Optimal Voting Turnouts

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I. Introduction

There is an extensive and far-ranging literature concerned with positive aspects of election turnouts, dealing with such questions as the number of people who vote, the composition of the voting population, whether it is narrowly 'rational' for those who vote to do so, and so on.[1] The normative properties of election turnouts and alternative institutional arrangements for influencing them have, by contrast, been largely neglected. A notable exception to this neglect is Gordon Tullock's recent sequence of papers, and Gärtner's associated comments on optimal poll taxes [Tullock, 1975, 1976 and 1977; see also Gärtner, 1976 and 1977]. Tullock's general conclusion is that turnouts should be restricted below the levels that would obtain if elections were financed out of general revenue, and hence that some poll tax is required.

The arguments which Tullock uses to support this conclusion are two fold. First, the voting process itself is costly in terms of the provision of polling booths, ballot papers, vote-counting machines, scrutineers and so on—and presumably, having greater numbers from the enfranchised population exercising their voting rights will increase this cost. At the optimal turnout, the net benefits from an extra individual's exercise of his vote must exactly offset the incremental processing cost.

Second, Tullock argues, since any individual's political "power," defined as the probability that his vote will be decisive, decreases with vote turnout, all voters are imposing a form of congestion cost on all others. When deciding whether to vote or not, an enfranchised person does not incorporate within his calculus the reduction in political power his voting would impose on all other voters. Thus, a negative vote-externality is generated.

The first of these arguments—that the cost of the voting process should influence optimal turnout—seems unexceptionable. The second we believe to be essentially misconceived, and to spring from a failure to place the discussion in the appropriate analytic setting. As Tullock himself emphasizes in another place decisions concerning alternative political institutions, including specifically those relating to voting rules, are most suitably treated as 'constitutional' questions, the answers to which emerge from the rational calculus of individuals in some sort of original position behind a 'veil of ignorance' [Buchanan and Tullock, 1962]. Thus, every individual may like best a situation in which he himself is dictator—yet have no liking at all for the institution of dictatorship *per se*, as evidenced by the fact that if the identity of the dictator is unknown each will exhibit a preference for some alternative political arrangement. Likewise, each individual may prefer the situation where he is the only voter (or one of a smaller number of voters), yet prefer an institutional setting in which the number of voters is large. It seems persuasive to us to argue that it is the choice between alternative *institutions*, rather than alternative situations for a particular in-period referendum, which constitutes the appropriate analytic base from which to assess the desirability, or otherwise, of different turnout levels and policies designed to

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[1] Some of this literature is cited in Dennis Mueller's recent survey of the public choice literature. See Dennis Mueller, 1976.

influence them.

Accordingly, the aim of this paper is to derive optimal turnouts, and optimal poll taxes/subsidies, from within the appropriate constitutional setting. In order to do so a very simple model of political decision-making is utilized to examine the effects of changing turnout levels on the expected utility of some typical voter-taxpayer, when he does not know either his particular preferences for specific public decisions in future periods, nor the preferences of those who do not vote. Our model of in-period elections involves the following assumptions:

1. All elections take the form of a referenda to determine the level of supply of a public good, G, where G is a pure public good in the Samuelson sense;
2. The cost-sharing arrangements as embodied in the tax system are fixed by some prior tax constitution;
3. The level of G supplied is determined by simply majority rule.

The first two assumptions are sufficient to ensure that preferences are single peaked. The third ensures that the outcome is determined by the preference of the median voter. We assume further that:

4. The median voter's preference coincides with the Samuelsonian optimum (where the sum of individual demand curves cuts the marginal cost curve). This last assumption is very restrictive, and we will proceed to relax it later, but for the time being it represents convenient simplification.

II. The Analysis

Now, since any voter does not know whether his demand for G under the tax constitution will exceed or be less than the median, and setting aside any Rawlsian-type preferences across income distributions (or alternatively assuming that any such desired restrictions on the income distribution can be effected in a way that leaves the desired level of public goods unaffected), the rational individual will prefer a level of public goods supply that maximizes expected net benefits–this level is the 'efficient' level. By assumption, here, this is the level of public goods supply that emerges under majority rule *when all vote.* Furthermore, there will be an expected loss in departing from this efficient outcome for *all* individuals, the aggregate of which will be the area under the aggregate demand curve for G minus the area under the cost curve over that range.

