# Supplementary Appendix to: *Identifying and Testing Models of Managerial Compensation*

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**Abstract**

This appendix includes extensions and supplemental materials for the model, identification, and estimation discussed in Gayle and Miller (2014). Section A presents a dynamic hybrid moral hazard (HMH) model and proves that the optimal long-term contract can be implemented as a sequence of short-term contracts, analogous to the static HMH contract analyzed in the main text. Section B analyses identification of the PMH1 model with shrinking contracts. Section C analyses identification in the PMH2 model. It derives identification and discusses how to use exclusion restrictions to potentially narrow the identified set. Section D extends identification results to accommodate unobserved heterogeneity with long panels. Section E gives details on the empirical implementation of the application presented in the main text. All proofs are collected in Section F.

## A A Dynamic HMH Model

This section develops the notation for a dynamic version of the HMH model, lays out the feasibility constraints for the optimization problem, and then shows that the optimal contract mimics the optimal contract for a static model under the parameter transformation given in the main text.

**Assumptions and notation** At the beginning of period $t$, the manager is paid compensation denoted by $w_t$ for his work the previous period, denominated in terms of period-$t$ consumption units. He makes his consumption choice, a positive real number denoted by $c_t$, and the board proposes a new contract. The board announces how managerial compensation will be determined as a function of what he will disclose about the firm's prospects, denoted by $r_t \in \{1, 2\}$, and its subsequent performance, measured by revenue $x_{t+1}$, revealed at the beginning of the next period. We denote this mapping by $w_{rt}(x)$, the subscript $t$ designating that the optimal compensation schedule may depend on current economic conditions, such as a bond prices. Then the manager chooses whether to be engaged by the firm or to be engaged outside the firm, either with another firm or in retirement. Denote this decision by the indicator $l_{t0} \in \{0, 1\}$, where $l_{t0} = 1$ if the manager chooses to be engaged outside the firm and $l_{t0} = 0$ if he chooses to be engaged inside the firm.

If the manager accepts employment with the firm, $l_{t0} = 0$, the prospects of the firm are now fully revealed to the manager but partially hidden from the shareholders. There are two states, and the probability the first state occurs is identically and independently distributed with probability $\varphi_1 \in (0, 1)$. For convenience we denote the probability of the second state occurring by $\varphi_2 \equiv 1 - \varphi_1$. We assume that managers privately observe the true state, $s_t \in \{1, 2\}$ in period $t$—gaining information that affects the distribution of the firm's next-period revenues—and reports the state $r_t \in \{1, 2\}$ to the board. If the manager discloses the second state, meaning $r_t = 2$, then the board can independently confirm or refute it; thus, if $s_t = 1$, he reports $r_t = 1$. If $s_t = 2$, the manager then truthfully declares or lies about the firm's prospects by announcing $r_t \in \{1, 2\}$, effectively selecting one of two schedules, $w_{1t}(x)$ or $w_{2t}(x)$, in that case.

The manager then makes his unobserved labor effort choice, denoted by $l_{stj} \in \{0, 1\}$ for $j \in \{1, 2\}$ for period $t$ which may depend on his private information about the state. There are two possibilities, to diligently pursue the shareholders objectives of value maximization by working, thus setting $l_{st2} = 1$, or to accept employment with the firm but follow the objectives he would pursue if he were paid a fixed wage by setting $l_{st1} = 1$, called shirking. Let $l_{st} \equiv (l_{t0}, l_{st1}, l_{st2})$. Since leaving the firm, working and shirking are mutually exclusive activities, $l_{t0} + l_{st1} + l_{st2} = 1$.

At the beginning of period $t + 1$, revenue for the firm, $x_{t+1}$, is drawn from a probability distribution that depends on the true state $s_t$ in period $t$ and the manager's action then, $l_{st}$. We denote the probability density function for revenue when the manager works diligently and the state is $s$ by $f_s(x)$. Similarly, let $f_s(x)g_s(x)$ denote the probability density function for revenue in period $t$ when the manager shirks. Thus, for both states $s_t \in \{1, 2\}$,

$$\int x f_s(x) g_s(x) \mathrm{d}x \equiv E_s \left[ x g_s(x) \right] < E_s[x] \equiv \int x f_s(x) \mathrm{d}x,$$

the inequality reflecting the shareholders' preference for diligent work over shirking. Since $f_s(x)g_s(x)$ is a density, $g_s(x)$ is positive and integrating $f_s(x)g_s(x)$ with respect to $x$ demonstrates $E_s \left[ g_s(x) \right] = 1$. As in the text, we assume

$$\lim_{x \to \infty} \left[ g_s(x) \right] = 0$$

for each $s \in \{1, 2\}$. We make similar assumptions about the weighted likelihood ratio of the second state occurring relative to the first given any observed value of excess returns, $x \in R$, by assuming

$$\lim_{x \to \infty} \left[ \varphi_2 f_2(x) / \varphi_1 f_1(x) \right] \equiv \lim_{x \to \infty} [h(x)] = \sup_{x \in R} [h(x)] \equiv \overline{h} < \infty. \tag{A-1}$$

The manager's wealth is endogenously determined by his consumption and compensation.

We assume a complete set of markets for all publicly disclosed events effectively attributes all deviations from the law of one price to the particular market imperfections under consideration. Let $b_t$ denote the price of a bond that pays a unit of consumption each period from period $t$ onwards, relative to the price of a unit of consumption in period $t$; to simplify the exposition, we assume $b_{t+1}$ is known at period $t$. Preferences over consumption and work are parameterized by a utility function exhibiting absolute risk aversion that is additively separable over periods and multiplicatively separable with respect to consumption and work activity within periods. In the model we estimate, lifetime utility can be expressed as

$$-\sum_{t=0}^{\infty}\sum_{j=0}^{J}\beta^t\widetilde{\alpha}_j l_{tj}\exp\left(-\widetilde{\gamma}c_t\right), \tag{A-2}$$

where $\beta$ is the constant subjective discount factor, $\widetilde{\gamma}$ is the constant absolute level of risk aversion, and $\widetilde{\alpha}_j$ is a utility parameter that measures the distaste from working at level $j \in \{0, 1, 2\}$. As in the main text, we assume $\widetilde{\alpha}_2 > \widetilde{\alpha}_1$ and normalize $\widetilde{\alpha}_0 = 1$.

**Feasibility constraints** The cornerstone of the constraint formulation that circumscribes the minimization problem shareholders solve is the indirect utility function for a manager choosing between immediate retirement and retirement one period hence. Lemma A.1 states this indirect utility function in terms of the utility received from retiring immediately. To state the lemma, let $r_t(s)$ denote the manager's disclosure rule about the state when the true state is $s \in \{1, 2\}$.

**Lemma A.1** *If the manager, offered a contract of $w_{rt}(x)$ for announcing $r$, retires in period $t$ or $t+1$ by setting $(1-l_{t0})(1-l_{t+1,0}) = 0$, upon observing the state $s$ and reporting $r_t(s)$, he optimally chooses $l_{st} \equiv (l_{t0}, l_{st1}, l_{st2})$ to minimize*

$$\sum_{s=1}^{2}\varphi_s\left\{\left(\frac{l_{st1}}{\widetilde{\alpha}_1}+\frac{l_{st2}}{\widetilde{\alpha}_2}\right)^{1/(b_t-1)} + E_s\left[\exp\left(-\frac{\widetilde{\gamma}w_{r_t(s),t}(x)}{b_{t+1}}\right)\left[g_s(x)l_{t1}+l_{t2}\right]\right]\right\}. \tag{A-3}$$

Had he truthfully disclosed the true state $s_t$ in period $t$, the manager would actually receive $w_{st}(x)$ as compensation if revenue $x$ is realized at the end of the next period, $t+1$. Suppressing for expositional convenience the bond price $b_{t+1}$, and recalling our assumption that $b_{t+1}$ is known at period $t$, we now let $v_{st}(x)$ measure how (the negative of) utility is scaled up by $w_{st}(x)$:

$$v_{st}(x) \equiv \exp\left(-\frac{\widetilde{\gamma}w_{st}(x)}{b_{t+1}}\right). \tag{A-4}$$

To induce an honest, diligent manager to participate, his expected utility from employment must exceed the utility he would obtain from retirement. Setting $(l_{t2}, r_t) = (1, s_t)$ in (A-3) and substituting in $v_{st}(x)$, the participation constraint is, thus

$$\left[\sum_{s=1}^{2}\int_{\underline{x}}^{\infty}\varphi_s v_{st}(x)f_s(x)\mathrm{d}x\right] \equiv E\left[v_{st}(x)\right] \le \widetilde{\alpha}_2^{-1/(b_t-1)}. \tag{A-5}$$

Given his decision to stay with the firm one more period, and to truthfully reveal the state, the incentive-compatibility constraint induces the manager to prefer working diligently to shirking. Substituting the definition of $v_{st}(x)$ into (A-3) and comparing the expected utility obtained from setting $l_{t1} = 1$ with the expected utility obtained from setting $l_{t2} = 1$ for any given state, we obtain the incentive-compatibility constraint for diligence:

$$0 \leq \int_{\underline{x}}^{\infty} \left( g_s(x) - (\widetilde{\alpha}_2/\widetilde{\alpha}_1)^{1/(b_t-1)} \right) v_{st}(x) f_s(x) \mathrm{d}x \equiv E_s \left[ \left( g_s(x) - (\widetilde{\alpha}_2/\widetilde{\alpha}_1)^{1/(b_t-1)} \right) v_{st}(x) \right], \quad \text{(A-6)}$$

for $s \in \{1, 2\}$.

