Overview

Every game has an extensive form, and knowing the extensive form is tantamount to knowing the game itself. Moreover many games can be easily solved using the extensive form. It is therefore fitting that we begin our analysis of strategic play by explaining the extensive form of a game. This chapter describes the elements that comprise the extensive form, and shows how a game can be invented by writing down its extensive form. Then we provide taxonomy of different classes of games, and indicate the chapters where they are analyzed.

Briefly, the elements comprising the extensive form are the players, the possible moves each player can make, the information the player has at the time, and the possible outcomes of the game. The information available to a player might include details about choices made by the other players, and if so this places restrictions on the order in which moves are made. Outcomes of the game determine how players will be rewarded, and are characterized by payoffs to the players at the conclusion of the game. There are, of course, many ways to categorize games, by the number of players, the choice set of each player, and so forth. One way is by the information that players have at their disposal about how the game has progressed when they make their respective moves. Our taxonomy of games is largely based on this criterion.

There are also many ways of describing the same game. Therefore it may not surprise you to discover that although every game has an extensive form, it is not necessarily unique. In the last part of this chapter we demonstrate this fact by example, showing that some games support two or more extensive forms, and in those cases we call them call equivalent extensive forms.

The exercises in this chapter seek to test your knowledge in recognizing and deriving from a general description of a game its extensive form. We also introduce you to different types of summaries the results of experiments in which subjects play extensive form games that you have designed. One question taken up at the end of this chapter is whether presenting subjects in experimental settings with equivalent extensive forms yields the same outcome, and how one might test this hypothesis.

Introductory Examples

Perhaps the fastest way of acquainting yourself with the extensive form of a game and discovering its usefulness is to review the examples below, design the extensive form for the game by copying the diagrams into some experimental software, using the software to conduct experiments indicated in the exercises, and analyzing the outcomes by answering the questions we have posed about each game. The examples we have chosen are mainly but not exclusively taken from the business and social worlds that managers inhabit.

Regional competition

In this first example there are three players, an entrant and two retailers, respectively called big monopolist and small monopolist, which currently hold regional monopolies in the localities they serve. The entrant decides which market to enter, or to stay out. In the absence of competition the big and small monopolies have present values of $20 million and $10 million respectively. If the entrant establishes itself in the large market, and the big monopolist holds a sale, the entrant loses $5 million and the profits of the big monopolist are cut in half. If the big monopolist colludes with the entrant, the newcomer makes $5 million and the big monopolist earns $15 million. In either case the profits of the small monopolist are unaffected. The payoffs to the firms when the entrant establishes itself in the small monopolist's territory are interpreted in a analogous fashion.

Figure 3.1 provides a schematic representation of this game, called its extensive form. There are eight nodes, comprising three decision nodes and five terminal nodes. The initial node is labeled entrant, and two other decision nodes are assigned to the big monopolist and the small monopolist respectively. Seven branches connect the nodes. Three of these join the entrant's decision node, and correspond to that player's choice set. The other four branches are attached to the decision nodes of the big and small monopolists, and correspond to their respective choice sets. Each terminal node shows the payoffs to each of the players that correspond to their preceding choices.

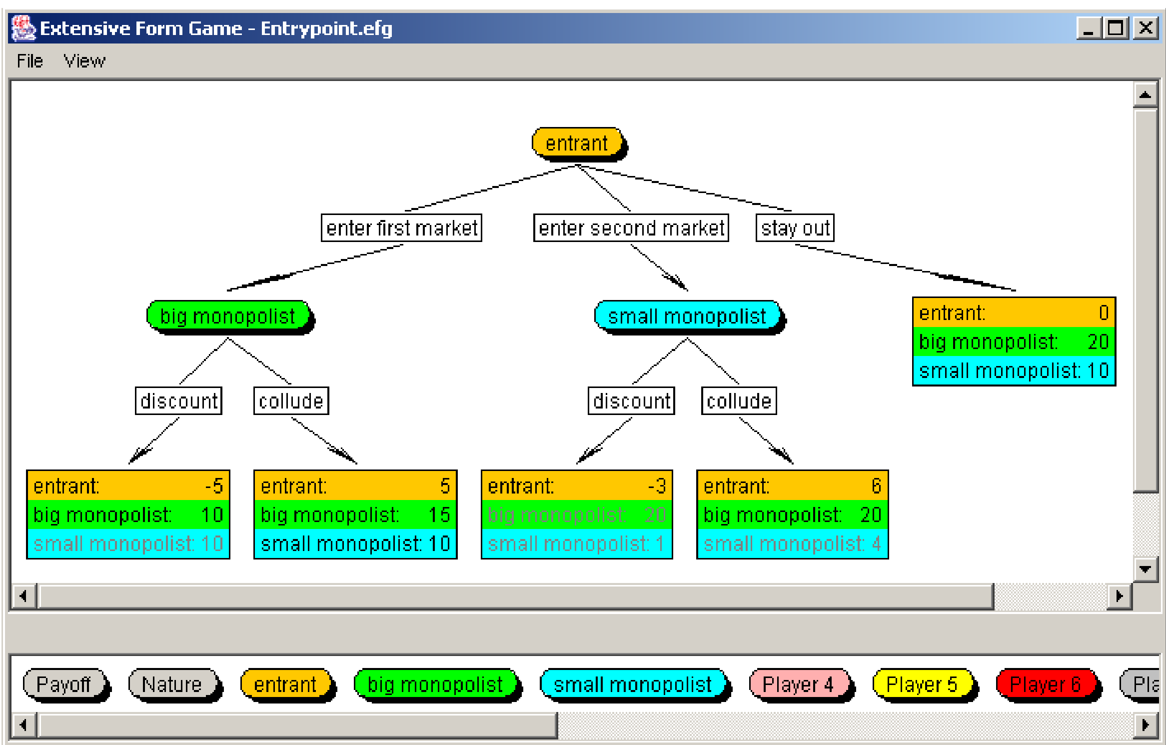

Figure 3.1
Regional competition

We can show how play proceeds by referring to the extensive form. First the entrant makes a choice between entering one of the two markets or staying out. Then if the entrant chooses to enter the monopolist affected decides whether to discount or collude. The bold outline circumscribing the big monopolist's choice node in Figure 3.2 indicates that it is his move. Apparently the entrant chose the big suburb.