Using the familiar Harberger measure of welfare loss [Harberger, 1964, Section II], the expected loss associated with a level of output q_i is given by:

$$W = 1/2(q_i - q_m)^2 \quad \frac{\partial \sum_i^n P_j}{\partial q} \tag{1}$$

where

$P_j =$ the j^{th} individual's marginal evaluation of the public good

$q_m =$ is the median voter's preference where m is the median (for the whole population, n), and

$q_m =$ by assumption the optimal level of public goods supply, occuring where

$\sum_i^n P_j =$ is equal to marginal cost.

Equation (1) can be rewritten as

$$W = \frac{C_q \cdot q_m}{2\epsilon} \cdot S_i^2 \tag{2}$$

where

$C_q =$ marginal cost,

$\epsilon =$ is the arc elasticity of the aggregate expected demand curve for G over the relevant range, and

$S_i =$ the proportionate deviation of output from optimality, or

$$S_i = \frac{q_i - q_m}{q_m} \tag{3}$$

Now consider the effects on the expected benefits from public goods provision as turnouts change. From the constitutional perspective, the set of individuals who exercise their voting rights is inherently a random sample of the total population, n. Of course, when everyone votes, the majoritarian outcome is q_m, the median preference for the population as a whole, which by assumption yields optimal output. As turnout falls, the probability that individual m will remain the median voter *in the smaller sample* falls, and output can be expected to deviate from the optimal output with consequent expected welfare losses, as given by (2) above.

More formally, for the given tax constitution and the associated cost-shares for individual voters, we can rank individual citizens in terms of the quantity of public goods they opt for: q_1 is the smallest quantity; and so on. Then, the probability that output q_i will emerge as a political equilibrium is the probability that individual i will be the median voter among those who turn out. Suppose for example that the turnout level is $K(\leq n)$. The median voter in this smaller sample is k where $K = 2k - 1$. Now the probability that the median voter is the i^{th} individual (that is, that the median selects q_i) is given by:

$$P_i(K) = \frac{K}{n} \binom{i-1}{k-1}\binom{n-i}{k-1}\binom{n-1}{2k-2}^{-1}$$

$$\text{if } k-1 < i < n-k+2$$

$$\text{and 0 otherwise} \qquad (4)$$

This probability is composed of two elements: first, the probability that i will vote (that is, the probability that i will belong to the sample K), which is K/n; and second, the probability that of the remaining $(K-1)$ voters, exactly one half prefer more G and one half less.[2]

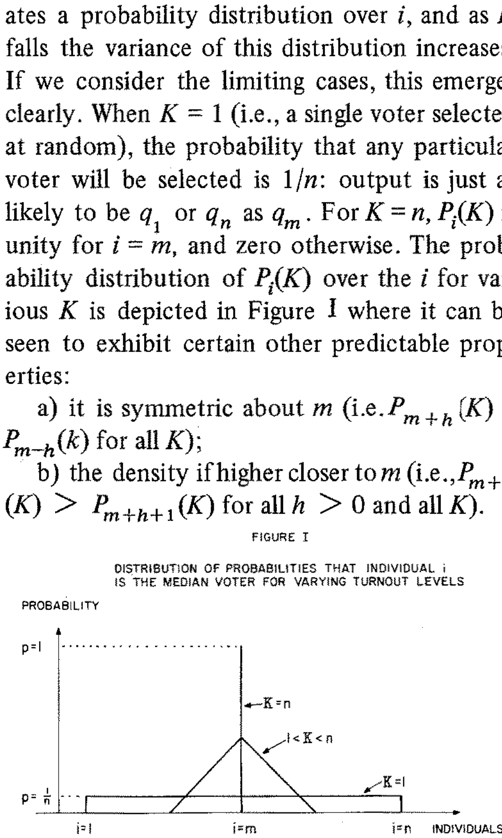
Our interest is in the way in which the probability of obtaining outlying quantities of G changes as K changes. Now, $P_i(K)$ in (4) generates a probability distribution over i, and as K falls the variance of this distribution increases. If we consider the limiting cases, this emerges clearly. When $K = 1$ (i.e., a single voter selected at random), the probability that any particular voter will be selected is $1/n$: output is just as likely to be q_1 or q_n as q_m. For $K = n$, $P_i(K)$ is unity for $i = m$, and zero otherwise. The probability distribution of $P_i(K)$ over the i for various K is depicted in Figure I where it can be seen to exhibit certain other predictable properties:

