

Lecture 4

Contracts

This lecture progress to more complicated environments, where the principal writing the contract knows less than the agents who sign it. This leads us to conclude with a statement of the revelation principle.

But first we review some of the results you generated on the entrepreneurial game!

Risk sharing

- ◆ The entrepreneur offers shares to **N** insiders.
- ◆ We label the share to the n^{th} insider by \mathbf{s}_n and the cost he incurs from becoming a partner by \mathbf{c}_n . Note that:
$$\sum_{n=1}^N s_n \leq 1, \text{ with } s_n \geq 0$$
- ◆ The project that yields the net payoff of \mathbf{x} , a random variable.
- ◆ Thus an insider accepting a share of \mathbf{s}_n in the partnership gives up a certain \mathbf{c}_n for a random payoff $\mathbf{s}_n \mathbf{x}$.
- ◆ The payoff to the entrepreneur is then:

$$x + \sum_{n=1}^N (c_n - s_n x)$$

The cost of joining the partnership

◆ We investigate two schemes.

1. The entrepreneur makes each insider an ultimatum offer, demanding a fee of c_n for a share of s_n . This pricing scheme is potentially nonlinear in quantity and discriminatory between partners.
2. The entrepreneur sets a price p for a share in the firm, and the N insiders buy as many as they wish. (Note that it is not optimal for the entrepreneur to ration shares by under-pricing to create over-subscription.) In this case

$$c_n = p s_n.$$

The merits of the two schemes

- ◆ The first scheme is more lucrative, since it encompasses the second, and offers many other options besides.
- ◆ However the first scheme might not be feasible:
 1. For example if trading of shares amongst insiders can trade or contract their shares with each other, then the solution to the first scheme would unravel.
 2. The first scheme may also be illegal (albeit difficult to enforce).

Two experiments

- ◆ In the experiments we will assume that the entrepreneur and the insiders have exponential utility functions.
- ◆ That is, for each $\mathbf{n} = 0, 1, \dots, \mathbf{N}$, given assets \mathbf{a}_n the utility of the player \mathbf{n} is:

$$u_n(a_n) = -\exp(-\gamma_n a_n)$$

where the entrepreneur is designated player 0.

- ◆ We also assume that \mathbf{x} is drawn from a normal distribution with mean and variance:

$$(\mu, \sigma^2)$$

Solving the discriminatory pricing problem

- ◆ There are two steps:
 1. Derive the optimal risk sharing arrangement between the insiders and the entrepreneur. This determines the number of shares each insider holds.
 2. Extract the rent from each insider by a nonnegotiable offer for the shares determined in the first step.

Optimal diversification between the players

- ◆ For the case of exponential utility, the technical appendix shows that

$$s_n^o = s \left[\sum_{k=0}^N \frac{\gamma_n}{\gamma_k} \right]^{-1}$$

- ◆ The more risk averse the person, the less they are allocated. If everyone is equally risk averse, then everyone receives an equal share (including the entrepreneur).
- ◆ Notice that in this case the formula does not depend on the wealth of the insider.

Optimal offers

- ◆ For the case of exponential utility, the certainty equivalent of the random payoff $s_n x$ is:

$$s_n \mu - \frac{\gamma_n (\sigma s_n)^2}{2}$$

- ◆ The more risk averse the insider, and the higher the variance of the return, the greater the discounting from the mean return.

Solving the uniform pricing problem

◆ There are three steps:

1. Solve the demand for shares that each insider would as a function of the share price.
2. Find the aggregate demand for shares by summing up the individual demands.
3. Substitute the aggregate demand function for shares into the entrepreneur's expected utility and optimize it with respect to price.

Demand for shares

- ◆ In the exponential case the demand for shares is

$$s_n(p) = \frac{\mu - p}{\gamma_n \sigma^2}$$

- ◆ Note that insider demand is

1. increasing in the net benefit of mean return minus price per share,
2. decreasing in risk aversion,
3. and decreasing in the return of the variance of the return too.