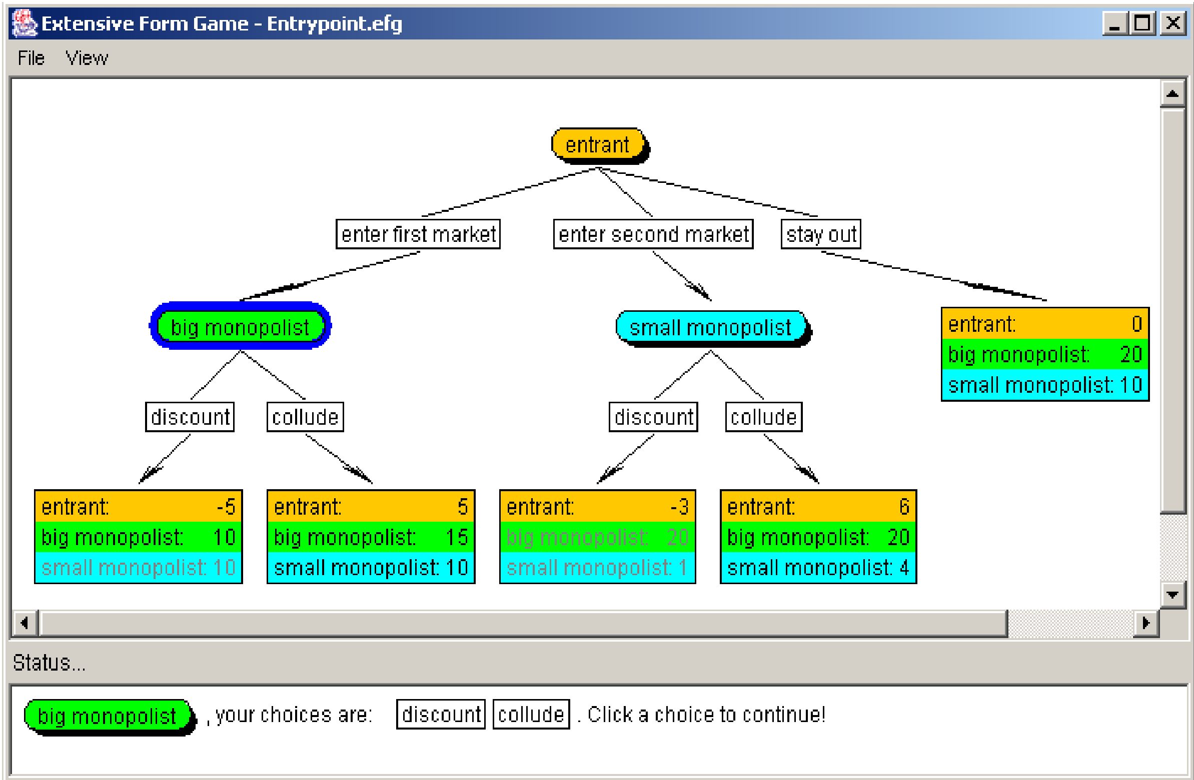

Figure 3.2
The choice node for the big monopolist

At this point the large monopolist may collude, and in that case Figure 3.3 shows a complete history of the game by indicating the outcome. Note there is only one path that could have produced this outcome. The diagram identifies this game history in boldface print, which corresponds to taking the left most branch wherever possible. As play proceeds, certain histories are eliminated as possibilities. For example if Node 2 is reached in Figure 1, then the histories corresponding to Nodes remain feasible but those identified with Nodes 7 through 8 are not.

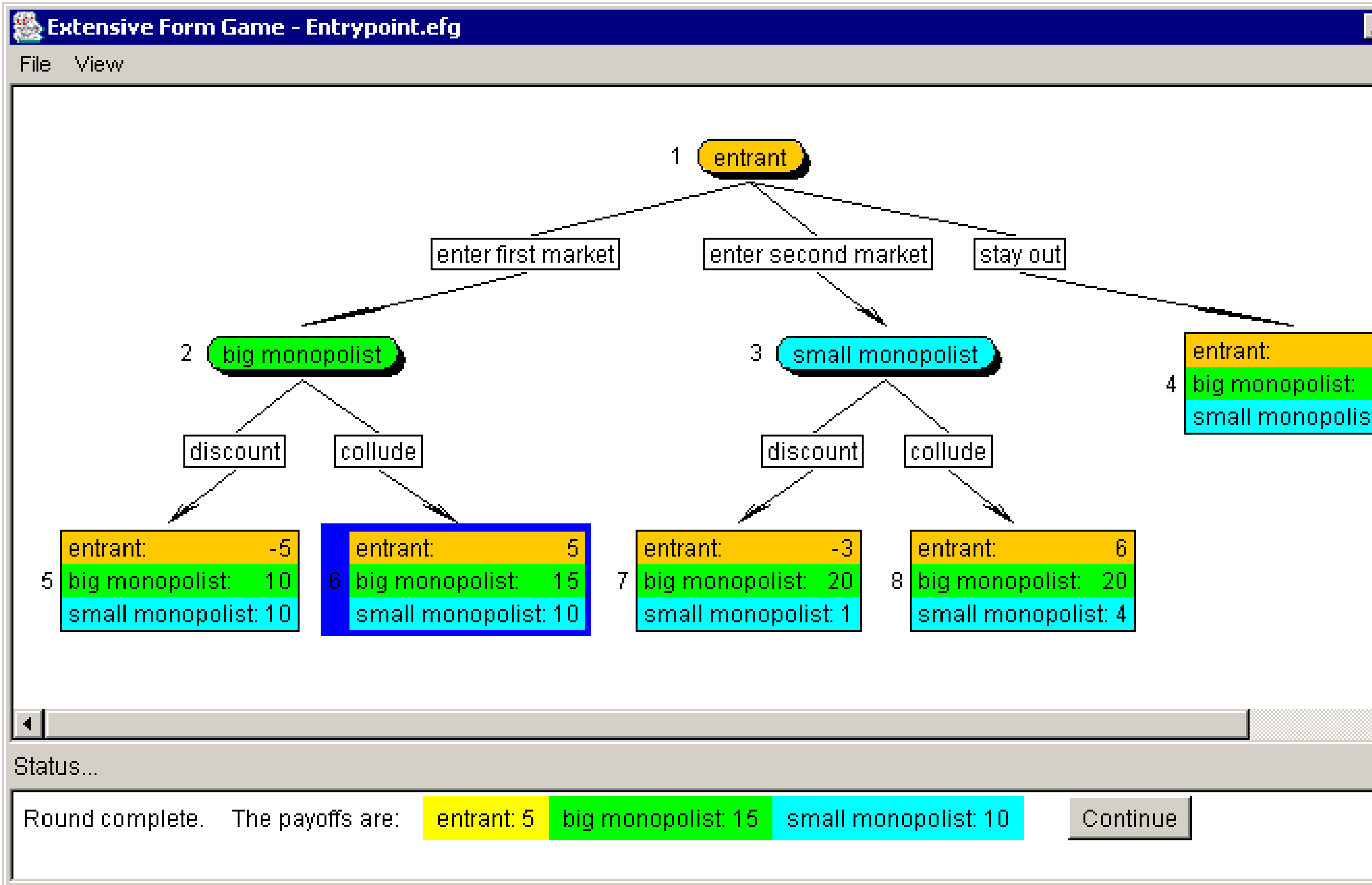

Figure 3.3
A complete history of the game

Design of the experiment

MBA and undergraduate students played several times the game presented in Figure 3.3. They were told that they will be randomly matched with another two people each time they connected to the game. The random matching appropriately represents one period game. However it allows us to observe how subjects change their behavior over time and do not build the reputation with the same player. The set up of the game is shown in Figure 3.4:

