Lecture 6 Reputation

Reputations arise in situations where there is an element of repetition, and also where coordination between players is possible. One definition of leadership is that it facilitates this coordination. This lecture develop these concepts. We analyze repeated games, and games with multiple equilibrium, showing where there might be a role for leadership, and how reputations might be established and maintained.

Multiple equilibrium in repeated games with a finite number of rounds

- Last week we discussed what happens when there is a unique solution to a finitely repeated game, and the role for leadership in games where there are multiple equilibrium.
- What happens when there are several equilibrium in the kernel game? We will see that the number of solutions in the repeated game increase dramatically.
- This result clearly opens the door for leadership, but in this case following the leader's advice may lead to the players acquiring a reputation with each other as well.

Coordinated advertising

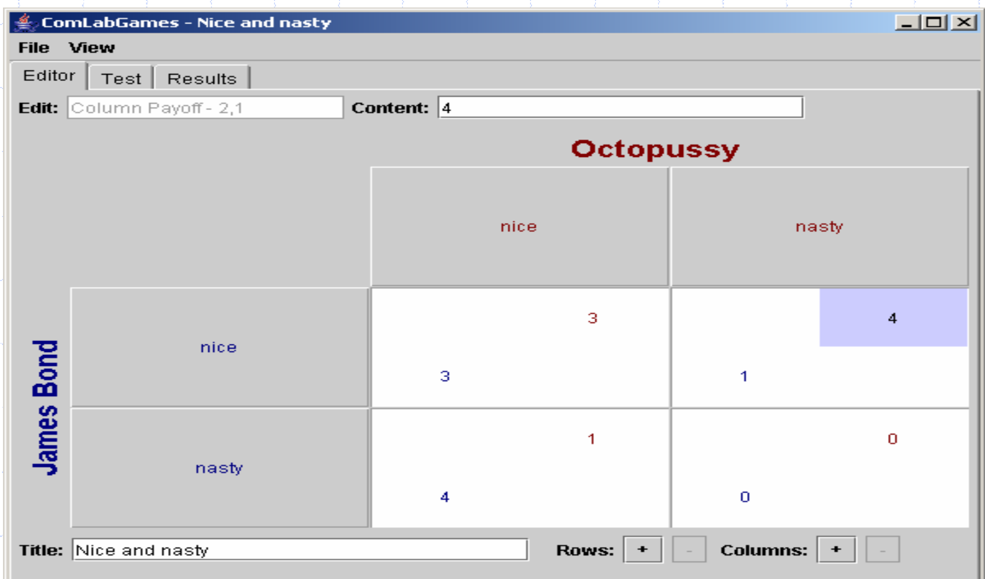
- Suppose there are only two firms in the industry, and that the size and loyalty of their customers depends on the nature of their advertising campaigns.

- Each firm simultaneously prepares an advertisement to run for one period. There are two types of advertising:
  1. Generic advertising (nice) increases the sales and net revenue of both firms.
  2. Differential advertising (nasty) increases the sales and net revenue of the differential advertiser at the expense of the other firm. Furthermore the net gain to the former is less than the net loss to the latter.

Recognizing possibilities for coordination

- In coffee break it was easy to identify the set of coordinated strategic profiles.

