracts and the terms of employment. We discuss why firms typically present their workers with the terms of employment, rather than the other way around, and why contracts tend to be multifaceted. Then we begin our examination of uncertainty, beginning with an insurance agency problem, followed by discussion of start ups. Next week we shall discuss other dimensions of dealing with uncertainty.

Different types of firms

- The legal definition of a firm type differs from country to country and even across states within the U.S. Roughly speaking there are 3 kinds of firms:

1. Sole proprietorships: Unlimited liability up to provisions allowed within personal bankruptcy. No special tax provisions and accounting requirements are minimal.
2. Partnerships: Same as above. In addition there are agreements between partners about revenue sharing, cost sharing and workload.
3. Corporations: Limited liability of shareholders. Firms subject to corporation tax, dividends are also taxed, and more rigorous accounting protocols.

Number and size of firms

- There are about 14 million sole proprietorships and partnerships, and 4 million corporations in the U.S.
- About 1,500 corporations hold about 70 percent of assets of all U.S. non-financial corporations.
- G.M. (still) has a workforce about the same size as those of smaller US states and European countries.
- Microsoft has an operating income comparable to the GDPs of many countries, with matching capitalized asset values.

Management objectives

- As a first approximation, it is is useful to think that:
  1. Sole proprietors maximize their expected utility from the firm, that is taking account of their other life cycle considerations.
  2. Partners bargain with each other, each partner maximizing her expected utility.
  3. Shareholders collectively maximize the expected value of the corporations they own.

The size of firms and the wealth of individuals

- But assuming that people are risk neutral and that they have unlimited access to capital markets at a constant interest rate is unreasonable.
- It is impossible to hold the CEO of medium size firms fully accountable for the firm's returns. His own total personal wealth is only a tiny fraction of the value of the firm he manages!
- Indeed that is why capital markets exist.
- But what about small firms? Here raising large amounts of capital is not an issue, and information problems might be even more important.

Labor demand

- Just over 10% of the workforce are self employed.
- The remaining 90% of workers receive wages, tips and other compensation from their employers.
- Thus, most demand for labor comes from private firms (75%) and the government sector (15%).

Employment contracts

- The same principles apply to hiring a worker. For example let y denote the income the worker receives for her labor.
- Let h denote her hours of labor supplies to the firm if she is employed by the firm.
- Let A denote the worker's non-wage wealth, and assume the worker's utility function takes the form

$$\log(A + y) + k \log(24 - h)$$

where k is a positive constant that measures her willingness to trade off goods for leisure.

- We also assume that if she is not employed with the firm her utility level is v.

The firm's optimization problem

- Suppose firm profits are :

  $$ph - y$$

  where p is the output price, h is the output of the firm (which night employ the worker to provide a service) and y is the wage earnings of the worker

- The firm chooses h and w to maximize profits subject to the participation constraint that the worker chooses to be employed.

The Lagrangian formulation

- Let λ denote the Lagrange multiplier associated with this optimization problem.
- The firm maximizes:

$$ph - y + \lambda[\log(A + w) + k \log(24 - h) - v]$$

- Denote the solution to this optimization problem by (y^o, h^o). An interior solution satisfies two first order conditions and the participation constraint with equality. The interior solution is then checked against the boundary point of $h = 0$.

Solution to employee problem

- The interior solution to the firm's problem is

$$y^o = \frac{p}{k}(24 - h^o) - A$$

$$h^o = 24 - p^{\frac{-1}{1+k}}(Ak)^{\frac{1}{1+k}}24^{\frac{k}{1+k}}$$

and in this case it is easy to show it is also the global solution if A and/or k are small enough.

Sales commission: the worker chooses her hours

- An alternative method of payment is for the firm to pay its employee a commission, denoted by s, on her output.
- In this case the worker chooses h to maximize

  $\log(A + sh) + k[\log(24 - h)]$.