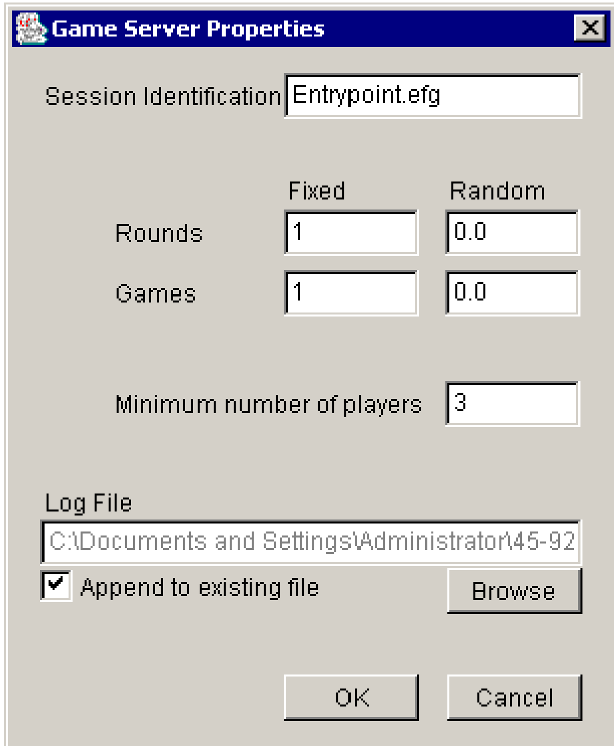

Figure 3.4: Experimental setup for Entry game

Experimental results for the Entry game:

A. Proportion of times each outcome is reached

Table 3.1 reports the frequencies that each of the possible outcome (i.e. terminal node) has been reached for the whole sample, for different programs and different level of experiences. Recall from Figure 3.3 that terminal nodes 4 to 8 are the only possible outcomes in this game.

Terminal Node Number	Whole Sample		Program				Round Number			
	Freq	Percent	Undergraduate		MBA		1		2 and more	
			Freq	Percent	Freq	Percent	Freq	Percent	Freq	Percent
4	3	4.62	1	3.33	2	5.71	1	3.70	2	5.13
5	4	6.15	3	10.00	1	2.86	4	14.81	0	0
6	15	23.08	6	20.00	9	25.71	8	29.63	7	17.95
7	9	13.85	5	16.65	4	11.43	2	7.41	7	17.95
8	34	52.31	15	50.00	19	54.29	12	44.44	23	58.97
N	65		30		35		27		39	

Table 3.1

The terminal nodes 6 and 8 are most often chosen for the whole sample as well as for the different programs and level of experience which will indicates that subjects were not indifferent between selecting their choices. Formally, we can test the hypothesis that subjects have no preferences in the choice of a terminal node, and that each terminal node would be chosen approximately equally likely (fifth of the time). Therefore,

$$H_0\colon p_4 = p_5 = p_6 = p_7 = p_8 = \frac{1}{5}$$

versus

$$H_a : \text{ at least one } p_i \text{ is different from } \frac{1}{5}$$

where p_i is the probability that a subject will choose terminal node $i = 4, 5, 6, 7,$ or 8. Therefore, if we choose α =0.05, we would reject the null hypothesis when $\chi^2 >$ 9.487 (The chi-square test statistic for this example has (k-1)=4 d.f.) Since χ^2 for the whole sample is 49.39 and is greater than the tabulated value of χ^2, 9.487, the null hypothesis is rejected and we can conclude that subjects have a preference for a particular choice that results in a particular terminal node. χ^2 for undergraduates was 19.33 and for the MBA students χ^2 was 31.14. Similarly, χ^2 for round 1 was 16.69 and for round 2 χ^2 was 21.70.

One possibility is that they were trying to pick the choices that will maximize the total sum of the payoffs to all the retailers. The sum of payoffs is 30 for terminal nodes

4, 6 and 8 and is much higher then the sum 15 that can be obtained in terminal node 5 or 18 in terminal node 7. Therefore,

$$H_0 : p_4 = p_6 = p_8 = \frac{1}{3}$$

$$H_a : \text{ at least one } p_i \text{ is different from } \frac{1}{3}$$

χ^2 for the whole sample is 28.19 we reject the null hypothesis at α =0.05 (5.99). Another option is that given that everybody should have a positive payoffs subjects would like to maximize the sum of total payoffs. In this case we would like to test the hypothesis:

$$H_0 : p = p_0 = 1$$

versus the alternative hypothesis:

$$H_0 : p < p_0$$

Test statistic:

$$z = \frac{p-1}{\sigma_p} = \frac{p-1}{\sqrt{\frac{pq}{n}}} \text{ with } p=\frac{x}{n}$$

where x is the number of successes in n binomial trials.

With α =0.05, we would reject H_0 when z < -1.645. With p=49/65=0.7538, the value of the test statistic is

$$z = \frac{p-1}{\sqrt{\frac{pq}{n}}} = \frac{0.7538-1}{\sqrt{\frac{(0.7538)\cdot(0.242)}{65}}} = \frac{-0.2462}{0.053} = -19.96$$

The calculated value of the test statistic falls in the rejection region, and we do reject the null hypothesis.

B. Testing learning

Exercise 1. *Design and run an experiment of the extensive form game of regional competition depicted in Figure 3.1.*

2. *What is the proportion of times each outcome is reached?*

3. *Which outcome maximizes the sum of the payoffs to the retailers? Is the probability of reaching this outcome significantly different from zero?*

4. *Which allocation has the most concentrated income distribution? Is* *there a higher propensity to reach this outcome than the others?*

5. *Can you provide an explanation for the behavior you observed in terms of the payoffs associated with each of the outcomes?*

Recruiting

A recruiting committee has just interviewing a pool of applicants, and found that only one of them is suitable for the job. If any of the others were hired, the firm would actually lose money, after taking into account their compensation. If they make an offer to best current applicant, the firm will gain $1 million if she accepts the offer and lose $0.5 million if she rejects the package. The compensation package itself pays $200,000 and her current salary is $150,000. An alternative to making an offer is to advertise the position again in the hope of finding a better qualified applicant willing to join the firm.