a) it is symmetric about m (i.e. $P_{m+h}(K) = P_{m-h}(k)$ for all K);

b) the density if higher closer to m (i.e., $P_{m+h}(K) > P_{m+h+1}(K)$ for all $h > 0$ and all K).

FIGURE I

DISTRIBUTION OF PROBABILITIES THAT INDIVIDUAL i IS THE MEDIAN VOTER FOR VARYING TURNOUT LEVELS



On this basis, we can associate with any electoral turnout K an expected welfare loss $E(W_k)$. This is determined by the sum across all i of the welfare loss associated with output q_i weighted by the probability that q_i will emerge as the majoritarian equilibrium in a sample of size K. Thus:

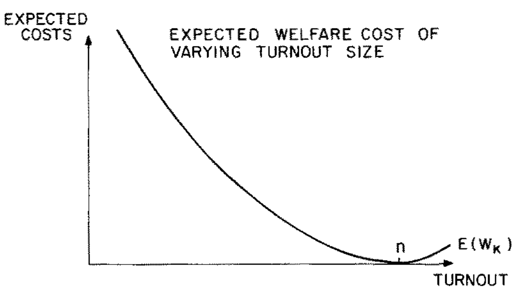
$$E(W_K) = \frac{C_q \circ q_m}{2\epsilon} \sum_{i=1}^{n} P_i(K) \circ S_i^2 \qquad (5)$$

[2] There are $\binom{i-1}{k-1}$ ways in which $(k - 1)$ voters can be selected from among the $(i - 1)$ individuals whose marginal evaluations of G lie below i's at q_i. Likewise, there are $\binom{n-i}{k-1}$ ways in which $(k - 1)$ voters can be selected from among the $(n - i)$ voters whose marginal evaluations of G lie above i's at q_i. And the number of ways of selecting $(K - 1) = (2k - 2)$ individuals from $(n - 1)$ is $\binom{n-1}{2k-2}$.

The only element in (5) that changes with K is the $P_i(K)$, and from the foregoing reasoning we know that $P_i(K)$ is larger for those S_i that are larger, the lower is K. We therefore know that as K falls, $E(W_k)$ rises–from zero, when $K = n$; to a maximum when $K = 1$. Conceptually, a turnout of greater than 100 percent could be achieved by assigning some individuals more than one vote. This would, however, if the extra voting power were to be assigned in a manner not known behind the veil of ignorance, serve to *increase* the variance of the distribution given by (4), and $E(W_k)$ would again become positive.

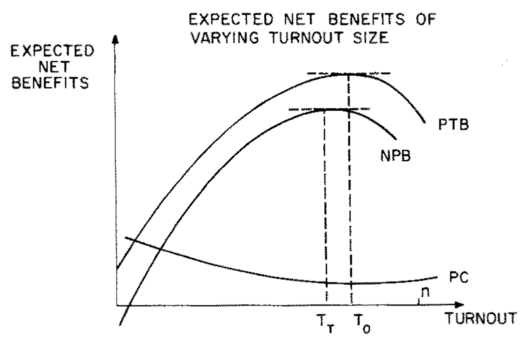
The central elements of the analysis so far can be captured diagrammatically in Figure II. The schedule labelled 'expected costs' in Figure IIa), indicates the behavior of $E(W_k)$ as K increases, and is obedient to the properties of (5) already discussed. In order to isolate the optimal outcome, two additional considerations are relevant. In the first place, these are the costs