In the HMH model, information hidden from shareholders further restricts the set of contracts that can be implemented. Comparing the expected value from lying about the second state and working diligently with the expected utility from reporting honestly in the second state and working diligently, we obtain the truth-telling constraint:

$$0 \leq \int \left[ v_{1t}(x) - v_{2t}(x) \right] f_2(x) \mathrm{d}x \equiv E_2 \left[ v_{1t}(x) - v_{2t}(x) \right]. \quad \text{(A-7)}$$

An optimal contract also induces the manager not to understate and shirk in the second state, behavior we describe as sincere. Comparing the manager's expected utility from lying and shirking with the utility from reporting honestly and working diligently, the sincerity condition reduces to

$$0 \leq \int \left[ (\widetilde{\alpha}_1/\widetilde{\alpha}_2)^{\frac{1}{b_t-1}} v_{1t}(x) g_2(x) - v_{2t}(x) \right] f_2(x) \mathrm{d}x \equiv E_2 \left[ (\widetilde{\alpha}_1/\widetilde{\alpha}_2)^{\frac{1}{b_t-1}} v_{1t} g_2(x) - v_{2t}(x) \right], \quad \text{(A-8)}$$

where $(\widetilde{\alpha}_1/\widetilde{\alpha}_2)^{1/(b_t-1)} v_{1t}(x)$ is proportional to the utility obtained from shirking and announcing the first state, and $f_2(x) g_2(x)$ is the probability density function associated with shirking when the second state occurs.

**Optimal contracting** We first prove that the short-term optimal contract for the dynamic model has a static analogue of the form we describe in the main text. We then show that the long-term contract decomposes to a sequence of short-term contracts. As in the static model, deriving $w_{st}(x)$ to minimize the expected compensation for inducing diligent work in both states subject to the five constraints is equivalent to choosing $v_{st}(x)$ to maximize

$$\sum_{s=1}^{2} \int_{\underline{x}}^{\infty} \varphi_s \ln \left[ v_{st}(x) \right] f_s(x) \mathrm{d}x \equiv E \left[ \ln v_{st}(x) \right] \quad \text{(A-9)}$$

subject to the same five constraints. To achieve diligent work and truth telling, shareholders maximize

$$\sum_{s=1}^{2} \varphi_\rho \int \left\{ \log\left[v_{st}(x)\right] + \eta_{0t} \left[1 - \widetilde{\alpha}_2^{1/(b_t-1)} v_{st}(x)\right] \right\} f_s(x) \mathrm{d}x$$
$$+ \sum_{s=1}^{2} \varphi_s \eta_{st} \int v_{st}(x) \left[\widetilde{\alpha}_1^{1/(b_t-1)} g_s(x) - \widetilde{\alpha}_2^{1/(b_t-1)}\right] f_s(x)\mathrm{d}x + \varphi_2 \eta_{3t} \int \left[v_{1t}(x) - v_{2t}(x)\right] f_2(x)\mathrm{d}x$$
$$+ \varphi_2 \eta_{4t} \int \left[\widetilde{\alpha}_1^{1/(b_t-1)} v_{1t}(x) g_2(x) - \widetilde{\alpha}_2^{1/(b_t-1)} v_{2t}(x)\right] f_2(x)\mathrm{d}x \quad \text{(A-10)}$$

with respect to $v_{st}(x)$, where $\eta_{0t}$ through $\eta_{4t}$ are the shadow values assigned to the linear constraints. Setting

$$\alpha_1 = \widetilde{\alpha}_1^{1/(b_t-1)} \qquad \alpha_2 = \widetilde{\alpha}_2^{1/(b_t-1)} \qquad \gamma = \widetilde{\gamma}/b_{t+1} \quad \text{(A-11)}$$

establishes by inspection that the solution to the static model solves the transformed problem as claimed in the text.

In this framework, there are no gains from a long-term arrangement between shareholders and the manager. Lemma A.2 verifies that Fudenberg, Holmstrom and Milgrom's (1990) assumptions are met, thus establishing that the long-term optimal contact decentralizes to a sequence of short-term contracts solved by the problem above.[1]

**Lemma A.2** *Denote by $\overline{\tau}$ the manager's date of retirement. The optimal long-term contract can be implemented by a $\overline{\tau}$-period replication of the optimal short-term contract.*

# B Identification in the PMH1 Model when Shirking Contracts are offered in Equilibrium

We now turn to the other two cases of possible effort choice by the principal not analyzed in the main text; when it is optimal for managers to shirk and when it is optimal for one type of principal to induce working and another type to induce shirking. In the case of shirking, Equation (11) in the main text holds, a fixed compensation is prescribed, so compensation does not depend on revenue. In this case the density $f(x)g(x)$ can be identified from observations on revenue, the compensation is constant at $w^{(1)}$, but nothing more can be gleaned from the data about the structure of the model. Loosely speaking, this variation on the model is under-identified, and is indistinguishable from a model where there are no moral-hazard considerations.

Now consider the final case, and suppose there exists some unobserved heterogeneity in the types of principals; some of them are just as we have described above and satisfy (10) in the main text, but the revenue generation process for the remainder is $f(x)g(x)$ regardless

[1] Malcomson and Spinnewyn (1988), Fudenberg, Holmstrom and Milgrom (1990), and Rey and Salanie (1990) have independently established conditions under which long-term optimal contracts can be implemented via a sequence of one-period contracts in dynamic models of generalized moral hazard, and the proof of Lemma A.2 draws extensively upon their results.

of whether the agent works or shirks. In equilibrium, the latter pay a fixed wage of $w^{(1)}$, and the former pay variable compensation of (8) in the main text. There is a discontinuity in the distribution function for compensation data at $w^{(1)}$, and the size of the jump determines the fraction of principals who induce shirking. The density $f(x)$ is identified from data on revenues to principals not paying $w^{(1)}$, and $f(x)g(x)$ is identified from data on revenues to principals paying $w^{(1)}$. Taking the quotient identifies $g(x)$. This only leaves $\alpha_1$, $\alpha_2$ and $\gamma$ to identify. In the optimal contract, the participation constraint for both types of principals holds with equality, as does the incentive-compatibility constraint for the principal who induces work. Thus (5), (6) and (7) in the main text reduce to

$$\alpha_1 E\left[e^{-\gamma w^o(x)} g(x)\right] = \alpha_1 e^{-\gamma w^{(1)}} = \alpha_2 E\left[e^{-\gamma w^o(x)}\right] = 1. \tag{A-12}$$

Define $\psi(\xi) \equiv E\left\{e^{-\xi\left[w^o(x)-w^{(1)}\right]} g(x)\right\}$. The first two equalities in (A-12) imply $\psi(\gamma) = 1$. By inspection, $\psi(0) = 1$, $\psi'(0) < 0$ and $\psi''(\xi) > 0$.[2] Thus, $\psi(\xi)$ is a convex function with a unique nonzero solution at $\psi(\gamma) = 1$. This equality identifies $\gamma$. Substituting the solution into the second two equalities of (A-12) identifies $\alpha_1$ and $\alpha_2$. Therefore, all the parameters are identified from the cost-minimization equations alone. Indeed this variant on the model is over-identified, because $w^o(x)$ must satisfy (8) and (9) in the main text for each $x$, a very strong exclusion restriction that relates the two types of principals to each other. However, relaxing the restriction would necessitate a separate analysis of the first two cases.

# C Identifying the PMH2 Model

When there is heterogeneity in the revenue probability distribution, additional restrictions on the set of observationally equivalent parameters can be imposed if preferences are invariant across states. Suppose there are two states denoted by $s \in \{1, 2\}$. We denote the revenue probability density function from working diligently in state $s$ by $f_s(x)$, and similarly express the corresponding likelihood ratio in $s$ as $g_s(x)$. We assume $f_1(x) \neq f_2(x)$ and $g_1(x) \neq g_2(x)$. The optimal contract—state dependent, but solved the same way as the one state model—is denoted by $w_s^o(x)$. We also write $\overline{w}_s$ for the limiting constant wage as $x \to \infty$ in state $s$. If the heterogeneity is observed, the data records the state $s_n \in \{1, 2\}$, revenue $x_n \in R$ and compensation $w_n \in R$ for each observation $n \in \{1, \ldots, N\}$.

Suppose the agent's risk-aversion parameter does not vary across states because, for example, the same type of agent works in both states. The solution to the cost-minimization problem of inducing diligence, now denoted by $w_s^o(x)$ to reflect the state dependence, is derived the same way as in Equation (8) in the main text. For each state $s \in \{1, 2\}$, we define $Q_s(\gamma)$ analogously to $Q_0(\gamma)$ by substituting $w_s^o(x)$ and $\overline{w}_s$ for $w^o(x)$ and $\overline{w}$, respectively, and by substituting $E_s[\cdot]$ for $E[\cdot]$ in Equation (25) in the main text to condition the expectations operator on the state.[3] Following the same reasoning as the derivation of Equation (25) in the main text, $Q_s(\gamma^*) \geq 0$ for $s \in \{1, 2\}$. More generally, increasing the states while maintaining

[2] To prove $\psi'(0) < 0$, first note that $\psi'(0) = w^{(1)} - E\left[w^o(x)g(x)\right]$. Second, $w^{(1)}$ is the certainty equivalent of $w^o(x)$ under the probability density $f(x)g(x)$. Hence $w^{(1)} < E\left[w^o(x)g(x)\right]$.

[3] We adopt the notation that $E_s[\cdot] \equiv E[\cdot|s]$ throughout.

the hypothesis that the risk-aversion parameter is invariant across states increases the number of inequalities from profit maximization by the same number.

Now suppose that, in addition, the nonpecuniary benefits from working diligently, $\alpha_2$, do not vary by state. Although there might only be one participation constraint ensuring that the agent's unconditional expected utility is at least as enticing as the outside alternative, it is straightforward to show that the participation constraint in Equation (6) in the main text holds with equality for each state $s \in \{1, 2\}$ in the optimal contract, implying $\alpha_2 E_s \left[e^{-\gamma w_s^o(x)}\right] = 1$. Defining

$$\Upsilon_2(\gamma) \equiv E_1 \left[e^{-\gamma w_1^o(x)}\right] - E_2 \left[e^{-\gamma w_2^o(x)}\right], \tag{A-13}$$

it follows that $\Upsilon_2(\gamma^*) = 0$.[4] Intuitively, a person's risk preferences cannot be identified from playing a single lottery if there are unobserved components to the reward from entering the lottery. When offered the chance to play two lotteries with different risk characteristics but the same unobserved nonpecuniary components, his risk preferences are partially revealed by the pecuniary compensating differential between them, which equalizes his expected utility from playing one versus the other.