In this game, the recruiting committee moves first choosing to offer the best current applicant the job or continue searching by placing another advertisement in the professional journals. If the current applicant is offered the job, she decides whether to accept the position or not. If the best current applicant is rejected and the firm continues searching, a random variable determines whether another applicant will show up. The search process has four fifth probability of being successful, and revealing a more suitable candidate. In this case the new candidate now has the choice of accepting the position or rejecting it. If a new candidate accepts the position her compensation package would be $210,000 while her current salary is $170,000. The firm will gain $1.3 million if the candidate accepts it, and lose $0.8 million if the offer is rejected. With one fifth probability the search process is not successful and the company looses $ 0.7 million, and the second applicant is offered a compensation package of $220,000 by another firm.

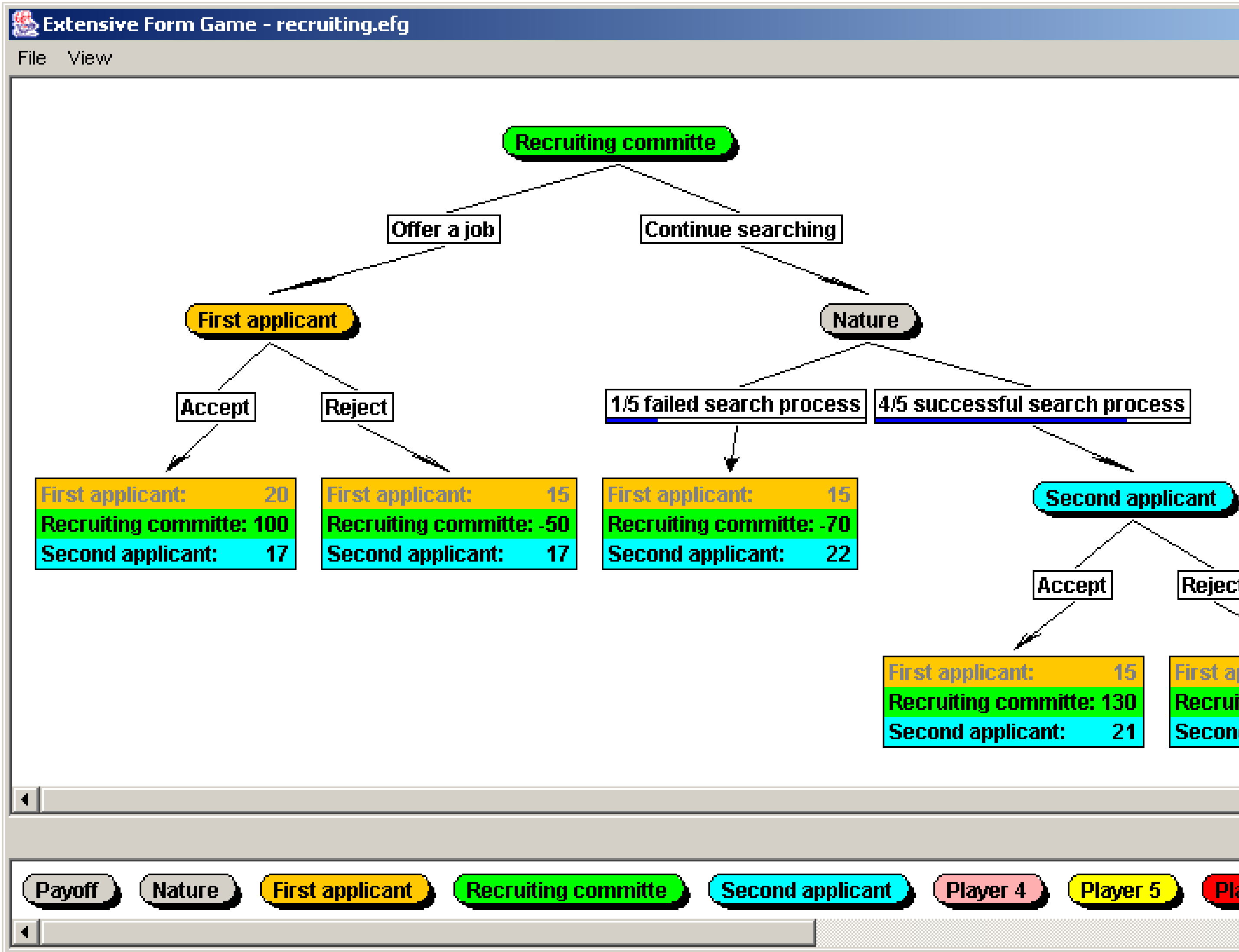

Figure 3.4
Recruiting

The extensive form for this example shows there are nine nodes and eight branches. Three of the nodes are decision nodes. The initial node is assigned to the committee, and the other two decision nodes are assigned to each of the potential recruits. In this game whether the second recruit is available or not depends on their alternative employment options, that is modeled as a random variable.

Exercise **1.** *Design and run a classroom experiment based on the recruiting game.*

2. *What is the relative frequency of reaching each terminal node?*

3. *Which of those relative frequencies are significantly different from zero?*

4. *Modify the extensive form of the recruiting game by raising the probability of a running a successful advertising campaign to 90 percent and* *run the modified game.*

5. *What are the relative frequencies of reaching each terminal node for the modified game with the original game*

6. *Are the relative frequencies for reaching the respective nodes significantly different from one another?*

7. *Modify the experiment of recruiting by changing the payoffs*

8. *Compare the proportion of each node reached with the original game*

9. *Are they significantly different from each other?*

10. *Reviewing the experiments above along with others that change the payoffs, can you draw any broad based conclusions about how the company adjusts to its changing opportunities?*

Matching pennies

In the game of regional competition, neither the big monopolist nor the small monopolist can move until they have seen which territory the entrant has established itself. There are many games where a player must move without knowing all the details about how play has proceeded up to that point. In the game of matching pennies, Player 1 places a penny flat on the table with his hand covering it, simultaneously choosing whether the coin's head or tail is facing up. Then Player 2 places a penny on the table either showing a head or a tail. Finally Player 1 removes his hand. If a tail appears on one coin and a head on the other Player 2 wins both coins. Otherwise Player 1 pockets them both.