FIGURE IIa)



of the voting process itself, mentioned earlier. As Tullock argues, these seem likely to level off at some level (i.e., the voting process is subject to diminishing marginal and average costs) and are drawn to exhibit this characteristic in Figure IIb). This is labelled "processing costs,", *PC.* The final piece of the model is the actual voting behavior of individuals within in-period elections: this presumably reflects the private benefits and costs which individuals sustain in voting, and can be depicted as a private net benefit schedule, *PTB*, in Figure IIb). This curve will achieve its maximum at some turnout level, T_o; this is the turnout which would prevail in the absence of any poll tax or subsidy. In the neighborhood of n, such net benefits will be declining sharply, reflecting the very substantial inconvenience and possibly psychic costs for the last few voters. It is, of course, conceivable that, where voters vote out of a sense of duty or for other quasi-ethical reasons, the curve *PTB* will already reflect some of the elements incorporated in the $E(W_k)$ curve. However, as elsewhere in the externality literature, we set this possibility aside and assume instead that, at the in-period stage, voters are motivated solely by the desire to secure political outcomes in accordance with their own individual preferences.

FIGURE IIb)



By deducting *PC* from *PTB*, we can derive a *net* private benefit curve, *NPB*, which will have its maximum at some turnout level T_T, to the left of T_O (unless marginal processing costs are actually zero). This turnout level, T_T, is the one finally isolated by Tullock as the optimal turnout, and the tax equal to marginal processing costs is the 'optimal poll tax' in Tullock's mode.

In fact, however, we need to include $E(W_k)$. By adding this vertically to *NPB* as in Figure IIIc), we obtain the *complete net* benefit curve, *CNB*, and the truly optimal turnout T_1 occurs where *CNB* achieves its maximum. There are several observations about this optimum that can be made:

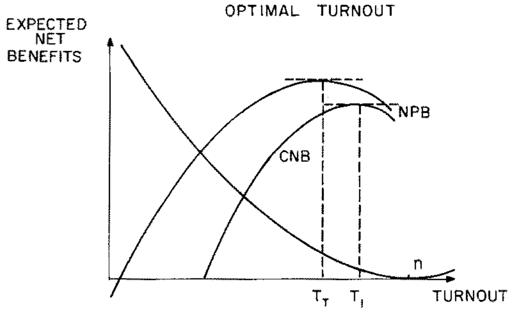
1. Optimal turnout will in general be less than total population, n, since $E(W_k)$ is zero in the neighborhood of n and *NPB* will in general be declining;

2. Optimal turnout will invariably be larger than Tullock's "optimum" T_T;

3. Equilibrium turnout T_O may be either too large or too small. It will be exactly correct if marginal processing costs are identical to marginal benefits from reducing $E(W_k)$. Given the presumptive evidence that marginal processing costs are likely to be close to zero over the relevant range, the possibility of a Pareto optimal poll *subsidy* (or perhaps a compulsory voting rule with appropriately low fines for non-compliance) seems rather more likely. In the absence of empirical evidence, however, one is simply not in a position to say.

FIGURE IIc)

OPTIMAL TURNOUT



III. Generalizations:

The foregoing analysis depends on the assumption that the median voter's preference determines a level of public goods output which is optimal. This is clearly a very special case, but it can be relaxed in a number of ways.

Let us suppose, for example, that the median voter's preference is expected to be non-optimal, but that the direction and magnitude of the likely deviation is unknown. Then we can define a subjective probability distribution of a variable t defined by:

$$t = \frac{q_m - \hat{q}}{q_m} \quad (6)$$

where $\hat{q}$ is the expected optimal level of G.

This probability distribution will, because the direction of the expected deviation is unknown, have a *mean* of zero. Now, using (2), the expected cost of choosing q_m is:

$$E[W_{q_m}] = E\left\{\frac{C_q \cdot q_m}{2\epsilon} \cdot t^2\right\} \quad (7)$$

$$= \frac{C_q \cdot q_m}{2\epsilon} \; var\ t \quad (8)$$

where *var t* is the variance of t given a probability distribution of $\hat{q}$.