Another potential restriction is that the nonpecuniary benefits from shirking, $\alpha_1$, do not vary by state. Since the incentive-compatibility constraint in Equation (7) in the main text also holds with equality in each state, Theorem 2.1 in the main text implies

$$\alpha_1^{-1} = \frac{1 - E_1 \left[e^{\gamma w_1^o(x) - \gamma \overline{w}_1}\right]}{E_1 \left[e^{-\gamma w_1^o(x)}\right] - e^{-\gamma \overline{w}_1}} = \frac{1 - E_2 \left[e^{\gamma w_2^o(x) - \gamma \overline{w}_2}\right]}{E_2 \left[e^{-\gamma w_2^o(x)}\right] - e^{-\gamma \overline{w}_2}}. \tag{A-14}$$

In this case, the restriction is based on two hypothetical lotteries, compensation from shirking in the different states. To incorporate these restrictions into the testing and estimation framework, we define

$$\Upsilon_1(\gamma) \equiv \frac{1 - E_1 \left[e^{\gamma w_1^o(x) - \gamma \overline{w}_1}\right]}{E_1 \left[e^{-\gamma w_1^o(x)}\right] - e^{-\gamma \overline{w}_1}} - \frac{1 - E_2 \left[e^{\gamma w_2^o(x) - \gamma \overline{w}_2}\right]}{E_2 \left[e^{-\gamma w_2^o(x)}\right] - e^{-\gamma \overline{w}_2}}. \tag{A-15}$$

From (A-13) and (A-14), Theorem 2.1 in the main text implies $\Upsilon_1(\gamma^*) = 0$ if $\alpha_1$ does not vary across states. A joint test of these restrictions can be based on the criterion function,

$$\min\{0, Q_1(\gamma)\}^2 + \min\{0, Q_2(\gamma)\}^2 + \Upsilon_1(\gamma)^2 + \Upsilon_2(\gamma)^2,$$

which attains a minimum of zero when all risk-aversion parameter values are observationally equivalent to $\gamma^*$.

To summarize, we provide an intuitive explanation of how the extension of the PMH1 model to two states PMH2 model affects identification. Consider as a baseline a framework with maximal heterogeneity, where the taste parameters for working and shirking, as well as the risk parameter, vary by state. With maximal heterogeneity, we obtain just two inequalities from the profit-maximization condition. The two risk parameter sets are separately determined state by state. If a single risk parameter satisfies both profit inequalities, then it

[4] If the equation were linear in $\gamma$, point identification could be easily determined, but it is not guaranteed.

must belong to the intersection of the individually determined sets. In this way, we can derive the set of risk parameters that are common across states, without imposing homogeneity on the other preference parameters. To impose homogeneity on the taste parameter for work as well, we would extract from the intersection derived above those risk parameters that induced the same taste parameter for work in both states. Alternatively, imagine permitting heterogeneity in the risk parameter across states, but imposing instead homogeneity on the taste parameter for work. We would seek to equalize the taste parameter for work across states using two different risk parameters that individually satisfy the profit inequalities for their respective states.

# D Identification with Unobserved Heterogeneity

To illustrate how our identification results can accommodate unobserved heterogeneity we discuss how the results in the PMH1 model would change if the risk aversion parameter, $\gamma_n$, is agent specific and time invariant for agents labelled as $n = 1, ..., N$. Since the changes to the HMH are similar, we do not explicitly discuss them in this supplementary appendix. We consider the following econometric model:

$$\widetilde{w}_{nt} = w(x_{nt}, \gamma) + \varepsilon_{nt} \quad t = 1, ..., T \tag{A-16}$$

where $\widetilde{w}_{nt}$ and $x_{nt}$ are observed compensation and revenue, $(\gamma_n, \varepsilon_{nt})$ denote unobservables, respectively time invariant and time varying, and $\gamma_n$ is countable. As in Section 2.2 of the main text $\varepsilon_{nt}$ is measurement error which is independent of all variables of interest including $x_t$, and $\gamma$, with $E[\varepsilon_t | x_t, \gamma] = 0$. As in the main text upper case notation is used to represent a random variable or vector and lower case notation is used for realizations. In some applications it would be reasonable to also assume that $\alpha_1$, $\alpha_2$, $f(x)$, and $g(x)$ are functions of $n$; we first assume here that they are not functions of $n$, and then briefly remark on how this assumption might be relaxed.

The econometric model is linked to the theoretical model by assuming that if the principal demands the agent work, then:

$$w(x_{nt}, \gamma_n) = \gamma_n^{-1}[\ln \alpha_2 + \ln\{1 + \theta[(\alpha_2/\alpha_1) - g(x_{nt})]\}] \tag{A-17}$$

where $\theta$ is the unique positive solution to

$$\int \left[ \frac{g(x) - (\alpha_2/\alpha_1)}{\alpha_2 + \theta[(\alpha_2/\alpha_1) - g(x)]} \right] f(x) dx = 0. \tag{A-18}$$

On the other hand, if the principal demands shirking:

$$w(x_{nt}\gamma_n) = \gamma_n^{-1} \ln \alpha_1. \tag{A-19}$$

We consider the function $w(x, \gamma_n)$ observed if observables are sufficient to calculate the following conditional expectation:

$$w_n(x) \equiv w(x, \gamma_n) = E_n[\widetilde{w}_{nt} | X_{nt} = x] \tag{A-20}$$

where $E_n[\cdot|\cdot]$ denotes the expectation operator over $x$ for a given manager $n$.

Our identification results in this section critically depend on environments in which $E_n[\cdot|\cdot]$ exist and can be consistently estimated from the distribution of observables. Sufficient conditions for the existence and consistent estimation of $E_n[\cdot|\cdot]$ are $N$ fixed and $T$ large or $T$ large relative to $N$; here we assume the former. Thus we assume that the equilibrium distribution of $(W, X)$ is identified, where $W$ and $X$ are $N \times 1$ dimensional random vectors. The identification problem reduces to whether the structure $[F, G, A_1, A_2, \{\Gamma_n\}_{n=1}^N]$ can be recovered from knowledge of distribution $(W, X)$.

Suppose $\gamma_n$ is known, and define the mappings $g(x, \gamma_n)$, $\alpha_1(\gamma_n)$, and $\alpha_2(\gamma_n)$ as:

$$g(x, \gamma_n) \equiv \frac{e^{\gamma_n \overline{w}_n} - e^{\gamma_n w_n(x)}}{e^{\gamma_n \overline{w}_n} - E\left[e^{\gamma_n w_n(x)}\right]} \tag{A-21}$$

$$\alpha_1(\gamma_n) \equiv \frac{1 - E\left[e^{\gamma_n w_n(x) - \gamma_n \overline{w}_n}\right]}{E\left[e^{-\gamma_n w_n(x)}\right] - e^{-\gamma_n \overline{w}_n}} \tag{A-22}$$

$$\alpha_2(\gamma_n) \equiv \left\{E\left[e^{-\gamma_n w_n(x)}\right]\right\}^{-1}. \tag{A-23}$$

As in Theorem 2.1 in the main text we can prove that if $\gamma_n$ is known, then $g(x)$, $\alpha_1$, and $\alpha_2$ are identified from the distribution of $(W, X)$. We now turn to the identification of $\gamma_n$. In contrast to the PMH1 model in the main text, identification may be achieved from the cost minimization conditions alone, that is if $\alpha_1$, $\alpha_2$ and $g(x)$ are not agent specific. Thus the equality of $\alpha_1(\gamma_n)$, $\alpha_2(\gamma_n)$, and $g(x, \gamma_n)$ across agents yield $3(N-1)$ restrictions for identifying $N+1$ parameters. The risk aversion parameters still might not be point identified because the restrictions are nonlinear, not necessarily invertible in $\gamma_n$.

Finally the profit maximization condition used in the text potentially adds another $N$ inequality restrictions. Hence even if $\alpha_1$, $\alpha_2$, and $g(x)$ are agent specific, there would still be $N$ inequalities left after using up the restrictions derived from (A-21), (A-22) and (A-23) to identify the remaining $N$ parameters, $\gamma_n$.

# E Implementation

This appendix extends the discussion of Section 5.2 on estimating and testing the PMH3 model to the dynamic PMH2, PMH3 and HMH models we analyzed in our empirical application. First, we show that the set of admissible $\widetilde{\gamma}$ shrinks when we impose the restrictions that tastes for working or shirking do not change with the state, and only vary with the bond price. Then we characterize the set of restrictions on $\widetilde{\gamma}$ implied by the fully restricted HMH model as defined in the main text.

## E.1 Restrictions in the Dynamic PMH model

In the PMH3 model, it follows from Section 5.2 that:

$$\widetilde{\Gamma}_3 = \{\widetilde{\gamma} > 0 : Q_3(\widetilde{\gamma}) = 0\},$$

where

$$Q_3(\widetilde{\gamma}) \equiv \sum_{t=1}^{T}\sum_{s=1}^{2}\min\{0, Q_{st}(\widetilde{\gamma})\}^2 . \tag{A-24}$$

With reference to equations (58) and (59) in the main text, we now define the taste parameters for the dynamic version of the pure moral hazard model as

$$\alpha_{1st}(\widetilde{\gamma}) \equiv \alpha_1 (\widetilde{\gamma}/b_{t+1})^{b_t - 1}$$
$$\alpha_{2st}(\widetilde{\gamma}) \equiv \alpha_2 (\widetilde{\gamma}/b_{t+1})^{b_t - 1} .$$

We investigated how the confidence region for $\widetilde{\gamma}$ shrinks when we impose the restrictions that $\alpha_{1st}(\widetilde{\gamma})$ and $\alpha_{2st}(\widetilde{\gamma})$ do not change with the state $s \in \{1, 2\}$ or with time $t \in \{1, \ldots, T\}$.