The extensive form of matching pennies is depicted in Figure 3.4. There are three decision nodes, four terminal nodes, and six branches representing the choices that the two players might make. Since Player 2 must make her choice after Player 1 has made his choice, there are two nodes at which he must choose, after Player 1 has placed a head face upwards, and also after Player 1 has placed a Tail face upwards. However the rules of the game prevent Player 2 from seeing the choice of Player 1 until after he has placed his coin on the table. Accordingly the decision nodes of Player 2 are connected by a dotted line, to indicate that both nodes belong to the same information set, preventing Player 2 from knowing whether he is at Nodes 2 or 3. It is therefore impossible for the second player to condition his choice on the first player's move.

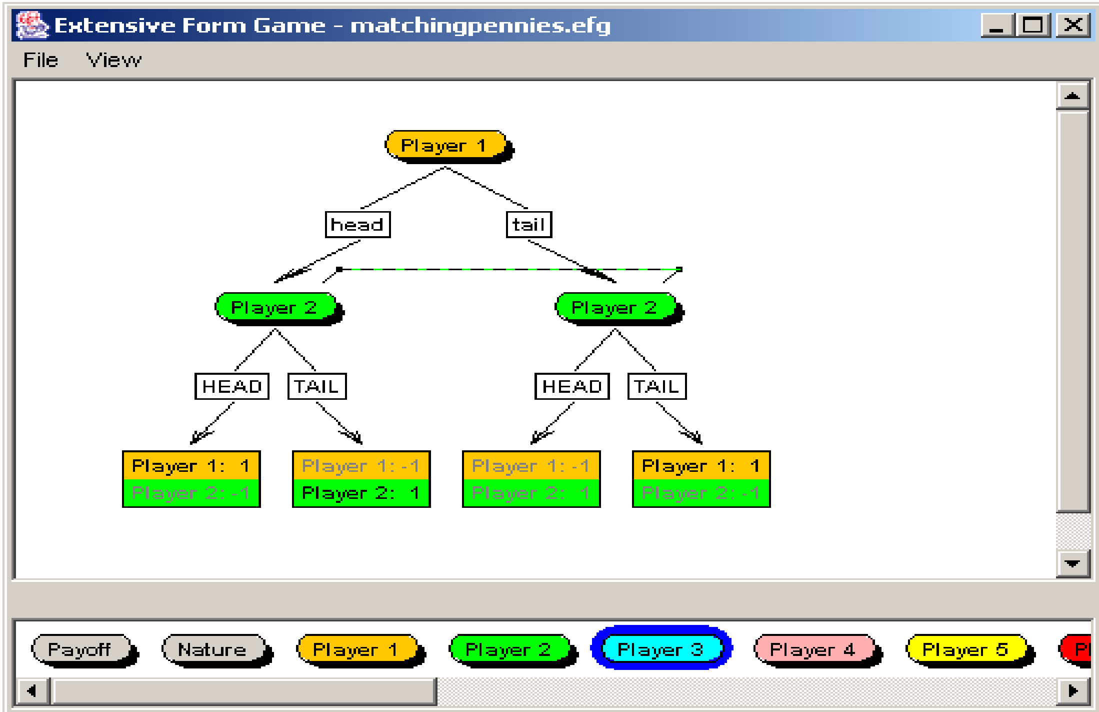

Figure 3.4
Matching pennies

Exercise **1.** *How many decision nodes, terminal nodes, branches and information sets are in the matching pennies game?*

2. *Design and run a matching pennies game.*

3. *Can you reject the hypothesis that players prefer being assigned to the role of first player rather than the second*

4. *Now suppose the second player can observe the outcome of the first player before making his choice. Show how this affects the extensive form.*

5. *Design and run the modified game. Are your results significantly different from the original matching pennies game?*

6. *In the modified game can you reject the hypothesis that players prefer being assigned to the role of first player rather than the second?*

7. *Now repeat the modified game so that players pay the same partner twice each, but do not rotate their roles. Are your results from your results with the outcomes from single games, are*

8. *Do players take turns winning to even out the payouts? Why or why not?*

9. *Does a comparison of the outcomes of the matching pennies game and* *its modification help explain why product development activities might be conducted in secret even when there is very little chance a rival can copy or replicate the discovery?*

Oil Extraction

In the matching pennies game the second player is equally as well informed about the play as the first. Neither of them observe anything about the move of the other player until it is over. By connecting Nodes 2 and 3 the informational advantage the second player has is eliminated. The next example also illustrates this phenomenon in a less dramatic fashion. There are two oil producers who decide how much oil to extract for sale. Each producer chooses the quantity without knowing what the other producer has chosen.