- This solution to this maximization problem is

$$h(s) = \begin{cases} \frac{24s - kA}{s(k+1)} \text{ if } \frac{s}{a} > \frac{k}{24} \\ 0 \text{ if } \frac{s}{A} > \frac{k}{24} \end{cases}$$

- The worker would prefer this arrangement since her utility typically exceeds v.

Sales commission: the firm chooses the commission

- Upon solving for h(s), the worker's supply of hours as function her commission, the firm chooses s to maximize:

$$(p - s)h(s)$$

- This solution to this maximization problem is found (numerically) by solving the first order condition to the firm's optimization problem:

$$(p-s)\frac{24}{s(k+1)} - \frac{(24s-kA)}{s^2(k+1)} = \frac{(24s-kA)}{s(k+1)}$$

- The total rent to both parties, and the firm's profits are lower under this scenario. However the firm still makes positive rents.

Freelance

- A third type of work contract is for the worker to approach the firm and propose an arrangement to the firm, which the firm can either accept or reject. This is quite close to outsourcing tasks that might have been undertaken within the firm.
- In this case the worker chooses both the payment y and hours or output h to maximize her utility

$$\log(A + y) + k \log(24 - h)$$

subject to the constraint that the firm accepts her proposal (does not make losses): $y \leqslant ph$

- The solution is almost identical to the employment contract problem, except that all the rent accrues to the worker.

Information relevant for contracting

- Note that the employment and sales commission contracts assume the employer:
  1. observes the alternative job or retirement opportunities of the employee
  2. knows how the employee values his leisure time, and the hardship of the job
  3. monitors the tasks undertaken by the employee on the job
- We have already relaxed the first assumption in our discussion of bargaining when there is incomplete information. Next week we relax the other two assumptions too.

The non-pecuniary value of work

- What happens when we relax the second assumption?
- Artists, writers, actors, researchers and professors, get considerable job satisfaction from their work, as well as being paid.
- If an employer knows how much job satisfaction his employees receive, he can offer a smaller wage subject to the participation constraint imposed by outside alternative employment opportunities.
- Thus professors of the same quality are typically paid more at weaker academic institutions than strong ones.

The value of leisure

- People also differ in the value they place on time off the job, that is leisure. It depends on:
    1. their household demographics (whether they live with a partner, whether the partner is employed, the number of children)
    2. interests outside work (such as time and energy consuming hobbies, such as sport participation)
    3. commuting time to and from work
- The more family attachments and demanding hobbies, the higher the value an employee places on leisure.
- Longer commutes reduce time left in the day, but may be selected by people who value their leisure less.

Some information can be verified

- Recruiters seeking to hire workers seek to extract the rents associated with their employees lifestyle, through lower wages and benefits.
- Similarly promotion and bonus schemes are sometimes designed to penalize those who have scheduling conflicts with outside interests.
- Eliciting information about the life outside the firm is a first step to extracting these rents.

Contracting under uncertainty

- Life is fraught with uncertainty:
  1. The benefits of human capital (schooling, on the job experience, children) are unpredictable.
  2. Personal health is another cause of great uncertainty. Insured can only be purchased against the most traumatic events (such as death and serious disability).
  3. Homeowners cannot usually diversify their housing assets without selling and renting.
  4. Entrepreneurs and small businessmen typically assume a lot of risk to their wealth.

Expected value maximization

- In 45-974 we took for granted that players were maximizing their expected value.
- Maximizing value is a useful assumption to start with, especially when thinking about the objectives of a publicly traded corporation. Shareholders:
  1. typically hold a small share in each company, and thus use the law of large numbers to reduce their exposure to risk
  2. can hold safer assets (such as bonds) if they choose. Consequently those with higher risk tolerance hold riskier portfolios, so the premium demanded for holding them is modest.

Evidence against value maximization

- But is value maximization a reasonable assumption in the situations facing individuals described above?
  1. The returns from (non-tradable) human capital are high relative to (tradable) physical capital.
  2. Homeowners (and drivers) partially insure their houses (and cars) at actuarially unfair rates .
  3. Individuals insure their health treatment costs at actuarially unfair rates.
  4. Entrepreneurs seek financial partners notwithstanding costs of the moral hazard and hidden information.