Price and quantity sold

- ◆ The optimal (uniform) price for a share, and the total quantity sold are respectively:

$$p^o = \mu - \frac{\sigma^2 \gamma_0}{\left[1 + \sum_{n=0}^N \frac{\gamma_0}{\gamma_n}\right]} \quad s(p^o) = 1 - \frac{2}{\left[1 + \sum_{n=0}^N \frac{\gamma_0}{\gamma_n}\right]}$$

- ◆ This discount from the mean return increases as the:
 1. variance of the return increases
 2. risk aversion of the insider partners and the entrepreneur increase.
- ◆ The total quantity of shares sold increases with the risk aversion of the entrepreneur but declines with the risk aversion of the insider partners.

Incomplete information

- ◆ As we indicated in the last lecture it is difficult to gather all information relevant for a contract that extracts the full rent from the worker.
- ◆ This leads us to investigate how firms should deal with contracts when their information on their employees is incomplete.
- ◆ We now consider a game between company headquarters and its research division, which has just made a discovery and is seeking to increase its budget so that it can proceed with product development.

Contracting with specialists

- ◆ Often the manager (or more generally the principal) knows less about the value of the work to the firm and its difficulty than the worker (or the agent).
- ◆ Professors are paid by students to explain what the important issues are, and how to analyze them.
- ◆ Medical doctors and specialists diagnose the illnesses.
- ◆ Strategic consultants evaluate firm performance.
- ◆ Building contractors know more about the job requirements than the the property owners.
- ◆ Auto service mechanics have the same kind of ambiguous relationship with their clients.

Research discoveries

- ◆ There are two types of discoveries, minor and major, denoted by $j = 1, 2$.
- ◆ Headquarters cannot observe whether a discovery has occurred, and if informed that there was a discovery does not know how to evaluate it.
- ◆ Given that a discovery has occurred, the probability of a minor one ($j = 1$), is p , and the probability of a major one ($j = 2$), is $1 - p$, where $0 < p < 1$.

Research funding policy

- ◆ The expected value of the new product line to the firm from a discovery is

$$f(x) - b$$

where x is an index of product development, and b is the research division's budget.

- ◆ We assume $f(x)$ is a concave increasing function and $f(0) = 0$.
- ◆ Headquarters forms a policy on funding product development, and then the research division announces whether it has made a discovery worthy of funding.

Product development

- ◆ After the policy formulation stage at headquarters, the division announces whether it has made a major or minor discovery.
- ◆ It does this by accepting a budget of b to develop new products to the level of x , netting divisional perks of

$$b - c_j(x)$$

where $c_j(x)$ is the total cost of product development.

- ◆ We assume $c_j(x)$ is convex increasing for each type j , that $c_j(0) = 0$, and that $c'_1(h) > c'_2(h)$ for all h .
- ◆ Alternatively the division acts as if there was no discovery. In this case its get its standard budget r .

Participation and incentive compatibility

- ◆ Define a research policy by (b_j, x_j) for each discovery type $j = 1$ and $j = 2$.
- ◆ If the division optimally accepts the budget allocation and development proposal designed for its discovery, then the following two constraints must be met:
 1. The participation constraint requires for each j :
$$b_j - c_j(x_j) \geq r$$
 2. Incentive compatibility constraint requires:
$$b_2 - c_2(x_2) \geq b_1 - c_2(x_1)$$
and vice versa.

Budget

- ◆ The participation constraint binds for the minor discovery ($j = 1$), but not for major ones. That is:

$$b_1 - c_1(x_1) = r$$

$$b_2 - c_2(x_2) \geq b_1 - c_2(x_1) > b_1 - c_1(x_1) \geq r$$

- ◆ For an optimal policy, the incentive compatibility constraint binds for major discoveries ($j = 2$), but not for minor ones. That is:

$$w_2 = w_1 + c_2(h_2) - c_2(h_1)$$

- ◆ This determines wages as a function of hours.