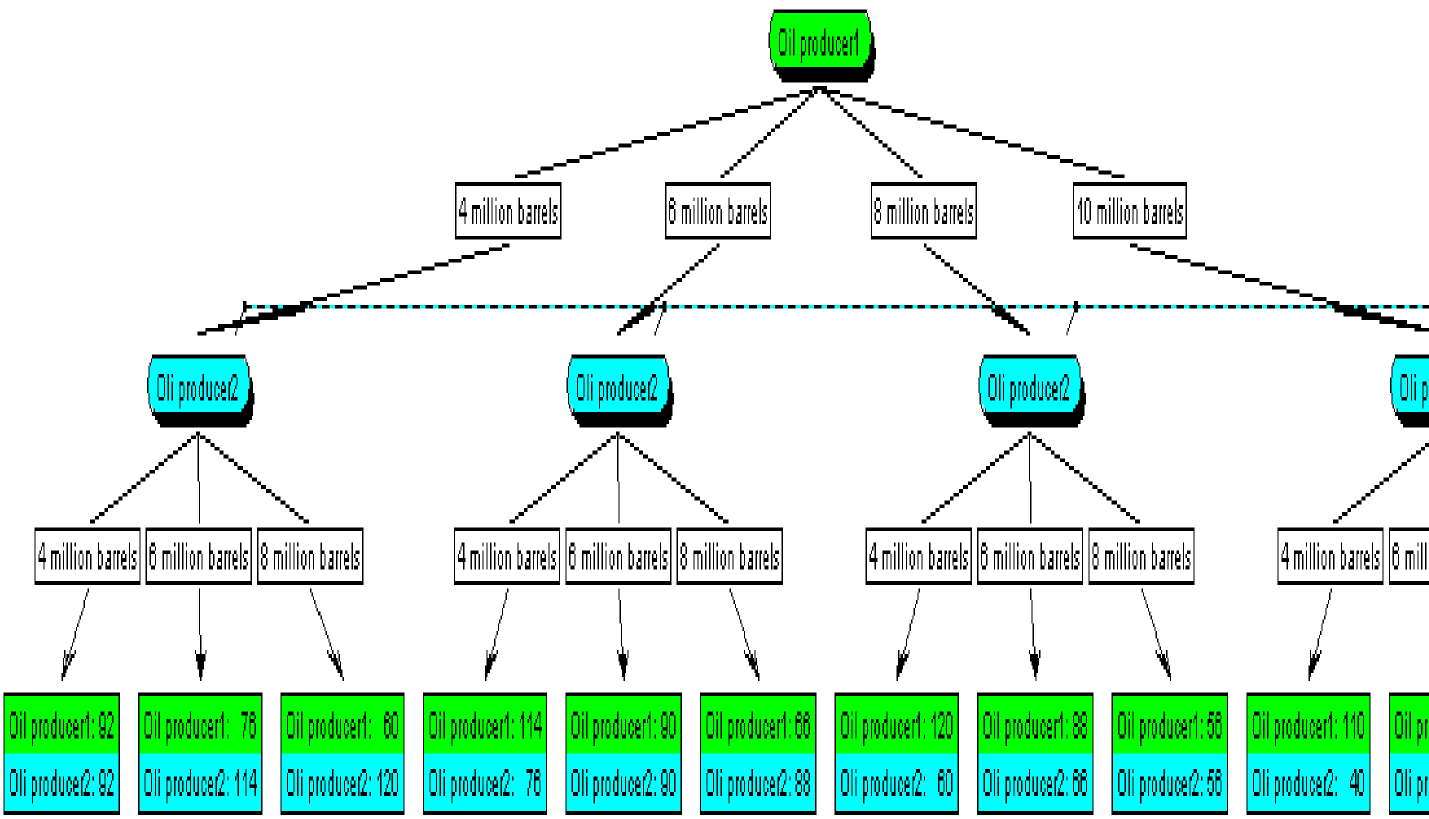

Figure 3.6
Oil extraction

The demand for oil is described by the equation

$$p(q) = a - bq$$

where

$$q = q_1 + q_2$$

and cost of extraction is

$$TC = cq_i \text{ where } i = 1, 2$$

In Figure 3.6 we $a = 40, b = 2$ and $c = 1$. The tabular presentation of the calculated payoffs are shown the the Table 3.1 for the Oil Producer 1. Payoffs for Oil Producer 2 are symmetric.

Oil producer1	Given 4 barrels produced by Oil producer 2	Given 6 barrels produced by Oil producer 2	Given 8 barrels produced by Oil producer 2
4 barrels	(40-2*8)*4-4=92	(40-2*10)*4-4=76	(40-2*12)*4-4=60
6 barrels	(40 –2*10)*6-6=114	(40 –2*12)*6-6=90	(40 –2*14)*6-6=66
8 barrels	(40 – 2*12)*8-8=120	(40 – 2*14)*8-8=88	(40 – 2*16)*8-8=56
10 barrels	(40 – 2*14)*10-10=110	(40 – 2*16)*10-10=70	(40 – 2*18)*10-10=30

Table 3.1: Payoff calculation for based on the demand equation $p(q) = 40 - 2q$ and TC=q_i where $i = 1, 2$.

Experimental results for the oil extraction game

Twentyfour subjects played this game twice and the results are presented in Figure 3.7 separately for each round and aggregated over the rounds. In the first round subjects most frequently selected six barrels each, eight barrels each and ten barrels if Oil producer 1 and six barrels if Oil producer 2. In round two, subjects who were assigned Oil producer 2 selected six barrels seventy percent of the times while oil producer 1 equally likely selected six barrel and eight barrels.

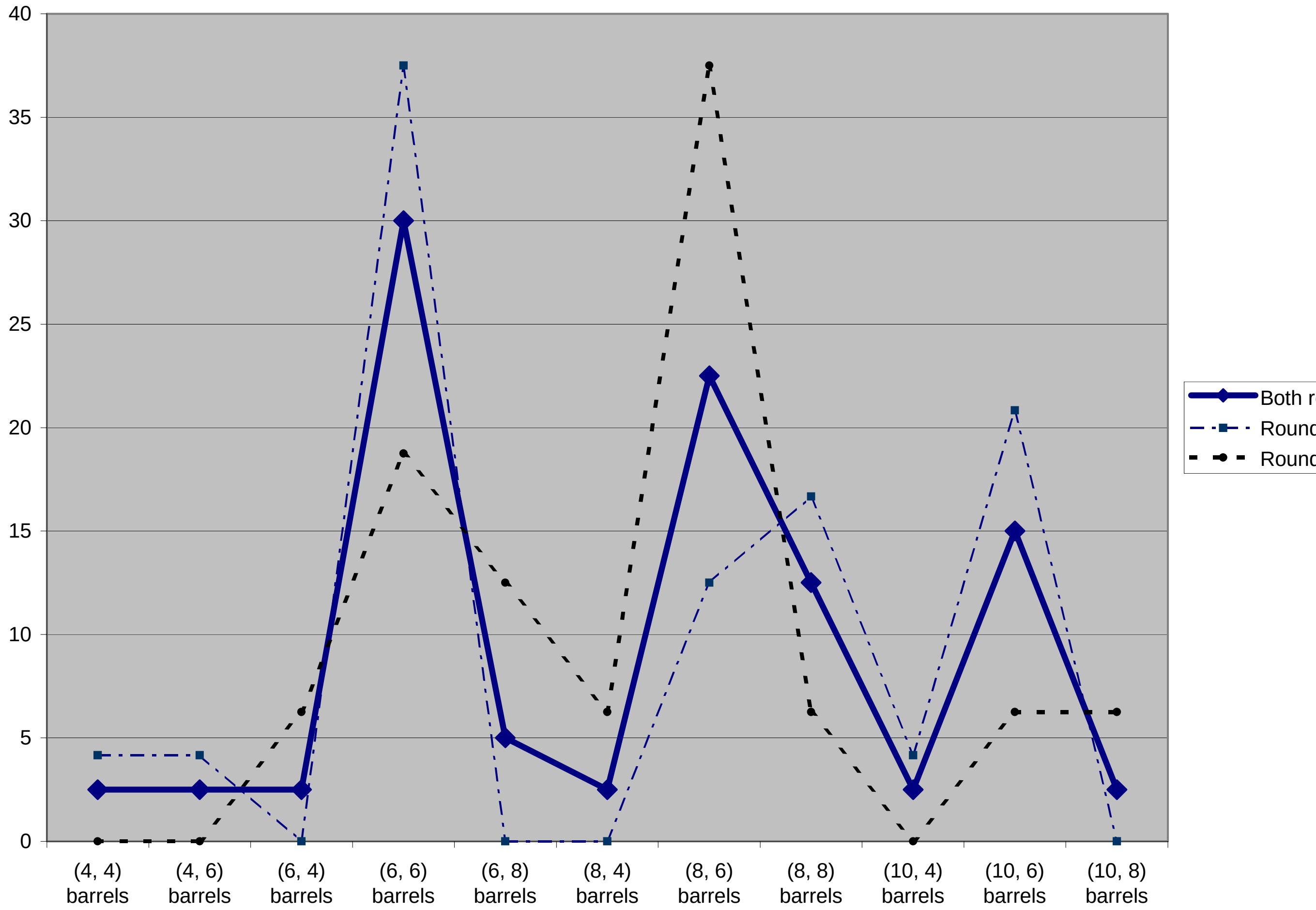

Figure 3. 7

Exercise *Design and run the game for different values of* a, b *and* c *and for different information sets. How does the cost and revenue structure change when*