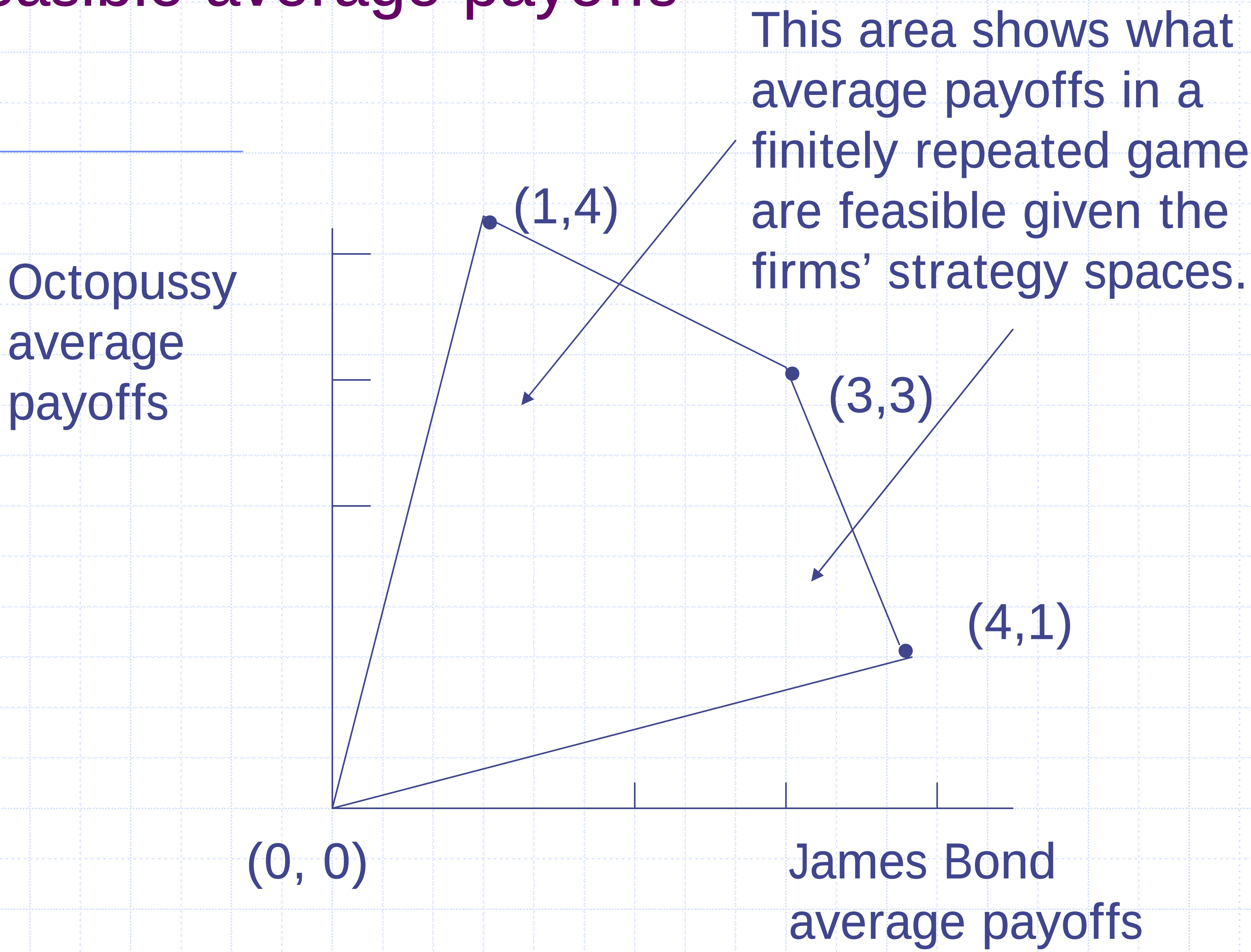
- But here it is not so obvious. Consider the following payoffs for the two firms, Bond and Octopussy.