Product development

- ◆ Having derived the optimal budget as a function of product development, we now choose the latter for both types to maximize the expected value of the firm, which is:

$$\begin{aligned} & p[f(x_1) - c_1(x_1)] + (1 - p) [f(x_2) - c_2(x_2)] \\ &= p[f(x_1) - r - c_1(x_1)] \\ &\quad + (1 - p) [f(x_2) - r - c_2(x_2) + c_2(x_1) - c_1(x_1)] \end{aligned}$$

- ◆ The marginal benefit of product development for a major discovery is equated with its marginal cost.
- ◆ When $c'_1(h) > c'_2(h)$, the marginal benefit of hours for a minor discovery exceeds its marginal cost!

Ignoring minor discoveries

- ◆ Note that it is not necessarily optimal for the product lines to be developed. For example minor discovery will be ignored if

$$p[f'(0) - c'_1(0)] < (1 - p) [c_1(0) - c_2(0)]$$

- ◆ If the left side of this inequality is positive. we conclude that a fully informed headquarters should develop a minor discovery.
- ◆ But if headquarters is not privy to the discoveries by its research division, then it should not develop minor ones because it will induce the division to hide the significance of its major discoveries, or end up budgeting too much for them.

Linear costs and logarithmic benefits

- ◆ For example suppose that costs are linear:

$$c_j(x) = \delta_j x$$

- ◆ Also assume the expected discounted value of new product lines developed to specifications x are

$$f(x) = \log(1 + x)$$

- ◆ We will assume that it is profitable to develop major discoveries, which is true if and only if

$$\delta_2 < 1$$

Solution to the example

- ◆ The first order condition yields an interior solution for a major discovery of

$$x_2^o = \frac{1}{\delta_2} - 1$$

- ◆ A minor discovery is ignored if

$$\delta_1 + \frac{(1-\theta)}{\theta} (\delta_1 - \delta_2) \geq 1$$

- ◆ Otherwise

$$x_1^o = \frac{1}{\delta_1 + \theta^{-1}(1-\theta)(\delta_1 - \delta_2)} - 1$$

Moral hazard

- ◆ Moral hazard arises when the unobserved choices of one player affect but do not completely determine the payoff received by another person.
- ◆ Since the player's choice is not observed, a contract cannot direct him to make a particular choice.
- ◆ However linking the player's payments to the consequences of his action, can help align his incentives with those of the other players, even though the consequences are only partly attributable to or caused by the action itself.

Examples of moral hazard

- ◆ Managers are paid to make decisions on behalf of the shareholder interests they represent. If they were paid a flat rate, why would they pursue the objectives of shareholders?
- ◆ Lawyers representing clients are more likely to win if they are paid according to their record, and also whether they win the case in question or not.
- ◆ The extent of warranties against product defects is limited by the uses the product is put, and how much care is taken.
- ◆ Insurance against accidents discourages care.

Settling up at the end

- ◆ Sometimes the unobserved action can be inferred exactly at some later point in time.
- ◆ In this case a moral hazard problem does not exist, providing the contract period is sufficiently long.
- ◆ For example:
 1. If an air-conditioning unit is installed during winter, the guarantee should extend to the summer, so that the owner is compensated if the unit malfunctions during peak usage.
 2. Software engineers can be paid a wage if there are also penalty provisions for coding errors.

. . . but settle up as soon as possible

- ◆ Credit entries are financial assets.
- ◆ Since debtors have an incentive to evade their liability (through bankruptcy, flight or death), such assets typically have a low or negative, pro-cyclical rate of return.
- ◆ Therefore non-banking institutions typically shun long term credit positions with others, unless there is new information about past performance that should be incorporated into the contract.
- ◆ For example retirement plans might include stock options if the manager's current decisions can affect the stock price at some future date.