Employment contract

Businessmen often know more about the demand for their product than many of their new employees. Employees have some flexibility in the type of employment contract they have. In this game the businessman knows whether the demand for the product is strong or weak, but the new hire does not; she believes the probability of it being strong is sixty percent, and in the past her beliefs have been well founded in terms of the outcomes they have predicted. The employer chooses between a high wage plan without benefits, and a lower wage package with more generous benefits should demand prove weak. The employee then accepts or rejects the contract. If she joins the firm, the project is undertaken and both parties learn if it has been a successful venture or not.

Figure 3.7 depicts the extensive form of the employment contract game. At the initial node, demand conditions

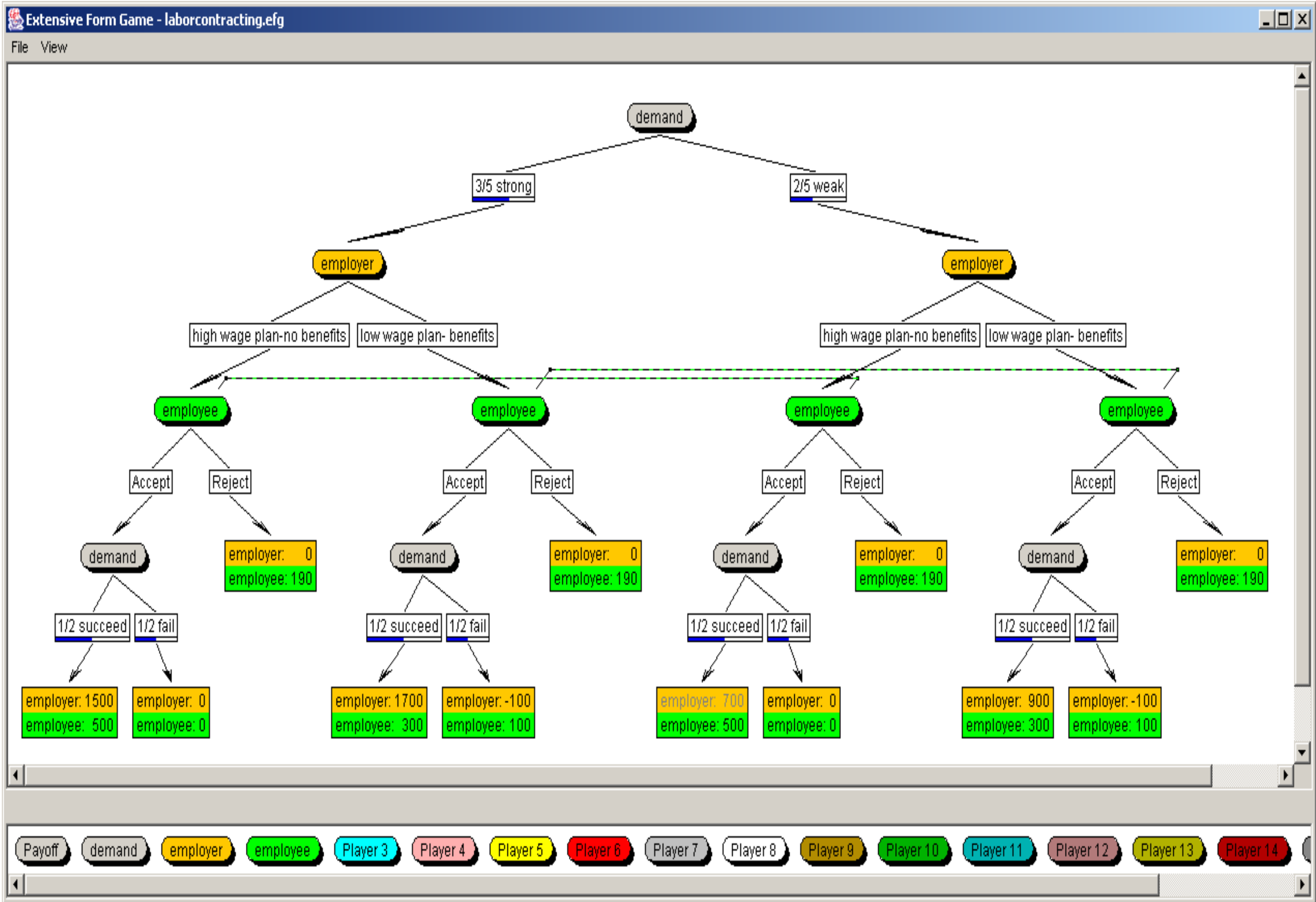

Figure 3.7
Employment contact

This might seem surprising to some readers, because one feature that seems to be missing from the extensive form is an sense of timing. For example

Exercise **1.** *Copy the employment contract on to some experimental software package and run a classroom experiment.*

2. *Compute the relative frequencies of the various terminal nodes. What is the average benefit to the employer from playing this game? And to the employee?*

3. *Which nodes are reached with*

4. *Observe that irrespective of the players choices, the choices of nature determine the probabilities of reaching certain sets of nodes. Delineate the sets and compute the probabilities of reaching them.*

5. *Can you reject the hypothesis that the outcomes of the experiments are significantly different from those probabilities (suggesting that the random* *number generating process in your experimental software package is flawed)?*

6. *compare with no dotted lines and*

7. *On the basis of your experimental outcomes, calculate the net benefit to both employer and employee from of the additional information*

Game trees

All the games we analyzed in the preceding section were illustrated by their extensive forms, a representation that shows branches that might be taken throughout the game. Which branch is taken at a node reached in play depends on what player is assigned to the node chooses or, in the case of a node controlled by nature, a random variable distributed over the different possible branches at that node. Connecting branches from the beginning of the game to its end is called a history. At the end of each is history is a terminal node showing the rewards players have earned from playing the game that way. Thus the history of play and the payoffs to the players are collectively determined by the choices of the players and by nature. Having introduced the concept of a game tree, described its main features, constructed several examples to illustrate how the extensive form is derived, and demonstrated how play can be depicted, we now turn to a more detailed discussion of each of its components.

