Lecture 2 in Competitive Equilibrium
Product Differentiation

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- The applications we have considered presuppose all the product or service units bought and sold within a market are identical.
- However the concept of competitive equilibrium can also be applied to markets where each supplier sells a uniquely differentiated product, and every demander value each differentiated product differently.
- In this lecture we derive the competitive equilibrium in a differentiated product market and analyzes its theoretical and experimental properties.

- The residential housing market is a product spectrum where no two houses are alike, and practically all demanders have special or idiosyncratic desires.
- Teams within a professional league put different values on any given player joining the league, depending on their current stock of players and their fan base, and each player brings a unique combination of skills to the league and the team he joins.
- Tourist attractions compete for the holiday maker's time, and specialized food and beverage products, often fall into this category, offering slightly different menus to cater to heterogeneous tastes.
- How are a pool of MBA graduates assigned to companies as employees?
- How are partners matched up across different law firms?

Some Notation

Wine Market

- Consider a market setting where there are I demanders and J suppliers.
- The product or service supplied by producer $j \in \{1, \ldots, J\}$ is indexed by a characteristic w_j, which we assume is a positive real number.
- Similarly demanders $i \in \{1, \ldots, I\}$ are distinguished by another characteristic denoted v_i, also a positive real number.
- For convenience producers are consumers are ranked by their respective characteristics from the lowest to the highest. Thus $v_i \leq v_{i+1}$ for all $i \in \{1, \ldots, I-1\}$ and $w_j \leq w_{j+1}$ for all $j \in \{1, \ldots, J-1\}$.
- The is a function of their characteristics, which we denote by $f(v_i, w_j)$.
- Thus if the i^{th} demander buys a service unit from the j^{th} supplier at price p_{ij}, then the demander attains a utility of $f(v_i, w_j) - p_{ij}$ and the supplier receives p_{ij}.

- The wine industry is also a good example of a product that is differentiated across suppliers and demanders.
- The following application analyzes the market for wine, an industry where production units are imperfect substitutes for each other in consumption.
- In deriving the competitive equilibrium price, we exploit the fact that it supports an efficient allocation of resources.
- That is, a competitive equilibrium yields an allocation that maximizes the potential gains from trade, that is sum of producer and consumer surplus.

Consumer Tastes and Product Quality

Wine Market

- Suppose the amount a person is willing to pay for a bottle of wine depends his tastes v, and let $F(v)$ denote a probability distribution function that characterizes wines tastes across the population.
- We assume $F(v)$ is strictly increasing (meaning that everyone has different tastes),
- Denote the quality of the wine w, and let $G(w)$ denote the probability distribution function characterizing wine quality supplied by wineries.
- Denote the reservation price of a consumer by the function $u(v, w)$.
- We assume $u(v, w)$ is increasing in both arguments; wine connoisseurs have higher values of v than beer drinkers and higher quality wines are associated with higher values of w.

Positive Assortive Mating

Wine Market

- We also assume there is positive assortive mating, that is for all $v_2 > v_1$ and $w_2 > w_1$:

$$u(v_2, w_2) + u(v_1, w_1) > u(v_2, w_1) + u(v_1, w_2)$$

- Positive assortive mating is satisfied if and only if the cross partial derivative of $u(v, w)$ is everywhere positive:

$$\begin{aligned} 0 &< u(v_2, w_2) - u(v_2, w_1) + u(v_1, w_1) - u(v_1, w_2) \\ &= \int_{w_1}^{w_2} u_2(v_2, w)\, dw - \int_{w_1}^{w_2} u_2(v_1, w)\, dw \\ &= \int_{v_1}^{v_2} \int_{w_1}^{w_2} u_{12}(v_2, w)\, dwdv \end{aligned}$$

- Under this assumption Pareto optimal allocations exhibit positive assortive matching.
- Those who like wine the most drink the highest quality wine if trading is efficient.

The Distribution of Tastes and Quality

Wine Market

- Positive assortive matching means that when people are ordered by the quality of wine they drink, the same ranking emerges as when they are ordered by their taste for wine.
- Denote by v_n the taste of the n^{th} consumer and by w_n the quality of the wine he would consumes.
- Then positive assortive mating is formally defined by the equation $F(v_n) = G(w_n)$.
- Since $F(v_n)$ is strictly monotone increasing it has an inverse $F^{-1}(x)$ for all $x \in (0, 1)$.
- Therefore wine of quality w' is bought by a consumer with taste $v' = F^{-1}[G(w')]$ in competitive equilibrium if the supplier of w' can make a profit.

Who drinks the worst wine and how much does he pay?

Wine Market

- Suppose the cost of growing vines and making wine does not depend on its quality, and is constant at c.
- Competitive pressure implies that the lowest quality wine offered to the market will be priced at cost c, and sold to the consumer who has the lowest valuation of all wine drinkers.
- Let $\underline{v}$ denote the threshold and the highest valuation amongst the the lowest by the cheapest wine will and that the quality of the wine. Similarly let $\underline{w}$ denote the lowest quality wine sold.
- Then $u(\underline{v}, \underline{w}) = c$.
- Since $F(\underline{v}) = G(\underline{w})$ we can solve for $\underline{w}$ from the equation

$$u\left(F^{-1}\left[G\left(\underline{w}\right)\right], \underline{w}\right) = c$$

Solving for Competitive Equilibrium

Wine Market

- Now consider the n^{th} consumer with valuation $v_n > \underline{v}$ reviewing the wine list $p(w)$, which indicates bottle price as a function of quality.
- He chooses w_n to maximize

$$u(v_n, w) - p(w)$$

- The first order condition for this optimization problem is

$$p'(w_n) = u_2(v_n, w_n)$$

- Substituting for $v_n = F^{-1}[G(w_n)]$ we obtain an equation for the price quality wine gradient

$$p'(w) = u_2\left(F^{-1}[G(w_n)], w\right)$$

- Integrating up from $\underline{w}$ we obtain

$$p(w) = c + \int_{\underline{w}}^{w} p'(x)\, dx = c + \int_{\underline{w}}^{w} u_2\left(F^{-1}[G(x)], x\right) dx$$

An Example

Wine Market

- Suppose tastes are uniformly distributed between 10 and 20.

$$F(v) = \frac{v - 10}{10}$$

- Suppose quality is uniformly distributed between 0 and 10.

$$G(w) = \frac{w}{10}$$

- Finally assume utility is the product of tastes and quality

$$u(v, w) = vw$$

- Then

$$u_2(v, w) = v$$

  and

$$v(p) \equiv F^{-1}(v) = 10 + 10p$$

- Substituting these expressions into the price equation and integrating to get the mark up in equilibrium

$$\begin{aligned} p(w) - c &= \int_{\underline{w}}^{w} u_2 \left(F^{-1} \left[G(x) \right], x \right) dx \\ &= \int_{\underline{w}}^{w} F^{-1} \left[G(x) \right] dx \\ &= \int_{\underline{w}}^{w} F^{-1} \left[\frac{x}{10} \right] dx \\ &= \int_{\underline{w}}^{w} (10 + x)\, dx \\ &= 10w + w^2/2 \end{aligned}$$