To impose the restriction that $\alpha_{1st}(\widetilde{\gamma})$ does not vary by state, we define the real valued functions $\Upsilon_{1t}(\widetilde{\gamma})$ as

$$\Upsilon_{1t}(\widetilde{\gamma}) \equiv \left\{\frac{1 - E_{1t}\left[\exp\left(\frac{\widetilde{\gamma}(w_{1t} - \overline{w})}{b_{t+1}}\right)\right]}{E_{1t}\left[\exp\left(\frac{-\widetilde{\gamma}w_{1t}}{b_{t+1}}\right)\right] - \exp\left(\frac{-\widetilde{\gamma}\overline{w}}{b_{t+1}}\right)}\right\}^{b_t - 1} - \left\{\frac{1 - E_{21}\left[\exp\left(\frac{\widetilde{\gamma}(w_{2t} - \overline{w})}{b_t}\right)\right]}{E_{2t}\left[\exp\left(\frac{-\widetilde{\gamma}w_{2t}}{b_t}\right)\right] - \exp\left(\frac{-\widetilde{\gamma}\overline{w}}{b_t}\right)}\right\}^{b_1 - 1}$$

and note that $\Upsilon_{1t}(\widetilde{\gamma}) = 0$ if and only if $\alpha_{11t}(\widetilde{\gamma}) = \alpha_{12t}(\widetilde{\gamma})$. Similarly, $\alpha_{21t}(\widetilde{\gamma}) = \alpha_{22t}(\widetilde{\gamma})$ if and only if $\Upsilon_{2t}(\widetilde{\gamma}) = 0$ where

$$\Upsilon_{2t}(\widetilde{\gamma}) \equiv \left\{E_2\left[\exp\left(\frac{-\widetilde{\gamma}w_{2t}}{b_{t+1}}\right)\right]\right\}^{1 - b_t} - \left\{E_1\left[\exp\left(\frac{-\widetilde{\gamma}w_{1t}}{b_t}\right)\right]\right\}^{1 - b_t} .$$

Thus, to find a confidence region for the risk parameter under the null hypothesis that tastes for shirking or working, $\alpha_{jst}(\widetilde{\gamma})$ for $j \in \{1, 2\}$, do not vary by state, we augment (A-24) and find those values of $\widetilde{\gamma}$ that achieve close to the lower bound of zero for a sample analog of

$$\sum_{t=1}^{T}\sum_{s=1}^{2}\left[\min\{0, Q_{st}(\widetilde{\gamma})\}^2 + \Upsilon_{jt}(\widetilde{\gamma})^2\right] . \tag{A-25}$$

The results from separately imposing these two sets of restrictions for are reported in Table 6 of the main text.

Essentially the same procedure can be used to constrain $\widetilde{\alpha}_{1st}$ or $\widetilde{\alpha}_{2st}$ to remain constant over time. Defining

$$\Upsilon_{1st}(\widetilde{\gamma}) \equiv \left\{\frac{1 - E_{st}\left[\exp\left(\frac{\widetilde{\gamma}(w_{st} - \overline{w})}{b_{t+1}}\right)\right]}{E_{st}\left[\exp\left(\frac{-\widetilde{\gamma}w_{st}}{b_{t+1}}\right)\right] - \exp\left(\frac{-\widetilde{\gamma}\overline{w}}{b_{t+1}}\right)}\right\}^{b_t - 1} - \left\{\frac{1 - E_{s1}\left[\exp\left(\frac{\widetilde{\gamma}(w_{s1} - \overline{w})}{b_2}\right)\right]}{E_{s1}\left[\exp\left(\frac{-\widetilde{\gamma}w_{s1}}{b_2}\right)\right] - \exp\left(\frac{-\widetilde{\gamma}\overline{w}}{b_2}\right)}\right\}^{b_1 - 1} ,$$

it immediately follows that $\widetilde{\alpha}_{1s1} = \widetilde{\alpha}_{1st}$ when $\Upsilon_{1st}(\widetilde{\gamma}) = 0$. Similarly, $\widetilde{\alpha}_{2s1} = \widetilde{\alpha}_{2st}$ when

$\Upsilon_{2st}(\widetilde{\gamma}) = 0$, where $\Upsilon_{2st}(\widetilde{\gamma})$ is defined as

$$\Upsilon_{2st}(\widetilde{\gamma}) \equiv \left\{ E_s \left[ \exp\left( \frac{-\widetilde{\gamma} w_{st}}{b_{t+1}} \right) \right] \right\}^{1-b_t} - \left\{ E_s \left[ \exp\left( \frac{-\widetilde{\gamma} w_{s1}}{b_2} \right) \right] \right\}^{1-b_1}.$$

This restriction implies that $\Upsilon_{2st}(\widetilde{\gamma}) = 0$ for all $t \in \{1, 2, \ldots, T\}$ and $s \in \{1, 2\}$. Thus, the confidence region for the risk parameter under the null hypothesis that $\alpha_{jst}(\widetilde{\gamma})$ does not vary over time could be found by constructing a sample analogue of

$$Q_2(\widetilde{\gamma}) = \sum_{t=1}^{T} \sum_{s=1}^{2} \left[ \min\{0, Q_{st}(\widetilde{\gamma})\}^2 + \Upsilon_{jst}(\widetilde{\gamma})^2 \right] \quad \text{(A-26)}$$

and, using the methods we describe below, selecting those $\widetilde{\gamma}$ that bring the criterion function close to zero. Formally the identified set for $\widetilde{\gamma}$ is now given by:

$$\widetilde{\Gamma}_2 = \{\widetilde{\gamma} > 0 : Q_2(\widetilde{\gamma}) = 0\}. \quad \text{(A-27)}$$

## E.2 Restrictions in the Dynamic HMH model

The restrictions in the HMH model are imposed similarly. Throughout our analysis of the HMH model, we maintain the null hypothesis that the taste parameters for working and shirking, both mappings of $\widetilde{\gamma}$, do not vary by state or time. These restrictions are maintained because the intersection of the estimated confidence intervals for $\widetilde{\gamma}$ for the 24 sectors under the null hypothesis is not empty.

To develop the notation for the econometric framework that accommodates a panel where bonds prices vary over time, as opposed to varying across sections or a steady state economy with constant interest rates, we extend our notation as follows. Similar to the dynamic PMH model we define taste parameters that are independent of the state

$$\widehat{\alpha}_{1t}(\widetilde{\gamma}) \equiv \widehat{\alpha}_1 (\widetilde{\gamma}/b_{t+1})^{b_t - 1}$$
$$\widehat{\alpha}_{2t}(\widetilde{\gamma}) \equiv \widehat{\alpha}_2 (\widetilde{\gamma}/b_{t+1})^{b_t - 1}.$$

Similarly, the likelihood ratio for the second state is defined as

$$\widehat{g}_{2t}(x, \widetilde{\gamma}) \equiv \widehat{g}_2 (x, \widetilde{\gamma}/b_{t+1}).$$

We then define the Lagrange multipliers $\eta_{1t}(\widetilde{\gamma})$ through $\eta_{4t}(\widetilde{\gamma})$ by substituting $\widetilde{\gamma}/b_{t+1}$ for $\gamma$, $\widehat{\alpha}_{1t}(\gamma)$ for $\widehat{\alpha}_1(\gamma)$, $\widehat{\alpha}_{2t}(\gamma)$ for $\widehat{\alpha}_2(\gamma)$ and $\widehat{g}_{2t}(x, \gamma)$ for $\widehat{g}_2(x, \gamma)$; hence, defining

$$\widehat{g}_{1t}(x, \gamma) \equiv \widehat{g}_{2t}(x, \gamma/b_{t+1}).$$

We are now in a position to define $\Gamma_{\mathrm{H}t}$ by substituting $\eta_{1t}(\widetilde{\gamma})$ through $\eta_{4t}(\widetilde{\gamma})$ for $\eta_1(\widetilde{\gamma})$ through $\eta_4(\widetilde{\gamma})$, $\Lambda_{it}(\widetilde{\gamma})$ for $\Lambda_i(\widetilde{\gamma})$, and $\Psi_{kt}(\widetilde{\gamma})$ for $\Psi_k(\widetilde{\gamma})$ in the definition of $\Gamma_{\mathrm{H}}$ and replacing $\gamma$ with $\widetilde{\gamma}$. To impose the restriction that none of the parameters vary over time, we take the intersection

$$\Gamma_H(T) \equiv \bigcap_{t=1}^{T} \Gamma_{\mathrm{H}t}$$
$$= \{\widetilde{\gamma} > 0 : Q_{\mathrm{H}}(\widetilde{\gamma}) = 0\}, \tag{A-28}$$

where

$$Q_{\mathrm{H}}(\widetilde{\gamma}) \equiv \sum_{t=1}^{T}\sum_{j=5}^{9} \min\left[0, \Psi_{jt}(\widetilde{\gamma})\right]^2 + \sum_{t=1}^{T}\sum_{j=6}^{7}\left[\Psi_{5t}(\widetilde{\gamma})\,\Psi_{jt}(\widetilde{\gamma})\right]^2$$
$$+ \sum_{t=1}^{T}\Psi_{4t}(\widetilde{\gamma})^2 + \sum_{t=1}^{T}\left[\Psi_{6t}(\widetilde{\gamma})\,\Psi_{8t}(\widetilde{\gamma})\right]^2 + \sum_{t=1}^{T}\sum_{k=3}^{5}\min\left[0, \Lambda_{kt}(\widetilde{\gamma})\right]^2. \tag{A-29}$$

Computing $\Lambda_{2t}(\widetilde{\gamma})$ and $\Lambda_{3t}(\widetilde{\gamma})$ requires us to solve for $w_s^{(1,0)}(x, \widetilde{\gamma}/b_{t+1})$ and $w_s^{(0,1)}(x, \widetilde{\gamma}/b_{t+1})$ for each candidate value of $\widetilde{\gamma}$, a nonlinear problem that includes two Lagrange multipliers. If the states $s \in \{1, 2\}$ and the effort level $(l_1, l_2)$ were observed by shareholders, then they would optimally offer $b_{t+1}\log\left[\widehat{\alpha}_{1t}(\widetilde{\gamma})\right]/(b_t - 1)\widetilde{\gamma}$ for shirking and $b_{t+1}\log\left[\widehat{\alpha}_{2t}(\widetilde{\gamma})\right]/(b_t - 1)\widetilde{\gamma}$ for diligence. The profits from this hypothetical arrangement are, therefore,

$$\Lambda'_{2t}(\widetilde{\gamma}) = \varphi_1\left\{E[x] - b_{t+1}\log\left[\widehat{\alpha}_{2t}(\widetilde{\gamma})\right]/(b_t - 1)\widetilde{\gamma}\right\}$$
$$+ \varphi_2 E_2\left\{x\widehat{g}_{2t}(x, \widetilde{\gamma}) - b_{t+1}\log\left[\widehat{\alpha}_{1t}(\widetilde{\gamma})\right]/(b_t - 1)\widetilde{\gamma}\right\},$$