Then, the expected welfare cost of choosing q_i where $q_i \neq q_m$ is:

$$E[W_{q_i}] = \frac{C_q \cdot q_m}{2\epsilon} \; E[q_i - \hat{q}]^2 \quad (9)$$

$$= \frac{C_q \cdot q_m}{2\epsilon} \; [S_i^2 + var\ t] \quad (10)$$

We can now amalgamate $E[W_{q_i}]$ with $P_i(K)$ as before, to obtain an expected welfare loss for turnout level K of:

$$E(W_k) = \frac{Cq \cdot qm}{2\epsilon} \sum_{i=1}^{n} P_i(K) \; [S_i^2 + var\ t] \quad (11)$$

which is minimized for $k = n$, and maximized for $k = 1$, as before. In this case, the $E(W_k)$ curve in Figure II will shift upwards, but its slope will not be affected and hence neither will optimal turnout.

The results can be generalized still further, if we assume that the q_i are symmetric about q_m. In this case, we can permit q_m to deviate from $\hat{q}$ in a systematic and known way. What remains true in this setting is that a random move from q_m will be in the direction *away from* $\hat{q}$ as often as it is *towards* $\hat{q}$, and if the q_i are symmetric about q_m, the expected cost of such a random move will remain positive.

Thus, in Figure III, $\hat{q}$ is the optimal level of public goods supply, and q_m is the median, both known with certainty. All voters also know their own individual demands for G. However, they do not know how a reduction in turnout will affect the equilibrium outcome: q_{m+h} and q_{m-h} are assumed to be equally likely. Then all will prefer q_m. Consider, for example, individual i. His expected net welfare loss in moving from q_m is

$$W_L = W(q_m) - \frac{1}{2}\,(W(q_{m+h}) + W(q_{m-h})) \quad (12)$$

In geometric terms, this is

$$W_L = ABq_m0 - \frac{1}{2}\,(ACq_{m-h}0 + AEq_{m+h}0) \quad (13)$$

where

$$AEq_{m+h}0 = ACq_{m-h}0 + BCq_{m-h}\,q_m + BEq_{m+n}q_m \quad (14)$$

and

$$ABq_m0 = ACq_{m-h}0 + BCq_{m-h}q_m \quad (15)$$

therefore

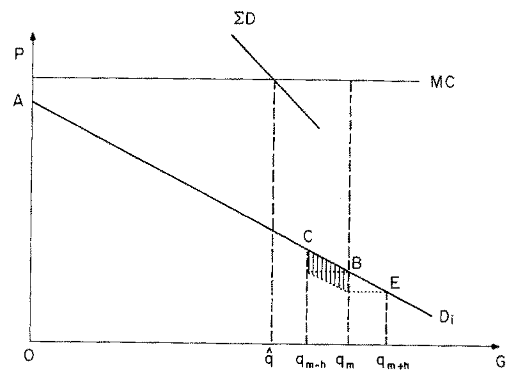
$$W_L = \frac{1}{2}\,(BCq_{m-h}q_m - BEq_{m+h}q_m) \quad (16)$$

which is positive regardless of the size of the deviation $q_mq_{m-h} = q_mq_{m+h}$. Furthermore, W_L is independent both of i's cost share, and of the quantity of G that i would prefer to have for that cost share: he will always prefer q_m to a symmetric distribution of q's around q_m. Therefore he will prefer a larger to a smaller turnout, other things being equal.[3]

Thus, we can go a good way towards relieving the foregoing analysis of some of the limiting assumptions, in relation both to the optimality of median voter/majoritarian outcomes and to the strict veil of ignorance assumptions. Providing only that demand curves slope downwards, all voters will prefer the certainty of q_m to a random distribution of outcomes around q_m. Increasing turnout will certainly reduce the variance of that distribution, and on this basis individuals can be expected, *ceteris paribus,* to prefer higher turnouts. It seems to us that this element is crucial in deriving the appropriate level of poll taxes or subsidies, and the corresponding level of voter turnout.[4] It is also an element that the relevant literature has so far ignored.

FIGURE III

EXPECTED NET COST OF RANDOM FLUCTUATIONS IN PUBLIC GOODS SUPPLY



[3] The basic elements of the reasoning used here are set out in Brennan and McGuire 1975.

[4] Although the analysis here has focused on the question of optimal turnouts, one could perhaps apply essentially the same apparatus to constitutional decisions concerning the extent of franchise and possibly other aspects of political and electoral instititions.

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