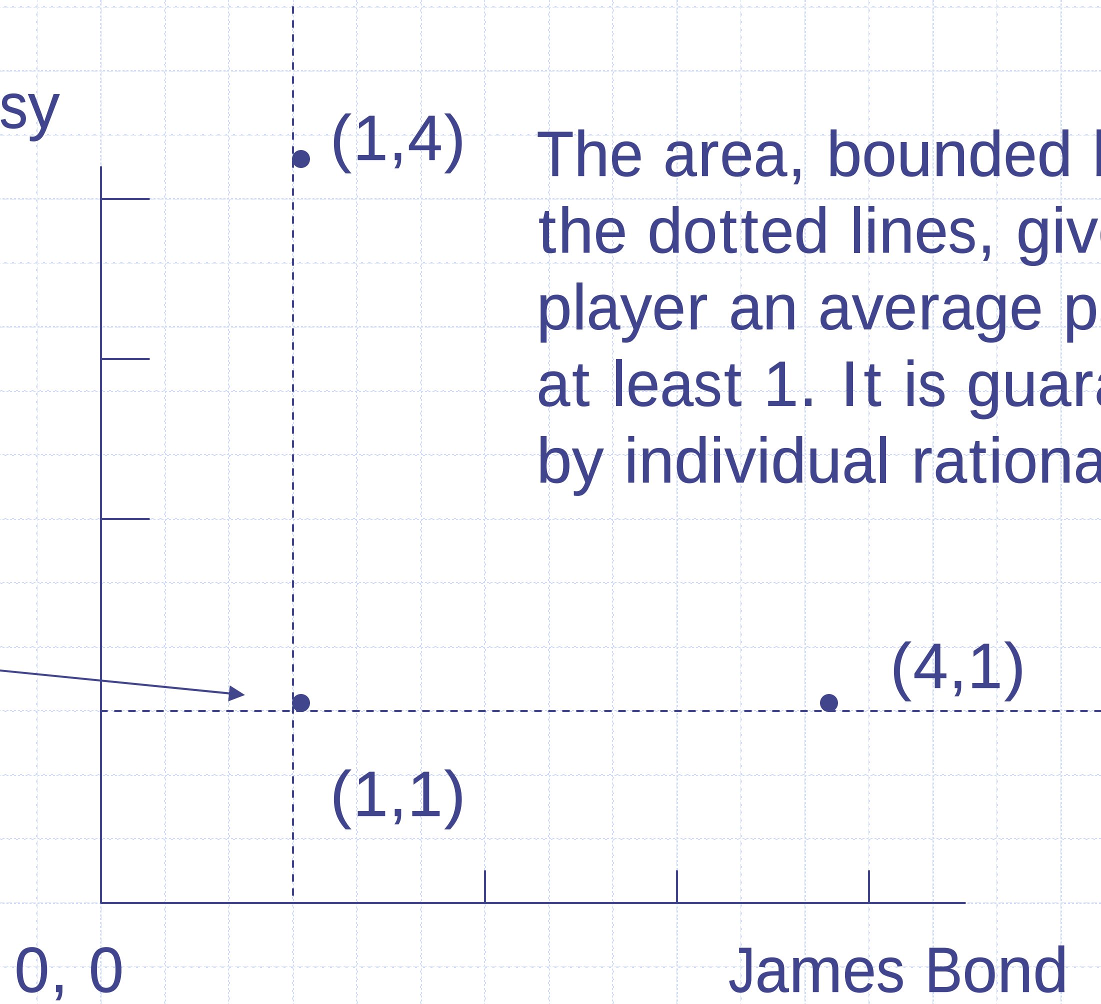
Some subgame perfect equilibrium paths

- (4,1), (4, 1)...   N times
- (1,4), (1,4)...   N times
- (4,1),(1,4)...   N times
- (3,3),(1,4),(4,1) (3 rounds)