Expected utility maximization

- A less restrictive assumption than value maximization is that individuals maximize the weighted sum of utilities from each each outcome, where the weights of the respective probabilities.
- Utility, as a function in wealth is increasing, and if individuals are risk averse, concave.
- In our discussion of contracting under uncertainty or where there is incomplete information we shall now assume that the expected utility formation of preferences applies.
- We can, however, test that assumption, and using experimental methods, construct utility functions for anybody obeying the expected utility hypothesis.

Pooling independent risks

- We can apply the basic rent extraction principle to problems involving risk sharing.
- Risk that it is independently distributed across households is often pooled by insurance agencies.
- For example cars, houses and other property are often insured, as well as health (costs) and life (earnings for distribution to loved ones in the event of death).

Housing insurance

- We consider a housing insurance problem. Let h be the value of the house and p the probability it is destroyed. Suppose the value of other assets are a, let c denote the cost of the insurance premium, and let x denote the size of the insurance policy.
- The insurance company maximizes its expected value: $c - px$
- The home owner maximizes her expected utility, which is: $(1 - p)u(h - c) + pu(x - c)$

  where $u(h - c)$ is the utility from having a house worth h and paying a premium of c, while $u(x - c)$ is the utility from having a house worth x and paying a premium of c.

Optimal insurance contract

- We choose c and x to maximize c – px subject to a participation constraint that the contract is at least as good as the competitor's contract yielding an expected utility of v to the household.
- The first order conditions from the Lagrangian for this problem imply that: $u'(x - c) = u'(h - c)$

  where u'(h) is derivative of u(h) with respect to h, and x^o and c^o denote the optimal choices.
- Therefore full insurance in optimal, meaning $x^o = h$, and c is chosen to equate the expected utility of the household with its best alternative.

Start up firms

- By definition newly created firms are the brainchild of one individual or a very small group of coworkers.
- When seeking to sell their idea, or attract outside funding in return for partial ownership. they must:
  1. prove to potential buyers or investors that their project is valuable (hidden information)
  2. simultaneously protect their idea or invention from theft by rivals with a lower cost of capital or some other advantage in development (adverse selection)
  3. prove they are motivated to ensure the project's success (moral hazard).

Venture capital for startup firms

- While hard data are difficult to obtain, it seems that:
  1. Less than 5% of of new firms incorporated annually are financed by professionally managed venture capital pools.
  2. Venture capitalists are besieged with countless business plans from entrepreneurs seeking funding.
  3. A tiny percentage of founders seeking financing attract venture capital.

Low probability of success

- Most new firms fail within two years. That is, most entrepreneurs starting new firms use up their own time and wealth to no avail (apart from the experience itself).
- Of the remainder, many new firms reward their founders with much toil for only modest wages.
- If founders were rational, we could infer that a relatively small proportion of new firms prove extremely lucrative for their founders.
- That is, entrepreneurship entails a huge gamble with the founder's time, and sometimes his or her initial wealth, for the prospect of very large rewards.

Private information about a new venture

- Suppose the expected value of a risky project is **E[v] = u**, but only the entrepreneur knows this value, and that venture capitalists view **u** as a random variable.
- Our work on bargaining and contracts explains why it is hard for entrepreneurs have difficulty funding their projects. As we shall argue later, no self financing, efficient bargaining mechanism exists!
- Thus the entrepreneur sells the project for less than **u**, or owns some of the project himself, thus accepting the risks inherent in it.

Insiders

- Because raising outside funds is very costly, entrepreneurs might exchange shares in their projects for labor and capital inputs to known acquaintances, called insiders.
- Marriage, kinship and friendship are examples of relationships that lead to inside contacts.

