Lecture 1 in Competitive Equilibrium
General Equilibrium

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Defining competitive equilibrium

Introduction

- A competitive equilibrium is a price, or a price vector, satisfying the condition that, when traders maximize their utility, or net gain, by optimally choosing their preferred buy and sell quantities at that price, the market clears.
- It is worth emphasizing that a competitive equilibrium price (vector) has two key features:
  1. Individual traders believe the terms of trade are beyond their control. They simply optimize by choosing the quantity they wish to trade at the given price.
  2. When all traders behave that way, there is no unplanned inventory or unfilled orders.

- Note the definition does not say how the competitive equilibrium price is formed, why every trader believes he or she cannot influence the price, or even whether a competitive equilibrium price exists.
- In this course on market stategy we ask:
    1. when is it reasonable to use competitve equilibrium as a tool for prediction?
    2. how should we analyze markets if competitive equilibrium does not provide much helpful guidance?

- Consider the supply and demand for holiday accommodation at a beach resort.
- Demand for accommodation by tourists range from one to three room units.
- Suppose there is a variable number of units avaliable
- What is (set of) the competitive equilibrium price(s)?
- What are the competitive equilibrium allocations? In other words what production and trade is predicted in competitive equilibrium?
- How much do the trading prices from limit order market outocmes differ from the competitive equilibrium?
- How closely do the equilibrium allocations of tourists and hoteliers resemble the competitive equilibrium allocations?

Production Sector

Consider the following example of the production side of the economy. There are three industries, housing construction, brickworks and forestry. The value of houses built by a firm in the industry is

$$p_h x_h^{1/3} y_h^{1/3}$$

where x_d is the amount of wood used, y_d is the number of bricks used, and p_h is the price of housing in terms of labor units.
Thus p_h^{-1} is the real wage in terms of housing units. Labor used in producing lumber is

$$x_f^2$$

and labor used in producing a bricks is

$$y_b^2$$

To summarize, there are two final goods in this economy, free time or leisure and housing, plus two intermediate goods, bricks and wood.

Profit maximization in building

- The firm supply of bricks and timber can be derived as follows. Profits to the brickyard in terms of labor units are

$$p_x x_b - x_b^2$$

Optimizing we obtain the first order condition

$$p_x = 2x_b$$

Hence the logarithm of a brickyard's supplies is

$$\log x_b = \log p_x - \log 2$$

Similarly the profits of a sawmill are

$$p_y y_f - y_f^2$$

from which we obtain

$$\log y_f = \log p_y - \log 2$$

Profit maximization in real estate

Let p_x denote the price of lumber in terms of a housing unit, and p_y denote the price of bricks in terms of a labor unit.
Then the value of a real estate development firm is

$$p_h x_h^{1/3} y_h^{1/3} - p_x x_h - p_y y_h$$

Taking derivatives to obtain the two first order conditions we obtain

$$\frac{1}{3} p_h x_h^{-2/3} y_h^{1/3} = p_x$$

In terms of logarithm

$$\log p_h - \frac{2}{3} \log x_h + \frac{1}{3} \log y_h - \log 3 = \log p_x$$

Similarly

$$\log p_h + \frac{1}{3} \log x_h - \frac{2}{3} \log y_h - \log 3 = \log p_y$$

Demand for primary inputs

Subtracting one equation from the other yields

$$\log y_h = \log p_x + \log x_h - \log p_y$$

Substituting for $\log y_h$ in the logged first order condition for x then gives us the (logarithm of the) demand for wood by a housing construction firm (as a function of input and output prices)

$$\log x_h = 3\log p_h - 2\log p_x - \log p_y - 3\log 3$$

By symmetry

$$\log y_h = 3\log p_h - 2\log p_y - \log p_x - 3\log 3$$

Competitive equilibrium

A competitive equilibrium is defined as a price vector (p_x, p_y), such that all the players maximize their profits at the competitive equilibrium prices and the aggregate supply of lumber and bricks is offset by the aggregate demand.
We suppose there are 4 builders, 8 brickworkers, and 12 lumberjacks, so in this case:

$$\begin{aligned} 4x_h &= 8x_b \\ &\Longrightarrow \log x_h = \log x_b + \log 2 \end{aligned}$$

and

$$\begin{aligned} 4y_h &= 12y_f \\ &\Longrightarrow \log y_h = \log y_f + \log 3 \end{aligned}$$

Equating the demand and supply of raw materials

To solve for the competitive equilibrium prices (p_x, p_y) we substitute for (x_h, x_b, y_h, y_f) in the logged demand and supply equations derived above to obtain

$$3 \log p_h - 2 \log p_x - \log p_y - 3 \log 3 = \log p_x$$

$$3 \log p_h - 2 \log p_y - \log p_x - 3 \log 3 = \log p_y - \log 2 + \log 3$$

Collecting terms

$$3 \log p_h - \log p_y - 3 \log 3 = 3 \log p_x$$

$$3 \log p_h - \log p_x - 4 \log 3 = 3 \log p_y - \log 2$$

Solving for p_x and p_y in terms of p_h using these two equations yields the equilibrium price of the raw materials as a function of the price of final goods.

Solving for the price of lumber

From the previous slide

$$\begin{aligned} 9 \log p_h - 3 \log p_y - 9 \log 3 &= 9 \log p_x \\ 3 \log p_y - \log 2 &= 3 \log p_h - \log p_x - 4 \log 3 \end{aligned}$$

Adding one equation to the other yields

$$9 \log p_h - 9 \log 3 - \log 2 = 8 \log p_x + 3 \log p_h - 4 \log 3$$

or

$$8 \log p_x = 6 \log p_h - \log 2 - 5 \log 3$$

Solving for the price of bricks

Also

$$\begin{aligned} 3\log p_h - \log p_y - 3\log 3 &= 3\log p_x \\ 9\log p_y - 3\log 2 &= 9\log p_h - 3\log p_x - 12\log 3 \end{aligned}$$

Adding the two equations together

$$3\log p_h + 8\log p_y - 3\log 3 - 3\log 2 = 9\log p_h - 12\log 3$$

or

$$8\log p_y = 6\log p_h + 3\log 2 - 9\log 3$$

Solving for competitive equilibrium

Having found $\log p_x$ and $\log p_y$ in terms of $\log p_h$ we substitute these equations into the supply equations for (x_b, y_f) and obtain the equilibrium quantities of the housing inputs, and hence, through the housing production function, housing itself, in terms of the housing price p_h. Equating a demand for housing equation (unspecified here) with the supply of housing equation would then solve out for the final goods market as well.