Managerial compensation

◆ Managerial compensation comes in the form of:

1. Cash and bonus
2. Stock and option grants
3. Abnormal return on stocks and options held by the manager
4. Pension and retirement benefits
5. Compensation for termination

A moral hazard problem

- ◆ To illustrate the nature of optimal contracting under moral hazard, we consider a wealth maximizing group of shareholders who contract with a risk averse CEO to manage their firm.
- ◆ The CEO has 3 choices He can:
 1. work for another firm ($j = 0$).
 2. accept employment with the shareholders' firm, but pursue his own interests rather than theirs ($j = 1$).
 3. accept employment with the shareholders' firm, and pursue their interests ($j = 2$).

Exponential Utility

- ◆ The utility function of the manager

$$u_j(w) \equiv -\alpha_j \exp(-\rho w)$$

- ◆ Given this utility function, the manager is willing to pay

$$w_2 - w_0 \geq \rho^{-1} \log\left(\frac{\alpha_2}{\alpha_0}\right)$$

for the luxury of shirking rather than working hard.

Signals about the managerial effort

- ◆ Suppose x is a signal about whether the manager is working diligently for the shareholders or not.
- ◆ That is the probability density function of x is $f_2(x)$ if the manager works “diligently”, but it is $f_1(x)$ if the manager “shirks”.
- ◆ We define the likelihood ratio of $f_1(x)$ and $f_2(x)$ as $g(x) = f_1(x)/f_2(x)$.
- ◆ If the shareholder observe the realization x^* then it is more likely that the manager shirked than worked diligently if $g(x^*) > 1$ and vice versa.

Optimal compensation

- ◆ To minimize the expected cost of compensation for hard work, the shareholders pay the manager :

$$w(x) = w_0 + \rho^{-1} \log\left(\frac{\alpha_2}{\alpha_0}\right) + \rho^{-1} \log\left[1 + \mu\left(\frac{\alpha_2}{\alpha_1}\right) - \mu g(x)\right]$$

- ◆ There are three components:
 1. The alternative wage at another firm
 2. The compensating differential that arises for the working conditions at the two firms
 3. A bonus/penalty system, with positive expected value, that depends on the signal through the likelihood ratio

An example

◆ Suppose

$$f_2(x) = 1/2 \text{ for } -1 \leq x \leq 1$$

$$f_1(x) = 3/4 \text{ for } -1 \leq x \leq 0 \text{ and } f_1(x) = 1/4 \text{ for } 0 \leq x \leq 1$$

$$g(x) = 3/2 \text{ for } -1 \leq x \leq 0 \text{ and } f_1(x) = 1/2 \text{ for } 0 \leq x \leq 1$$

◆ Then the optimal compensation to the manager is:

$$w(x) = w_0 + \rho^{-1} \log\left(\frac{\alpha_2}{\alpha_0}\right) + \begin{cases} \rho^{-1} \log\left[1 + \mu\left(\frac{\alpha_2}{\alpha_1}\right) - \frac{3\mu}{2}\right] & \text{if } x \in [-1, 0] \\ \rho^{-1} \log\left[1 + \mu\left(\frac{\alpha_2}{\alpha_1}\right) - \frac{\mu}{2}\right] & \text{if } x \in (0, 1] \end{cases}$$

where u is the positive value of:

$$\frac{2 \pm 2 \sqrt{1 + \alpha(3 - 2\alpha)/(2\alpha - 1)}}{(3 - 2\alpha)}$$

Importance of moral hazard

◆ There are three ways of measuring how important moral hazard is managerial compensation:

1. How much is a firm willing to pay to eliminate moral hazard?

$$\rho^{-1} E \left\{ \log \left[1 + \mu \left(\frac{\alpha_2}{\alpha_1} \right) - \mu g(x) \right] \right\}$$

2. How much are managers willing to pay to indulge in their objectives rather than the firm's?

$$\rho^{-1} \log \left(\frac{\alpha_2}{\alpha_0} \right)$$

3. How much would shareholders lose if they wrote contracts that ignore the moral hazard problem?

$$E \{ f_2(x) [1 - g(x)] \}$$