from shirking in the second state and working diligently in the first, and

$$\Lambda'_{3t}(\widetilde{\gamma}) = \varphi_2\left\{E[x] - b_{t+1}\log\left[\widehat{\alpha}_{2t}(\widetilde{\gamma})\right]/(b_t - 1)\widetilde{\gamma}\right\}$$
$$+ \varphi_1 E_1\left[x\widehat{g}_{1t}(x, \widetilde{\gamma}) - b_{t+1}\log\left[\widehat{\alpha}_{1t}(\widetilde{\gamma})\right]/(b_t - 1)\widetilde{\gamma}\right],$$

from shirking in the first state and working diligently in the second. Since neither cost-minimization problem imposes the truth telling, sincerity or incentive-compatibility constraint, but they have the same objective function, it now follows that $\Lambda'_{2t}(\widetilde{\gamma}) \geq \Lambda_{2t}(\widetilde{\gamma})$ and $\Lambda'_{3t}(\widetilde{\gamma}) \geq \Lambda_{3t}(\widetilde{\gamma})$. Let $\Gamma_{\mathrm{H}}(\widetilde{\gamma}, T/\Lambda_{2t}, \Lambda_{3t})$ denote the set of $\widetilde{\gamma}$ formed from excluding $\Lambda_{2t}(\widetilde{\gamma})$ and $\Lambda_{3t}(\widetilde{\gamma})$ for all $t$. By construction,

$$\Gamma_{\mathrm{H}}(\widetilde{\gamma}, T) \subseteq \Gamma_{\mathrm{H}}(\widetilde{\gamma}, T/\Lambda_{2t}, \Lambda_{3t}).$$

Now let $\Gamma_{\mathrm{H}}(\widetilde{\gamma}, T/\Lambda_{2t}, \Lambda_{3t}\Lambda'_{2t}, \Lambda'_{3t})$ denote the set of $\widetilde{\gamma}$ formed from intersecting $\Gamma_{\mathrm{H}}(\widetilde{\gamma}, T/\Lambda_{2t}, \Lambda_{3t})$ with $\Lambda'_{2t}(\widetilde{\gamma})$ and $\Lambda'_{3t}(\widetilde{\gamma})$ for all $t \in \{1, 2, \ldots, T\}$. Since $\Lambda'_{2t}(\widetilde{\gamma}) \geq \Lambda_{2t}(\widetilde{\gamma})$ and $\Lambda'_{3t}(\widetilde{\gamma}) \geq \Lambda_{3t}(\widetilde{\gamma})$, it immediately follows that

$$\Gamma_{\mathrm{H}}(\widetilde{\gamma}, T/\Lambda_{2t}, \Lambda_{3t}\Lambda'_{2t}, \Lambda'_{3t}) \subseteq \Gamma_{\mathrm{H}}(\widetilde{\gamma}, T).$$

Thus,

$$\Gamma_{\mathrm{H}}(\widetilde{\gamma}, T/\Lambda_{2t}, \Lambda_{3t}\Lambda'_{2t}, \Lambda'_{3t}) \subseteq \Gamma_{\mathrm{H}}(\widetilde{\gamma}, T) \subseteq \Gamma_{\mathrm{H}}(\widetilde{\gamma}, T/\Lambda_{2t}, \Lambda_{3t}). \tag{A-30}$$

In our empirical application, we found that the confidence region for $\widetilde{\gamma}$ obtained from imposing $\Gamma_{\text{H}}(\widetilde{\gamma}, T/\Lambda_{2t}, \Lambda_{3t})$ coincided with the confidence region obtained from imposing $\Gamma_{\text{H}}(\widetilde{\gamma}, T/\Lambda_{2t}, \Lambda_{3t}, \Lambda'_{2t}, \Lambda'_{3t})$. In other words, imposing the restrictions $\Lambda'_{2t}(\widetilde{\gamma})$ and $\Lambda'_{3t}(\widetilde{\gamma})$ for all $t \in \{1, 2, \ldots, T\}$ did not shrink $\Gamma_{\text{H}}(\widetilde{\gamma}, T/\Lambda_{2t}, \Lambda_{3t})$, implying from (A-30) that

$$\Gamma_{\text{H}}(\widetilde{\gamma}, T/\Lambda_{2t}, \Lambda_{3t}\Lambda'_{2t}, \Lambda'_{3t}) = \Gamma_{\text{H}}(\widetilde{\gamma}, T) = \Gamma_{\text{H}}(\widetilde{\gamma}, T/\Lambda_{2t}, \Lambda_{3t}).$$

In this way, we computed the confidence region, $\Gamma_{\text{H}}(\widetilde{\gamma}, T)$, without solving for $w_s^{(1,0)}(x, \widetilde{\gamma}/b_{t+1})$ or $w_s^{(0,1)}(x, \widetilde{\gamma}/b_{t+1})$.

## E.3 Measurement Error

Abnormal returns to the firm are defined as the residual component of returns that cannot be priced by aggregate factors the manager does not control. More specifically, let $V_{nt}$ denote the equity value of firm $n$ at time $t$ on the stock market, and let $\widetilde{x}_{nt}$, net abnormal returns, defined as the financial return on its stock net of the financial return on the market portfolio in period $t$. Gross abnormal returns for the $n^{\text{th}}$ firm in period $t$ attributable to the manager's actions are defined as net abnormal returns plus compensation as a ratio of firm equity:

$$x_{nt} \equiv \widetilde{x}_{nt} + \frac{w_{nt}}{V_{n,t-1}}. \tag{A-31}$$

In an optimal contract, compensation depends on $x_{nt}$, not $\widetilde{x}_{nt}$. If $w_{nt}$ were observed without error, then we could deduce $x_{nt}$ directly from $(\widetilde{x}_{nt}, w_{nt}, V_{n,t-1})$ and apply the estimator to obtain $w_{nt}$ for each $z_{nt}$. In that case, ignoring dynamic concerns, we could compute the test statistics described in Section 5.2 of the main text.

However the series we construct on executive compensation, $w_{nt}$, is assumed to be measured with error, rendering the estimator described in Section 5.2 inconsistent. Measured compensation, denoted $\widetilde{w}_{nt}$, is the sum of true compensation $w_{nt}$ plus an independently distributed disturbance term $\varepsilon_t$, assumed orthogonal to the other variables of interest:

$$\widetilde{w}_{nt} = w_{nt} + \varepsilon_{nt}. \tag{A-32}$$

Although $(\widetilde{w}_{nt}, \widetilde{x}_{nt})$ rather than $(w_{nt}, x_{nt})$ is observed for each $(n, t)$, we can nevertheless construct consistent estimates of $(w_{nt}, x_{nt})$ from $(\widetilde{w}_{nt}, \widetilde{x}_{nt})$ by exploiting a premise of the model that the manager is risk averse under a mild regularity condition, that net abnormal returns to shareholders increase with gross abnormal returns; in other words, the manager does not appropriate all the increase in the firm value.

**Theorem E.1** *For all* $(x_1, x_2) \in R^2$

$$w_{nt} = E_t[\widetilde{w}_{nt} | \widetilde{x}_{nt}, r_{nt}, V_{n,t-1}] \tag{A-33}$$

This theorem implies that the compensation schedule is the conditional expectation of measured compensation given net abnormal returns and lagged firm size. Pointwise-consistent estimates of compensation, $w_{nt}$, can be obtained for each observation with Kernel estimators of successive cross sections. From our estimates of $w_{nt}$, we then construct a consistent estimator of the gross abnormal return, which we denote

$$x_{nt}^{(N)} \equiv \widetilde{x}_{nt} + w_{nt}^{(N)}/V_{n,t-1}. \tag{A-34}$$

## E.4 Estimation

In the PMH2, PMH3, HMH models, the components of $Q_2(\widetilde{\gamma})$, $Q_3(\widetilde{\gamma})$, $Q_{\mathrm{H}}(\widetilde{\gamma})$ are formed from the probability density functions characterizing abnormal returns—conditional on the firm's characteristics, $z$, and the manager's report—, $f_r(x,z)$, and the nonlinear regression function of compensation on abnormal returns and the same set of variables, denoted by $w_{rt}(x,z)$. Below we described our estimates of the compensation scheme, $w_{rt}^{(N)}(x,z)$, the probability densities, $f_r^{(N)}(x,z)$, and the probabilities, $\varphi_s^{(N)}(z)$. From these estimated functions, we directly form the estimated weighted ratio $h^{(N)}(x,z)$. Our structural analysis inputs vectors of the form $\left(w_{nt}^{(N)}, x_{nt}^{(N)}, r_{nt}, z_{nt}, V_{n,t-1}\right)$, and the subsampling methods we use to obtain test statistics compute the vectors in each subsample. Denote by $f_r(x,z)$ the conditional density of abnormal returns $x$ given the true state $s$ and the firm's characteristics $z$.

**Compensation schedules** We estimate $w_r^o(x,z)$ with the nonparametric kernel regression:

$$w_{nrt}^{(N)}(x,z) = \frac{\sum_{m=1}^{N} \boldsymbol{I}\{r_{mt}=r,\ z_{mt}=z\} K\left(\frac{x-\widetilde{x}_{mt}}{\varsigma_{1N}}, \frac{V_{n,t-1}-V_{m,t-1}}{\varsigma_{2N}}\right) \widetilde{w}_{nt}}{\sum_{m=1}^{N} \boldsymbol{I}\{r_{mt}=r,\ z_{mt}=z\} K\left(\frac{x-\widetilde{x}_{mt}}{\varsigma_{1N}}, \frac{V_{n,t-1}-V_{m,t-1}}{\varsigma_{2N}}\right)}, \tag{A-35}$$

where $K\left(\frac{x-\widetilde{x}_{mt}}{\varsigma_{1N}}, \frac{V_{n,t-1}-V_{m,t-1}}{\varsigma_{2N}}\right)$ is a bivariate kernel, $\varsigma_{1N}$ and $\varsigma_{2N}$ are the bandwidths associated with $x$ and $V_{n,t-1}$ respectively, and $\boldsymbol{I}\{r_{mt}=r,\ z_{mt}=z\}$ is an indicator function that takes the value one if $r_{nt}=r$ for firm type $z$ and zero otherwise.