Feasible average payoffs

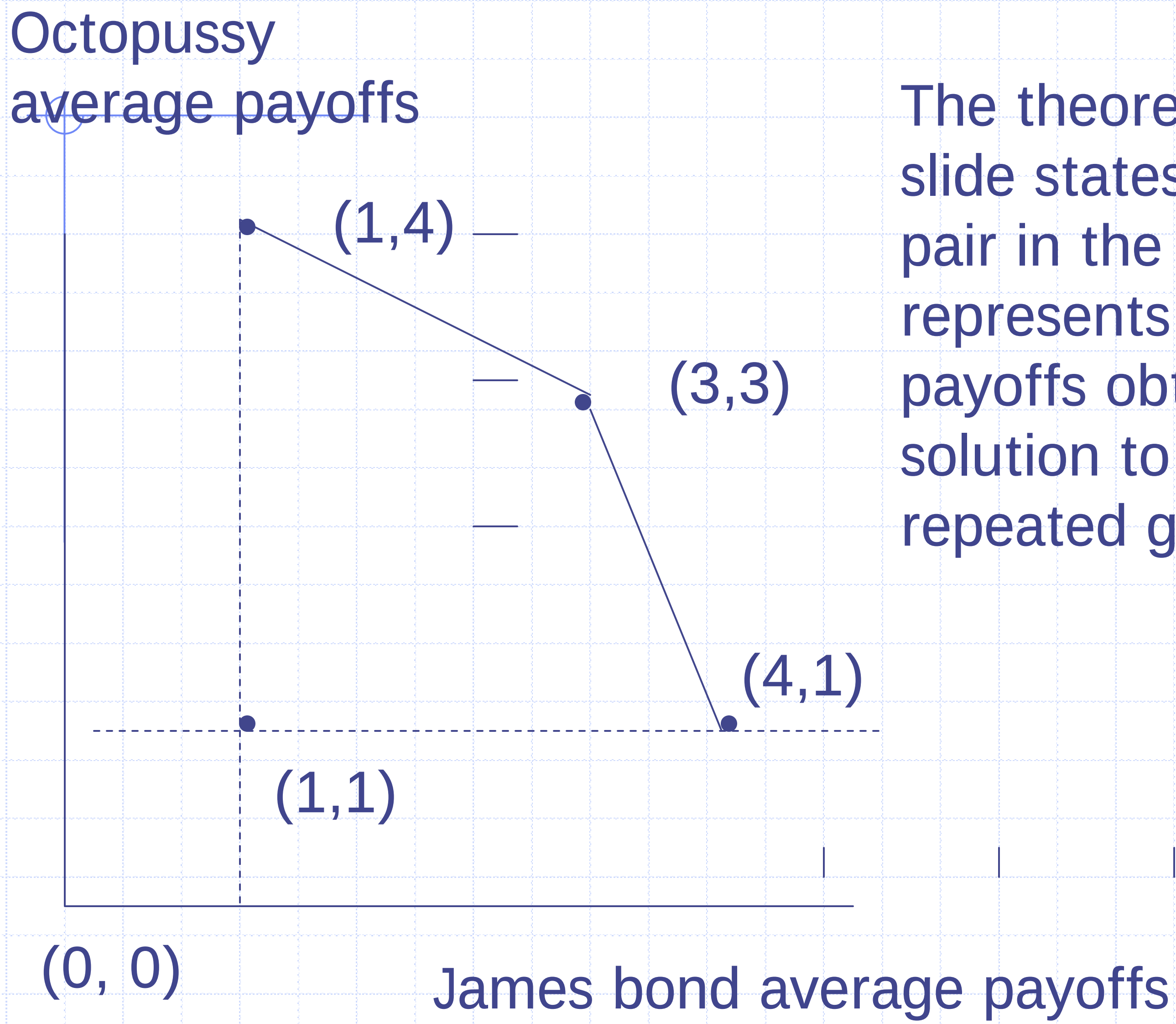


Individual rationality



The area, bounded below by the dotted lines, gives each player an average payoff of at least 1. It is guaranteed by individual rationality.

Average payoffs in equilibrium



The theorem in the next slide states that every pair in the enclosed area represents average payoffs obtained in a solution to the finitely repeated game.

Folk theorem

- Let w_1 be the worst payoff that player 1 receives in a solution to the one period kernel game, let w_2 be the worst payoff that player 2 receives in a solution to the one period kernel game, and define $w = (w_1, w_2)$

- In our example $w = (1,1)$

- Folk theorem for two players: Any point in the feasible set that has payoffs of at least w can be attained as an average payoff to the solution of a repeated game with a finite number of rounds.