Risk sharing

- The entrepreneur offers shares to **N** insiders.
- We label the share to the nth insider by $\mathbf{s_n}$ and the cost he incurs from becoming a partner by $\mathbf{c_n}$. Note that:

$$\sum_{n=1}^{N} s_n \leq 1, \; with \; s_n \geq 0$$

- The project that yields the net payoff of **x**, a random variable.
- Thus an insider accepting a share of $\mathbf{s_n}$ in the partnership gives up a certain $\mathbf{c_n}$ for a random payoff $\mathbf{s_n}\,\mathbf{x}$.
- The payoff to the entrepreneur is then:

$$x + \sum_{n=1}^{N} \left(c_n - s_n x \right)$$

The cost of joining the partnership

- We investigate two schemes.
  1. The entrepreneur makes each insider an ultimatum offer, demanding a fee of $\mathbf{c_n}$ for a share of $\mathbf{s_n}$. This pricing scheme is potentially nonlinear in quantity and discriminatory between partners.
  2. The entrepreneur sets a price **p** for a share in the firm, and the **N** insiders buy as many as they wish. (Note that it it not optimal for the entrepreneur to ration shares by under-pricing to create over-subscription.) In this case

$$\mathbf{c_n = p\, s_n}.$$

The merits of the two schemes

- The first scheme is more lucrative, since it encompasses the second, and offers many other options besides.
- However the first scheme might not be feasible:
    1. For example if trading of shares amongst insiders can trade or contract their shares with each other, then the solution to the first scheme would unravel.
    2. The first scheme may also be illegal (albeit difficult to enforce).

Two experiments

- In the experiments we will assume that the entrepreneur and the insiders have exponential utility functions.
- That is, for each **n = 0,1, . . . ,N**, given assets $\mathbf{a_n}$ the utility of the player **n** is:

$$u_n(a_n) = -\exp(-\gamma_n a_n)$$

where the entrepreneur is designated player 0.

- We also assume that **x** is drawn from a normal distribution with mean and variance:

$$(\mu, \sigma^2)$$

Solving the discriminatory pricing problem

- There are two steps:
  1. Derive the optimal risk sharing arrangement between the insiders and the entrepreneur. This determines the number of shares each insider holds.
  2. Extract the rent from each insider by a nonnegotiable offer for the shares determined in the first step.

Optimal diversification between the players

- For the case of exponential utility, the technical appendix shows that

$$s_n^o = s\left[\sum_{k=0}^{N} \frac{\gamma_n}{\gamma_k}\right]^{-1}$$

- The more risk averse the person, the less they are allocated. If everyone is equally risk averse, then everyone receives an equal share (including the entrepreneur).
- Notice that in this case the formula does not depend on the wealth of the insider.

Optimal offers

- For the case of exponential utility, the certainty equivalent of the random payoff $s_n x$ is:

$$s_n \mu - \frac{\gamma_n (\sigma s_n)^2}{2}$$

- The more risk averse the insider, and the higher the variance of the return, the greater the discounting from the mean return.

Solving the uniform pricing problem

- There are three steps:
  1. Solve the demand for shares that each insider would as a function of the share price.
  2. Find the aggregate demand for shares by summing up the individual demands.
  3. Substitute the aggregate demand function for shares into the entrepreneur's expected utility and optimize it with respect to price.

Demand for shares

- In the exponential case the demand for shares is

$$s_n(p) = \frac{\mu - p}{\gamma_n \sigma^2}$$

- Note that insider demand is
  1. increasing in the net benefit of mean return minus price per share,
  2. decreasing in risk aversion,
  3. and decreasing in the return of the variance of the return too.

Price and quantity sold

- The optimal (uniform) price for a share, and the total quantity sold are respectively:

$$p^o = \mu - \frac{\sigma^2 \gamma_0}{\left[1 + \sum_{n=0}^{N} \frac{\gamma_0}{\gamma_n}\right]} \qquad s(p^o) = 1 - \frac{2}{\left[1 + \sum_{n=0}^{N} \frac{\gamma_0}{\gamma_n}\right]}$$

- This discount from the mean return increases as the:
  1. variance of the return increases
  2. risk aversion of the insider partners and the entrepreneur increase.
- The total quantity of shares sold increases with the risk aversion of the entrepreneur but declines with the risk aversion of the insider partners.