**Probability densities** We use the simple frequency estimator of $\varphi_r(z)$ defined by

$$\varphi_1^{(N)}(z) = (TN)^{-1} \sum_{t=1}^{T} \sum_{n=1}^{N} \boldsymbol{I}\{r_{nt}=1, z_{nt}=z\}. \tag{A-36}$$

The probability density, $f_r(x,z)$, is nonparametrically estimated by

$$f_r^{(N)}(x,z) = \frac{\sum_{t=1}^{T} \sum_{n=1}^{N} \boldsymbol{I}\{r_{nt}=1, z_{nt}=z\} K_1\left(\frac{x-x_{nt}^{(N)}}{\varsigma_{3N}}\right)}{\varsigma_{3N} \sum_{t=1}^{T} \sum_{n=1}^{N} \boldsymbol{I}\{r_{nt}=1, z_{nt}=z\}\}}, \tag{A-37}$$

where $K_1\left(\frac{x-x_{nt}}{\varsigma_{3N}}\right)$ is the kernel $\varsigma_{3N}$ is the bandwidth associated with $x$, and $x_{nt}^{(N)}$ is formed by substituting $w_{nrt}^{(N)}(x,z)$ into Equation (A-34). In the HMH model, $h(x,z)$ is estimated by

$$h^{(N)}(x,z) \equiv (1-\varphi_1^{(N)}(z)) f_2^{(N)}(x,z) / \varphi_1^{(N)}(z) f_1^{(N)}(x,z). \tag{A-38}$$

**Boundary conditions** We also require an estimate of $\overline{w}_{rt}$ to form estimates of $\overline{v}_{rt}(\gamma) \equiv \exp\left[-\widetilde{\gamma}\overline{w}_{rt}/b_{t+1}\right]$. However, in the presence of measurement error, the simple boundary estimator used in the body of the paper may not be consistent. We use the fact that although $\overline{w}_{rt}$ is unknown, $w_{rt}(x)$ is a locally nondecreasing function in $x$ in the limit as $x \to \infty$, to define an alternative boundary estimator that is robust to the presence of measurement error in observed compensation. Following Brunk (1958), given firm type, for each state $r \in \{1,2\}$ and period $t \in \{1,\ldots,T\}$, we rank the observations on returns in decreasing order by $x_{rt}^{(1)}$, $x_{rt}^{(2)},\ldots$ and so on, denoting by $w_{rt}^{(1)}$, $w_{rt}^{(2)},\ldots$ the corresponding (estimated) compensations. Letting $w_t(q)$ correspond to the $q^{\text{th}}$ highest value of $w_{nt}$ within the subset of data formed from observations for which $r_{nt} = r$, we estimate $\overline{w}_{rt}$ with:

$$\overline{w}_{rt}^{(N)}(z) \equiv \max_q \left[q^{-1} \textstyle\sum_{n=1}^{N} \boldsymbol{I}\left\{w_{nt} \geq w_t(q)\right\} \boldsymbol{I}\left\{r_{nt} = r, z_{nt} = z\right\}\right] \tag{A-39}$$

for each state $r \in \{1,2\}$.

Finally, we require estimates of $g_s(x,z)$, which we denote by $g_s^{(N)}(\gamma,x,z)$. Note from Theorem 3.1 in the main text that $g_2^{(N)}(\gamma,x,z)$ can be directly found from $\overline{w}_{st}^{(N)}$, but that $g_1^{(N)}(\gamma,x,z)$ also requires an estimate of $\overline{h}(z)$. L'Hospital's rule yields

$$\overline{h}(z) = \frac{\varphi_2(z)}{\varphi_1(z)}\left\{\lim_{x\to\infty}\left[\frac{f_2(x,z)}{f_1(x,z)}\right]\right\} = \frac{\varphi_2(z)}{\varphi_1(z)}\left\{\lim_{x\to\infty}\left[\frac{1-F_2(x,z)}{1-F_1(x,z)}\right]\right\}.$$

Ranking excess returns realized in the first state achieved at the end of any period $t \in \{1,\ldots,T\}$, we obtain the decreasing sequence $x^{(1)}$, $x^{(2)},\ldots$. Again following Brunk (1958), we estimate $\overline{h}(z)$ with

$$\overline{h}^{(N)}(z) \equiv \max_q \left[q^{-1}\sum_{t=1}^{T}\sum_{n=1}^{N} 1\left\{x_{nt}^{(N)} \geq x^{(q)}\right\} 1\left\{r_{nt}^{(N)} = 2, z_{nt} = z\right\}\right]. \tag{A-40}$$

## E.5 Construction of Confidence Intervals

To impose the restrictions embodied in the dynamic version of PMH2 and PMH3 models, we form nonparametric estimators $Q_{st}^{(N)}(\widetilde{\gamma})$ and $\Upsilon_{jt}^{(N)}(\widetilde{\gamma})$ for $Q_{st}(\widetilde{\gamma})$ and $\Upsilon_{jt}(\widetilde{\gamma})$ from estimates of their components. From our estimates of the compensation scheme, $w_{rt}^{(N)}(x,z)$, maximum compensation, $\overline{w}_{st}^{(N)}$, and the probability densities, $f_r^{(N)}(x,z)$ we directly form the estimated $Q_{st}^{(N)}(\widetilde{\gamma})$, and $\Upsilon_{jt}^{(N)}(\widetilde{\gamma})$ for $j \in \{1,2\}$ using the definitions of $\Upsilon_{jt}(\widetilde{\gamma})$ given in the previous section for firm type $z$. Defining the sample analogue to $Q_2(\widetilde{\gamma})$ and $Q_3(\widetilde{\gamma})$ as

$$Q_2^{(N)}(\widetilde{\gamma}) = \sum_{t=1}^{T}\sum_{s=1}^{2}\left[\min\left\{0, Q_{st}^{(N)}(\widetilde{\gamma})\right\}^2 + \Upsilon_{jst}^{(N)}(\widetilde{\gamma})^2\right] \tag{A-41}$$

and

$$Q_3^{(N)}(\widetilde{\gamma}) = \sum_{t=1}^{T}\sum_{s=1}^{2}\left[\min\left\{0, Q_{st}^{(N)}(\widetilde{\gamma})\right\}^2\right].$$

To impose the restrictions embodied in the dynamic version of the HMH, we form non-parametric estimators $\Psi_{jt}^{(N)}(\widetilde{\gamma})$ and $\Lambda_{kt}^{(N)}(\widetilde{\gamma})$ for $\Psi_{jt}(\widetilde{\gamma})$ and $\Lambda_{kt}(\widetilde{\gamma})$ from estimates of their components. From our estimates of the compensation scheme, $w_{st}^{(N)}(x)$, the probability densities, $f_s^{(N)}(x)$, and the probabilities, $\varphi_s^{(N)}$, we directly form the estimated weighted ratio, $h^{(N)}(x,z)$, $Q_{st}^{(N)}(\widetilde{\gamma})$, and $\Psi_{jt}^{(N)}(\widetilde{\gamma})$ for $j \in \{1,2,5\}$ using the definitions of $\Psi_{jt}(\widetilde{\gamma})$ given in the previous section. In addition to $\overline{w}_{st}^{(N)}$, we also require an estimate of $\overline{h}(z)$ and $\overline{h}^{(N)}$ which we obtain using the estimators described above. We define the sample analogue to $Q_{\mathrm{H}}(\widetilde{\gamma})$ as

$$Q_{\mathrm{H}}^{(N)}(\widetilde{\gamma}) \equiv \sum_{t=1}^{T}\sum_{j=5}^{9}\min\left[0,\Psi_{jt}^{(N)}(\widetilde{\gamma})\right]^2 + \sum_{t=1}^{T}\sum_{j=6}^{7}[\Psi_{5t}^{(N)}(\widetilde{\gamma})\,\Psi_{jt}^{(N)}(\widetilde{\gamma})]^2 + \sum_{t=1}^{T}\Psi_{4t}^{(N)}(\widetilde{\gamma})^2 + \sum_{t=1}^{T}[\Psi_{6t}^{(N)}(\widetilde{\gamma})\,\Psi_{8t}^{(N)}(\widetilde{\gamma})]^2 + \sum_{t=1}^{T}\sum_{k=3}^{5}\min\left[0,\Lambda_{kt}^{(N)}(\widetilde{\gamma})\right]^2. \quad \text{(A-42)}$$

Given appropriate regularity conditions, the law of large numbers implies $Q_{st}^{(N)}(\gamma)$, $\Upsilon_{jt}^{(N)}(\widetilde{\gamma})$, $\Psi_{jt}^{(N)}(\widetilde{\gamma})$, and $\Lambda_{kt}^{(N)}(\widetilde{\gamma})$ converges to their population counterparts, and we denote their rate of convergence by $N^a$. Let $\Gamma_{2\delta}^{(N)}$, $\Gamma_{3\delta}^{(N)}$, and $\Gamma_{\mathrm{H}\delta}^{(N)}$ denote the set of risk-aversion parameters that asymptotically cover the observationally equivalent sets of $\widetilde{\gamma} > 0$ under the three models with probability $1-\delta$. Let $c_{2\delta}^{(N)}$, $c_{3\delta}^{(N)}$, $c_{\mathrm{H}\delta}^{(N)}$ denote a consistent estimators for $c_{2\delta}$, $c_{3\delta}$, and $c_{\mathrm{H}\delta}$ respectively, critical values associated with tests of size $\delta$, and define $\Gamma_{2\delta}^{(N)}$, $\Gamma_{3\delta}^{(N)}$, and $\Gamma_{\mathrm{H}\delta}^{(N)}$ as

$$\Gamma_{2\delta}^{(N)} \equiv \left\{\widetilde{\gamma} > 0 : Q_2^{(N)}(\widetilde{\gamma}) \le c_{2\delta}^{(N)}\right\}, \quad \text{(A-43)}$$

$$\Gamma_{3\delta}^{(N)} \equiv \left\{\widetilde{\gamma} > 0 : Q_3^{(N)}(\widetilde{\gamma}) \le c_{3\delta}^{(N)}\right\}, \quad \text{(A-44)}$$

and

$$\Gamma_{\mathrm{H}\delta}^{(N)} \equiv \left\{\widetilde{\gamma} > 0 : Q_{\mathrm{H}}^{(N)}(\widetilde{\gamma}) \le c_{\mathrm{H}\delta}^{(N)}\right\}. \quad \text{(A-45)}$$

Then $\Gamma_{2\delta}^{(N)}$, $\Gamma_{3\delta}^{(N)}$, and $\Gamma_{\mathrm{H}\delta}^{(N)}$ are a consistent estimators of the identified sets $\Gamma_2$, $\Gamma_3$, and $\Gamma_{\mathrm{H}}$.