Can Bond and Octopussy both earn more than 6 in a three period game?

- The outcome {(3,3), (1,4), (4,1)} comes from playing:

  $\{(\text{nice}_1, \text{nice}_1), (\text{nice}_2, \text{nasty}_2), (\text{nasty}_3, \text{nice}_3)\}$.

- Is this history the outcome of a solution strategy profile to the 3 period repeated game?

Strategy for Bond

Round 1: nice_1

Round 2: $(\ldots, \text{nice}_1) \rightarrow \text{nice}_2$

otherwise $\rightarrow \text{nasty}_2$

Round 3: $(\text{nasty}_1, \ldots) \rightarrow \text{nice}_3$

otherwise $\rightarrow \text{nasty}_3$

Bonds should be nice in the first round. If Octopussy is nice in the first round, Bond should be nice in the second round too. If Octopussy is nasty in the first round, Bond should be nasty in the second. Bond should be nasty in the final round, unless he was nasty in the first round.

Strategy for Octopussy

Round 1: $nice_1$

Round 2: $(\ldots, nasty_1) \rightarrow nice_2$

otherwise $\rightarrow nasty_2$

Round 3: $(nasty_1, \ldots) \rightarrow nasty_3$

otherwise $\rightarrow nice_3$

Octopussy should be nice in the first round. Then if she followed her script in the first round, she should be nasty in the second. However if she forgot her lines in the first round and was nasty, then she should be nice in the second round. If Bond has was nasty in the first round, Octopussy should be nasty in the final round, but nice otherwise.

Verifying this strategy profile is a solution

- Note that the last two periods of play, taken by themselves, are solutions to the kernel game, and therefore strategic form solutions for all sub-games starting in period 2.

- To check whether being nice is a best response for James bond given that Octopussy chooses according to her prescribed strategy we compare:

Checking for deviations by Bond in the first round

Compare

1.	$(nice_1, nice_1)$	3
2.	$(nice_2, nasty_2)$	1
3.	$(nasty_3, nice_3)$	4

		8

with

1.	$(nasty_1, nice_1)$	4
2.	$(nice_2, nasty_2)$	1
3.	$(nice_3, nasty_3)$	1

		6

Since 8 > 6 Bond does not profit from deviating in the first period. A similar result holds for Octopussy.

Therefore, by symmetry, the strategy profile is a SPNE.

Unforgiven

- What is the lowest sum of payoffs in the 9 period repeated game that can be supported by a SPNE?
- Consider the outcome of receiving (0,0) 5 periods followed by (1,4), (4,1), (1,4), (4,1) in the final 4 periods.
- It is induced by playing (nasty, nasty) 5 times followed by (nice, nasty), (nasty,nice), (nice,nasty) and (nasty,nice).
- Can this outcome be supported by a SPNE?

Unforgiven as a solution strategy

- Strategy for Eastwood:

  If Hackman plays deviates from profile prescribed in previous slide, play nasty for all the remaining slides. Otherwise follow prescribed strategy.

- Strategy for Hackman:

  If Eastwood plays deviates from profile prescribed in previous slide, play nasty for all the remaining slides. Otherwise follow prescribed strategy.

- Using the same methods as before one can show this is also a solution strategy profile for the three period game.

Checking for a solution

- More generally by punishing any deviation from the equilibrium path with the unfavorable kernel equilibrium repeated until the end of the game guarantees any payoff pair that averages more than the value given by individual rationality.

- This raises an interesting question about the wisdom of acquiring a reputation for threatening to destroy the business of rivals, and also why certain types of managers (and politicians . . . Winston Churchill?) are hired (empowered) at one time and retired (voted out) at other times.

Results from finitely repeated games

- To summarize:
  1. If the kernel game has a unique solution, then the solution to the repeated game is to play the solution of the kernel in each round.
  2. If a kernel game for two players has multiple solutions, then the area enclosed by the payoffs and the individual rationality constraints determines the set of average payoffs that can be attained.
  3. Leaders choose amongst multiple solutions to achieve coordination between players. The less the potential for coordination between players, the greater the rent that leaders can extract.

Infinite horizon repeated games

- Now we will analyze games that last indefinitely, continuing with some positive probability period after period.
- In this class of repeated games, the horizon is not fixed in advance at a finite number of rounds. Instead the game never ends, or the game ends with some probability after each round.
- We refer to both cases as infinite horizon repeated games.
- If the game lasts forever, payoffs in the future are discounted relative to the present. Otherwise it is hard to define the sum of total payoffs.

An expanded strategy space

- When players realize that their relationship does not have a foreseeable terminal node, new possibilities for cooperation and mutual benefit emerge.
- Cooperative behavior between group members can sometimes be enforced despite their individually conflicting objectives, by threatening to use strategies that punish actions that harm the collective interest.
- In this way we extend the results we found for the principal agent game on rent extraction, where they are multiple solutions to finitely repeated kernel games.

An example showing how play proceeds

		Column Player	
		ht	lt
Row Player	Ht	R R	D d
	Lt	d D	r r

If the game has lasted t rounds, at that time the Row Player picks H_t or L_t, and the Column Player simultaneously picks h_t or l_t. At the end of the period, players accumulate the payoff implied by their collective choices. Then a random variable determines whether play will continue at least one more period.

Discounting the future

- Now consider the following multi-period extension, in which the future payoffs are discounted by:

$$\beta = \frac{1}{1+i}$$

where i is the interest rate.

- The payoffs to the row player are now:

$$\sum_{t=0}^{T} \beta^t u_1(s_{1t}, s_{2t})$$

Unique equilibrium in the Kernel game

Suppose D > R and r > d.

	Column Player: ht	Column Player: lt
Row Player: Ht	R, R	d, D
Row Player: Lt	D, d	r, r

In the kernel (one period) game there is a unique Nash equilibrium, (L,l) which is dominance solvable.

Finite horizon case

The total payoffs to the column player at the end of the game may be expressed as:

$$\sum_{t=0}^{T} \beta^t u_2(s_{1t}, s_{2t})$$

where:

- there are T rounds or periods in the game;
- s_{1t} is the period t move of the row player (either H or L)
- s_{2t} is the period t move of the column player (either l or r)
- $u_2(s_{1t}, s_{2t})$ is the period t payoff to the column player evaluated in period t currency units
- B is the discount factor that gives the exchange rate between period t payoffs relative to payoffs in period 1

Solving finite horizon games

- The arguments we discussed for finite horizon games extends in a simple way to this class of games.

- If there is a unique solution to the kernel game(s), the solution to a finite round game formed from the kernel game(s) is to sequentially play the unique solution(s) of the composite kernel games.

- In this case the unique solution to this game is:

$$\{(L_t, l_t)\}_{t=0}^{T}$$

Infinite horizon case

- What happens if T = ∞?
- Or equivalently what happens when there is a positive probability at the end of each round that the game will continue one more round?
- As before, one possibility is:

$$\{(L_t, l_t)\}_{t=0}^{T}$$

Are there any other solutions?

- Consider the following strategy profile:

  - For the row player, in period u:

$$\{s_{it}, s_{2t}\}_{t=0}^{u-1} = \begin{Bmatrix} \{(H_t, h_t)\}_{t=0}^{u-1} \text{—>} H_t \\ otherwise \text{—>} L_t \end{Bmatrix}$$

  - For the column player, in period u:

$$\{s_{it}, s_{2t}\}_{t=0}^{u-1} = \begin{Bmatrix} \{(H_t, h_t)\}_{t=0}^{u-1} \text{-->} h_t \\ otherwise \quad \text{-->} l_t \end{Bmatrix}$$

Trigger strategies

- Each player picks the high (collusive) price, unless the player has evidence that either of them have cheated in the past, in which case they pick the low price.
- This is called a *"trigger strategy"* .