### E.5.1 Rate of convergence

We now derive the rate of convergence and numerically compute the critical value using the subsampling procedure of Chernozhukov, Tamer and Hong (2007). The rates of convergence and the asymptotic distributions of $Q_{st}^{(N)}(\gamma)$, $\Upsilon_{jt}^{(N)}(\widetilde{\gamma})$, $\Psi_{jt}^{(N)}(\widetilde{\gamma})$, and $\Lambda_{kt}^{(N)}$ are determined by their most slowly converging components. The regularity condition about the upper bound $\overline{x}_r$ plays a role in determining the rate of convergence of $\overline{w}_r^{(N)}$ to $\overline{w}_r$, which is in turn determines the rate of convergence of $Q_{st}^{(N)}(\gamma)$, $\Upsilon_{jt}^{(N)}(\widetilde{\gamma})$, $\Psi_{jt}^{(N)}(\widetilde{\gamma})$, and $\Lambda_{kt}^{(N)}$. Suppose there exists a finite $\overline{x}_r$ such that $F_r(\overline{x}_r, z) < 1$ and if $x > \overline{x}_r$, then $g_r(x,z) = 0$. In that case the derivative of $w_{rt}(x,z)$ at $\overline{x}_r$ is zero, and following Parsons (1978), $\overline{w}_r^{(N)}$ converges to $\overline{w}_r$ at $N^{1/2}$. Appealing to Parsons (1978), $\overline{h}^{(N)}(z)$ converges at rate $N^{1/2}$ to $\overline{h}(z)$ under condition stated in Equation (28) in main text. Although $w_r^{(N)}(x,z)$ and $f_r^{(N)}(x,z)$ converge pointwise more slowly than $N^{1/2}$, the results in Newey and McFadden (1994) imply, for a given $\widetilde{\gamma} > 0$, then $Q_{st}^{(N)}(\gamma)$, $\Upsilon_{jt}^{(N)}(\widetilde{\gamma})$, $\Psi_{jt}^{(N)}(\widetilde{\gamma})$, and and $\Lambda_{kt}^{(N)}$ are asymptotic normal with $\sqrt{N}$ convergence rate.

Alternatively, we can relax the assumption about the existence of a finite $\overline{x}_r$, and assume less restrictively, that $\lim_{x\to\infty} g_r(x,z) = 0$. Then as Wright (1981) shows, $\overline{w}_r^{(N)}$ converges to $\overline{w}_r$ at rate $N^{1/3}$, and $N^{1/3}$ is now the convergence rate. Although the regularity assumption about $\overline{x}_r$ does not affect the estimation of the model or the identification results, it affects the convergence rate of the estimates, and the formula for variance-covariance matrix. As a practical matter we adopt the weaker assumption in our empirical work.

### E.5.2 Subsampling Procedure

Below we give a brief summary of the subsampling procedure used. So, consider all subsets of the data with size $N_b < N$, where $N_b \longrightarrow \infty$, but $N_b/N \longrightarrow 0$, and denote the number of subsets by $B_N$. Define $c_{j0}$ and $\Gamma_{j0}^{(N)}$ for $j = \{2, 3, \mathrm{H}\}$ as

$$c_{j0} \equiv \inf_{\widetilde{\gamma} > \widetilde{\gamma}_N} \left[ N^{1/3} Q_j^{(N)}(\gamma) \right] + \kappa_N$$
$$\Gamma_{j0}^{(N)} \equiv \{\gamma \geq \gamma_N : N^{1/3} Q_j^{(N)}(\gamma) \leq c_{j0}\},$$

where $\kappa_N \propto \ln N$ and $\gamma_N$, a strictly positive sequence, converges to zero at a rate faster than $N^a$. For each subset $i \in \{1, ..., B_N\}$ of size $N_b$ define

$$C_j^{(i,N_b)} \equiv \sup_{\gamma \in \Gamma_{j0}^{(N)}} \left[ (N_b)^{1/3} Q_j^{(i,N_b)}(\gamma) \right],$$

and denote by $c_{j\delta}^{(N)}$ the $\delta-$quantile of the sample $\left\{ C^{(1,N_b)}, \ldots, C^{(B_N,N_b)} \right\}$.

## E.6 Estimation of Welfare Measures

The expected gross output loss to the firm for switching from the distribution of abnormal returns for working to the distribution for shirking is defined as:

$$\triangle_1^j(z, \widetilde{\gamma}) = \sum_{r=1}^{2} \varphi_r(z) \int x(1 - g_r(x, z, \widetilde{\gamma})) f_r(x, z) dx \tag{A-46}$$

for $j = \{3, H\}$ for the PMH3 and HMH models respectively. For the PMH3 model $g_r(x, z, \widetilde{\gamma})$ is given by:

$$g_r(x, z, \widetilde{\gamma}) \equiv \frac{e^{\widetilde{\gamma}/b_{t+1}\overline{w}_{rt+1}(z)} - e^{\widetilde{\gamma}/b_{t+1}w_{rt+1}^o(x,z)}}{e^{\widetilde{\gamma}/b_{t+1}\overline{w}_{rt+1}(z)} - E\left[ e^{\widetilde{\gamma}/b_{t+1}w_{rt+1}^o(x,z)} | z \right]} \tag{A-47}$$

for for each reported state $r = 1, 2$. In the HMH model we define $v_{rt}(x, z, \widetilde{\gamma}) \equiv e^{\widetilde{\gamma}/b_{t+1} w^o_{rt+1}(x,z)}$ and $\overline{v}_{rt}(z, \widetilde{\gamma}) \equiv \sup_x \left[e^{\widetilde{\gamma}/b_{t+1} w^o_{rt+1}(x,z)}\right]$ then $g_2(x, z, \widetilde{\gamma})$ is given by

$$g_2(x, z, \widetilde{\gamma}) \equiv \frac{\overline{v}_{2t}(z, \widetilde{\gamma})^{-1} - v_{2t}(x, z, \widetilde{\gamma})^{-1}}{\overline{v}_{2t}(z, \widetilde{\gamma})^{-1} - E_2\left[v_{2t}(x, z, \widetilde{\gamma})^{-1}\right]}, \tag{A-48}$$

and $g_1(x, z, \widetilde{\gamma})$ is sequentially defined as:

$$g_1(x, z, \widetilde{\gamma}) \equiv \frac{\overline{v}_{1t}(z, \widetilde{\gamma})^{-1} - v_{1t}(x, z, \widetilde{\gamma})^{-1} + \eta_3(z, \widetilde{\gamma})\left[\overline{h}(z) - h(x, z)\right] - \eta_4(z, \widetilde{\gamma}) g_2(x, z, \widetilde{\gamma}) h(x, z) \frac{\widehat{\alpha}_1(z,\widetilde{\gamma})}{\widehat{\alpha}_2(z,\widetilde{\gamma})}}{\eta_1(z, \widetilde{\gamma})} \tag{A-49}$$

where:

$$\eta_4(z, \widetilde{\gamma}) \equiv \frac{\frac{E_1[v_{1t}(x,z,\widetilde{\gamma})|z]}{E[v_{rt}(x,z,\widetilde{\gamma})|z]} - 1 - E_1\left[v_{1t}(x, z, \widetilde{\gamma}) h(x, z)|z\right]\left\{E_2\left[v_{2t}(x, z, \widetilde{\gamma})|z\right]^{-1} - E\left[v_{rt}(x, z, \widetilde{\gamma})|z\right]^{-1}\right\}}{\frac{\widehat{\alpha}_1(z,\widetilde{\gamma})}{\widehat{\alpha}_2(z,\widetilde{\gamma})} E_1\left[v_{1t}(x, z, \widetilde{\gamma}) g_2(x, z, \widetilde{\gamma}) h(x, z)|z\right] - E_1\left[v_{1t}(x, z, \widetilde{\gamma}) h(x, z)|z\right]} \tag{A-50}$$

$$\eta_3(z, \widetilde{\gamma}) \equiv E_2\left[v_{2t}(x, z, \widetilde{\gamma})|z\right]^{-1} - \eta_4(z, \widetilde{\gamma}) - E\left[v_{rt}(x, z, \widetilde{\gamma})|z\right]^{-1} \tag{A-51}$$

$$\eta_1(z, \widetilde{\gamma}) \equiv \frac{\widehat{\alpha}_1(z, \widetilde{\gamma})}{\widehat{\alpha}_2(z, \widetilde{\gamma})}\left\{\overline{v}_{1t}(z, \widetilde{\gamma}) - E\left[v_{rt}(x, z, \widetilde{\gamma})|z\right]^{-1} + \eta_3(z, \widetilde{\gamma})\overline{h}(z)\right\} \tag{A-52}$$

$$\widehat{\alpha}_2(z, \widetilde{\gamma}) \equiv \left[\int \sum_{s=1}^{2} \varphi_s v_{st}(x, z, \widetilde{\gamma}) f_s(x, z) \mathrm{d}x\right]^{-1} \tag{A-53}$$

$$\widehat{\alpha}_1(z, \widetilde{\gamma}) \equiv \widehat{\alpha}_2(z, \widetilde{\gamma})\left\{\frac{\overline{v}_{2t}(z, \widetilde{\gamma})^{-1} - E_2\left[v_{2t}(x, z, \widetilde{\gamma})^{-1}|z\right]}{\overline{v}_{2t}(z, \widetilde{\gamma})^{-1} - E_2\left[v_{2t}(x, z, \widetilde{\gamma})|z\right]^{-1}}\right\}. \tag{A-54}$$

The manager's compensating differential from shirking versus working, $\triangle^j_2(z, \widetilde{\gamma})$, is defined as the difference between the certainty equivalent wages for working and shirking. It is given by:

$$\triangle^j_2(z, \widetilde{\gamma}) = w^{(2)}_j(z, \widetilde{\gamma}) - w^{(1)}_j(z, \widetilde{\gamma}) \tag{A-55}$$

for $j = \{3,\text{H}\}$ for the PMH3 and HMH models respectively. For the PMH3 model $w^{(1)}_j(z, \widetilde{\gamma})$ is given by:

$$w^{(1)}_{3r}(z, \widetilde{\gamma}) = \frac{b_{t+1}}{\widetilde{\gamma} b_t} \ln\{E_r\left[e^{-\widetilde{\gamma}/b_{t+1} w^o_{rt+1}(x,z)}|z\right]^{-1}\} \tag{A-56}$$

for each reported state $r = \{1, 2\}$ and for the HMH model:

$$w^{(1)}_{\text{H}}(z, \widetilde{\gamma}) = \frac{b_{t+1}}{\widetilde{\gamma} b_t} \ln\left\{\left[\int \sum_{s=1}^{2} \varphi_s v_{st}(x, z, \widetilde{\gamma}) f_s(x, z) \mathrm{d}x\right]^{-1}\right\}. \tag{A-57}$$

For the PMH3 model $w^{(2)}_j(z, \widetilde{\gamma})$ is given by:

$$w^{(2)}_{3r}(z, \widetilde{\gamma}) = \frac{b_{t+1}}{\widetilde{\gamma} b_t} \ln\left(\frac{1 - E_r\left[e^{\widetilde{\gamma}/b_{t+1} w^o_{rt+1}(x,z) - \widetilde{\gamma}/b_{t+1} \overline{w}_{rt+1}(z)}|z\right]}{E_r\left[e^{-\widetilde{\gamma}/b_{t+1} w^o_{rt+1}(x,z)}|z\right] - e^{\widetilde{\gamma}/b_{t+1} \overline{w}_{rt+1}(z)}}\right) \tag{A-58}$$

for each reported state $r = \{1, 2\}$ and for the HMH model:

$$w_{\mathrm{H}}^{(2)}(z, \widetilde{\gamma}) = \frac{b_{t+1}}{\widetilde{\gamma} b_t} \ln\left(\frac{\overline{v}_{2t}(z, \widetilde{\gamma})^{-1} - E_2\left[v_{2t}(x, z, \widetilde{\gamma})^{-1}|z\right]}{E[v_{rt}(x, z, \widetilde{\gamma})|z]\{\overline{v}_{2t}(z, \widetilde{\gamma})^{-1} - E_2\left[v_{2t}(x, z, \widetilde{\gamma})|z\right]^{-1}\}}\right). \tag{A-59}$$

Finally the the risk premium from agency, $\triangle_3^j(z, \widetilde{\gamma})$, is given by:

$$\triangle_3^j(z, \widetilde{\gamma}) = \begin{cases} E[w_{rt+1}^o(x, z)|z] - \sum_{r=1}^2 \varphi_r w_{3r}^{(2)}(z, \widetilde{\gamma}) & \text{for } j = 3 \\ E[w_{rt+1}^o(x, z)|z] - w_{\mathrm{H}}^{(2)}(z, \widetilde{\gamma}) & \text{for } j = \mathrm{H} \end{cases} \tag{A-60}$$

for $j = \{3,\mathrm{H}\}$ for the PMH3 and HMH models respectively. Using the above formula we calculate the confidence interval using the empirical counterpart of $w_{rt+1}^o(x, z)$, $\overline{w}_{rt+1}(z)$, $f_r(x, z)$, $\varphi_r$, $h(x, z)$, and $\overline{h}(z)$ estimated in Section E.4 from the estimated confidence interval for $\widetilde{\gamma}$.

# F Proofs

**Proof of Lemma A.1.** Let $\lambda_{t'}$ be the date-$t$ price of a contingent claim made on a consumption unit at date $t'$, implying the bond price is defined as

$$b_t \equiv E_t\left[\sum_{t'=t}^{\infty} \lambda_{t'}\right],$$

and let $q_t$ denote the date-$t$ price of a security that pays off the random quantity

$$q_t \equiv E_t\left[\sum_{t'=t}^{\infty} \lambda_{t'}\left(\ln \lambda_{t'} - t' \ln \beta\right)\right].$$

From Equation (15) on page 680 of Margiotta and Miller (2000), the value to a manager with current wealth endowment $e_{nt}$ of announcing state $r_t(s)$ in period $t$ when the true state is $s$ and choosing effort level $l_{st2}$ in anticipation of compensation $w_{r_t(s)t}(x)$ at the beginning of period $t+1$ when he retires one period later is

$$-b_t \widetilde{\alpha}_2^{1/b_t}\left\{E\left[\exp\left(-\frac{\widetilde{\gamma} w_{r_t(s)t}(x)}{b_{t+1}}\right)\right]\right\}^{1-1/b_t} \exp\left(-\frac{q_t + \widetilde{\gamma} e_{nt}}{b_{t+1}}\right).$$

The corresponding value from choosing effort level $l_{st1}$ is

$$-b_t \widetilde{\alpha}_1^{1/b_t}\left\{E_t\left[\exp\left(-\frac{\widetilde{\gamma} w_{r_t(s)t}(x)}{b_{t+1}}\right)\left[g_s(x)\right]\right]\right\}^{1-1/b_t} \exp\left(-\frac{q_t + \widetilde{\gamma} e_{nt}}{b_{t+1}}\right),$$

whereas from their Equation (8) on page 678, the value from retiring immediately is

$$-b_t \exp\left(-\frac{q_t + \widetilde{\gamma} e_{nt}}{b_{t+1}}\right).$$

Dividing each expression through by the retirement utility, it immediately follows that the manager chooses $l_{st} \equiv (l_{t0}, l_{st1}, l_{st2})$ to minimize the negative of expected utility:

$$l_{t0} + (\widetilde{\alpha}_1 l_{st1} + \widetilde{\alpha}_2 l_{st2})^{1/b_t} \left\{ E\left[ \exp\left( -\frac{\widetilde{\gamma} w_{r_t(s)t}(x)}{b_{t+1}} \right) [g_s(x) l_{st1} + l_{st2}] \right] \right\}^{1-1/b_t}$$
$$= l_{t0} + \left\{ (\widetilde{\alpha}_1 l_{st1} + \widetilde{\alpha}_2 l_{st2})^{1/(b_t-1)} E_t \left[ \exp\left( -\frac{\widetilde{\gamma} w_{r_t(s)t}(x)}{b_{t+1}} \right) [g_s(x) l_{st1} + l_{st2}] \right] \right\}^{(b_t-1)/b_t}.$$

Since $l_{t0} \in \{0, 1\}$ and $b_t > 1$, the solution to this optimization problem also solves

$$l_{t0} + (\widetilde{\alpha}_1 l_{st1} + \widetilde{\alpha}_2 l_{st2})^{1/(b_t-1)} E_t \left[ \exp\left( -\frac{\widetilde{\gamma} w_{r_t(s)t}(x)}{b_{t+1}} \right) [g_s(x) l_{st1} + l_{st2}] \right].$$

Multiplying through by the factor $(\widetilde{\alpha}_1 l_{st1} + \widetilde{\alpha}_2 l_{st2})^{1/(b_t-1)}$ and summing over the two states $s \in \{1, 2\}$ yields the minimand in Lemma A.1. ■

**Proof of Lemma A.2.** In our model, the proof of Proposition 5 in Margiotta and Miller (2000) can be simply adapted to show that Theorem 3 of Fudenberg, Holmstrom and Milgrom (1990) applies, thus demonstrating that the long-term optimal contract can be sequentially implemented. An induction completes the proof by establishing that the sequential contract implementing the optimal long-term contract for a manager who will retire in $\overline{\tau}$ periods replicates the one-period optimal contract. In the optimal short-term contract, the participation constraint is satisfied with strict equality, which implies that at the beginning of period $\overline{\tau} - 1$ the expected lifetime utility of the manager is determined by setting $t = \overline{\tau} - 1$ in the equation

$$-b_t \exp\left( -\frac{a_t + \widetilde{\gamma} e_t}{b_t} \right). \tag{A-61}$$

Suppose that at the beginning of all periods $t \in \{\tau + 1, \tau + 2, \ldots, \overline{\tau} - 1\}$; the expected lifetime utility of the manager is given by Equation (A-61). We first show the expected lifetime utility of the manager at $\tau$ is also given by Equation (A-61). From Lemma 3.1 in the main text, the problem shareholders solve at $\tau$ is identical to the short-term optimization problem solved in the text. In the solution to each cost-minimization subproblem for the four $(L_{1t}, L_{2t})$ choices, the manager's participation constraint is met with equality. Consequently, the manager achieves the expected lifetime utility given by Equation (A-61), as claimed. Therefore the problem of participating at time $\tau$ and possibly continuing with the firm for more than one period reduces to the problem of participating at time $\tau$ for one period at most, solved in Lemma A.1. The induction step now follows. ■

**Proof of Theorem E.1.** For notational convenience, and without loss of generality, we suppress the dependence of compensation $w_{nt}$ on $(s_{nt}, b_t, b_{t+1})$. Let $\widetilde{x}$ denote the net excess returns, $x$ gross excess returns, $w(x)$ the compensation schedule as a mapping from gross excess returns, and $V$ the value of the firm at the beginning of the period. By our definition of net and gross excess returns,

$$\widetilde{x} = x - w(x)/V. \tag{A-62}$$

Suppose there exists for some $(\widetilde{x}_0, V_0)$, two distinct values of net excess returns, denoting $x_1 \in R$ and $x_2 \in R$, satisfying Equation (A-62). Then,

$$\widetilde{x}_0 = x_i - w(x_i)/V_0$$

for $i \in \{1, 2\}$ which implies

$$V_0(x_2 - x_1) = w(x_2) - w(x_1).$$

But this possibility is ruled out in the theorem's premise. Therefore, a unique solution to the relation defined by Equation (A-62) exists for each pair $(\widetilde{x}, V)$, and we can denote the solution mapping by $x \equiv X(\widetilde{x}, V)$. Substituting $X(\widetilde{x}, V)$ for $x$ in $w(x)$, we define $\Lambda(\widetilde{x}, V) \equiv w[X(\widetilde{x}, V)]$. The theorem now follows because the measurement error on compensation is assumed to be independent of $(\widetilde{x}, V)$, so $E[\widetilde{w} | \widetilde{x}, V] = \Lambda(\widetilde{x}, V)$. ■

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