Is the trigger strategy profile a solution?

- To determine whether the trigger strategies are a solution, we only need to check whether the sub-games are solved by them.

- There are two kinds of sub-games, depending on whether somebody has cheated in the past or not.

The punishment phase

- Note that regardless of the history up until now the strategy profile:

$$\{s_{it}, s_{2t}\}_{t=u}^{\infty} = \{L_t, l_t\}_{t=u}^{\infty}$$

is subgame perfect.

- It immediately follows that if cheating has occurred at some point in the recent or distant past, it is a the subgame is solved by continuing the punishment phase forever.

The cooperative phase

- All that remains to check is whether whether playing (H_u, h_u) is a best response in period u if nobody has cheated up until now, and the history is

$$\{s_{it}, s_{2t}\}_{t=0}^{u-1} = \{H_t, h_t\}_{t=0}^{u-1}$$

Cooperating

Using the formula for summing a geometric series, that says:

$$\sum_{t=0}^{\infty} \beta^t = 1/(1-\beta)$$

we obtain the value of continuing to cooperate by charging the high price:

$$\sum_{t=u}^{\infty} \beta^t R = R\beta^u / (1-\beta)$$

Defecting

- Now consider the value of defecting by charging a low price in the current period. Since the other player charges a high price the payoff this period is D. But from next period onwards, both players will charge the low price because the punishment phase will begin (and never end). In this case the player gets:

$$\beta^{u} D + \sum_{t=u+1}^{\infty} \beta^{t} r$$

$$= \beta^{u} D + \beta^{u+1} r / (1 - \beta)$$

Which is more profitable?

Taking the difference cooperation can be sustained as a solution to this repeated game if the expression:

$$\beta R^{u} / (1-\beta) - \beta^{u} D - \beta^{u+1} r / (1-\beta)$$

$$= \beta R^{u} + \beta R^{u+1} / (1-\beta) - \beta^{u} D - \beta^{u+1} r / (1-\beta)$$

$$= [(R-D) + (R-r)\beta /(1-\beta)]\beta^{u}$$

is positive.

A further simplification

- Thus cooperation can only occur if the punishment from deviating offsets its immediate gain:

Punishment from deviating

Immediate gain from deviating

$$\frac{\beta(R-r)}{1-\beta} > D-R$$

or

$$\beta/(1-\beta) > (D-R)/(R-r)$$

A numerical example

In this case:

$R = 10$

$D = 20$

$r = 5$

Thus:

$D - R = 10$

$R - r = 5$

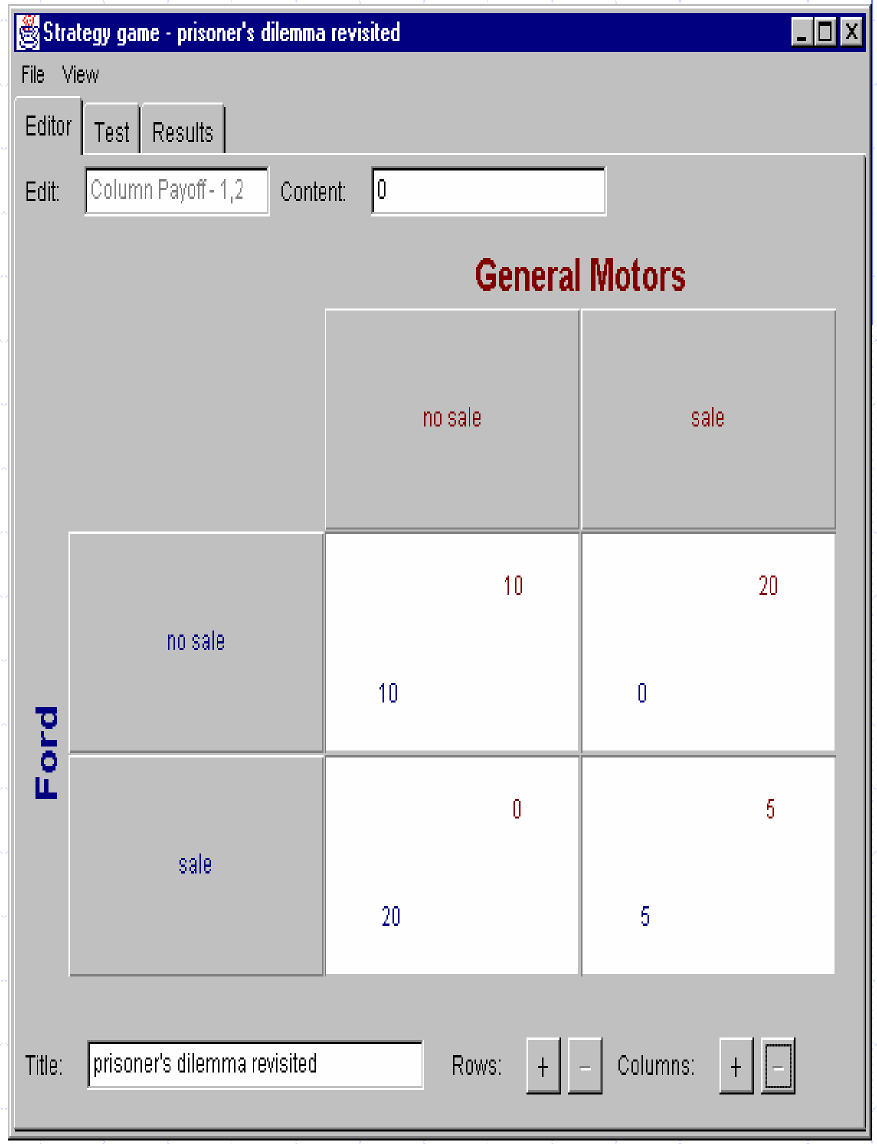
And therefore:

$(D - R)/(R - r) = 2$

So if:

$B/(1-B) > 2$

cooperation at the non-sale price can be sustained as a solution to this repeated game.



Factors determining cooperation

- Our discussion highlights three factors that determine whether a relationship between strategic partners is cooperative or adversarial.
- To recapitulate, the three factors are :
    1. The gains of maintaining, compared to the benefit of defecting from, a cooperative arrangement
    2. The losses from destroying a cooperative arrangement and reverting to an adversarial relationship
    3. The duration of the relationship measured in discounted time (to reflect the probability of survival and the interest rate).

Stable relationships

- In the applications above, one of two possibilities emerges:
  1. Cooperation is maintained for the whole game, and the credible punishment threat is never administered.
  2. Cooperation is never achieved.
- We can extend this result. Suppose the three factors described above are known for all future periods. Then:
  1. Either cooperation is established at some point and maintained thereafter.
  2. or cooperation is never established.
- Cooperation never breaks down when all the factors are known in advance and the players are rational.

Volatile relationships

- There are several reasons why cooperative relationships can break down or revert to adversarial confrontations:
  1. When the activities of one or more players cannot be monitored
  2. When one or more of the three factors described above is a time dependent stochastic process
  3. If some of the payoff relevant information to one player is hidden from the other one.
- We conclude this lecture with experiments that illustrate these scenarios.

Results from infinitely repeated games

- If a kernel game is uniquely solved, there is a unique solution to a game that repeats the kernel a finite number of times. However there may be multiple solutions if the kernel is repeated indefinitely.
- Opportunities for coordination depend on the payoff parameters and the probability of repetition (or the discount factor.)
- In a trigger strategy solution, playing what would be the best reply in the kernel yields less than the long term benefits obtained by cooperating with the other players.