Players

The players in the game are typically parties whose interests do not necessarily coincide but whose actions affect each other. The number of players in the game does not have to match the number of people being modeled. For example a large firm owned by many shareholders might be represented by a player who maximizes the expected value of the firm. Under some conditions extensively discussed in the microeconomics literature on production, this objective would be unanimously supported by all the employees and the shareholders, thus justifying the assumption of a single decision maker acting on behalf of the firm. Another examples of aggregation

aggregating firms in a competitive industry, to get an industry supply curve

aggregating consumers to get an representative consumer, a composite consumer

To take the other extreme, it might not be prudent for households to be treated as

Arrow's paradox illustrates the futility of representing the decisions of a committee as coming from a

The number of players

Payoffs

The number of players in the game is partly determined by the nature of the payoffs-capture all the consequences of the game

-represent the utility to each of the players from the game ending at a specified terminal node.

-show how resources are allocated to all the players contingent on a terminal node being reached. Nothing in the game we have said so far would lead a player to prefer one outcome to another. Preferences over outcomes are labeled at the end of the game.

At each terminal node the outcome from playing that game history is summarized by the payoffs received by all the players. The payoffs are defined in terms of what the agent values. In classroom experiments this is typically money or grade points. How should these payoffs be determined to portray they application they are meant to model? In the applications the classroom experiments are meant to model, the payoffs are harder to define and measure.

It might be reasonable to argue that the chief executive officers of large publicly trading corporations maximize the value of their respective firms, in which the payoffs to such firms are represented by the share return, the increase in its value plus the dividend payments.

union boss versus rank and file

Consumers obviously prefer paying less than more for any product, but to make headway an analysis that involves product quality, it is necessary for us to know how much he is willing to give up to have products of higher quality

Branches and Nodes

An intuitive way to define and characterize a game is to present it in extensive form. The extensive form of a game looks like a root system, or an inverted tree, which explains why the extensive form is often described as a game tree. A root system has a very special structure that the extensive game form follows. Starting from the trunk, roots fan out and subdivide, never to rejoin. The initial node is at the beginning of the root system, or the bottom of the tree trunk. The point at which a root divides into two or more is called a node, and the length of root between two consecutive nodes is called a branch. As in a root system, branches sprout from each node, and each branch connects precisely two nodes. Play proceed from the initial node to one of the terminal nodes never revising direction, rendering it impossible to return to a node that has already been visited.

All nodes are decision nodes, nature’s nodes, or terminal nodes. Thus all nonterminal nodes are designated to a particular player or to nature. Each decision node is assigned to a player, who chooses one of the branches connecting the node consistent with the flow of play to determine the directions the play will flow from that point. Each branch is uniquely labeled by an action that signifies a portion of the history if play proceeds down the branch. In this way the decision nodes show whose turn it is. At a node assigned to nature, a random variable determines which branch is taken. When play reaches a terminal node no further choices are made. at each terminal node every player receives a (possibly negative) payoff. chooses

-have only one predecessor to each successor node to indicate the history of play thus far.

-display possible choices (random outcomes) at each node for the player whose turn it is to move (nature).

The players are thus defined as the set of people whose names appear on at least one of the non terminal nodes. Not all non terminal nodes need to be labeled. If no one is responsible for choosing how play should proceed at a particular node, then probabilities are assigned to each of the possible directions, and play at that node follows a random direction in accordance with the law of motion defined by the assigned probabilities.

At each terminal node, a root tip, payoffs are assigned to all the players in the game. Every other node is assigned to a player, or to nature, and labeled accordingly. The branches are also labeled by actions they represent. One of the nodes a branch connects indicates which player is responsible for the action or in the case of nature what led to it. A player is assigned to each decision node.

Exercise *Explain which of the following objects have the same structure as a game tree. Sketch the tree structure where possible, indicating the initial and terminal nodes:*

1. *artery and veins systems in humans, considered separately*
2. *combined artery and veins systems in humans*
3. *airline network with nonstop flights connecting all cities*
4. *spider web*
5. *octopus*
6. *parking garage*
7. *Kansas City street map*
8. *World Cup schedule*
9. *assembly manual for bicycle*
10. *river system*

Histories

The course of history and the outcome playing a game is determined by the direction play takes at the initial node, and then at each of the subsequent non terminal node that are reached. The rules of the game specify who, if anyone, decides the direction of play. When a person is designated, following the terminology of card games and board games, we say he must make a move, to take his turn. How the direction of play is chosen is depicted in the extensive form game tree by labeling each node by the player who will pick which branch will follow. Play proceeds from the unique initial node to one of several terminal nodes, the assigned player directing the course of play at decision nodes, and nature drawing an outcome from the Information sets define which parts of the history a player can see. The direction or action taken at nodes labeled by nature is determined randomly according to a probability distribution that is part of the game's specification. Thus playing a game is like tracing a path from the top of the root structure, the initial node, and tracing down a path to one extremity of the root structure, called a terminal node without letting the pencil leave the paper or retracing any branches. Tracing a path from the trunk to an extremity of the root system is called a history of play, each of which is uniquely identified by its terminal node.

Exercise **1.** *Several of the examples with a tree structure also have a chronological direction of play associated with them. Is the direction of play consistent with your labeling of the initial and terminal nodes?*

2. *A predecessor for a node is another node that must be passed through to reach the node in question. Prove that for all games two nodes with the same predecessor cannot be visited in the same play.*

Information sets

Each non-terminal decision node is associated with an information set. Dotted lines (or their absence) define the information set. Information sets show what paths of play a person making a choice can distinguish between.

Information in the extensive form is represented by joining or circling nodes assigned to the same player who cannot . Since each path to any given decision node is unique, a player assigned to move at a node which is not connected to any other, a singleton, can deduce exactly how play has evolved up to that point in the game.

In many board games, such as chess, drafts and monopoly each player can review the course of play before making a move. In many card games this is not so. For example in the game of bridge 4 players are dealt 13 cards each from a shuffled pack of 52, and then take turns bidding. Although they hear each bid, each player sees only his own cards. The initial node of the extensive form of bridge is not assigned to any player thus each person can only distinguish between 13 of

In many problems imposing the assumption of perfect information would be unreasonable.

It follows

Perfect information games

The hallmark of a perfect information game is that it supports an extensive form which has as many information sets as there are decision nodes. Schematically, there are no dotted lines. This means that whenever a player takes his turn to move, he knows exactly what has happened in the game up to that point. Chapters IV and V are devoted to this topic. Within the class of perfect information games there is a further distinction to be made between games in which nature or chance plays any role, and games in which all the nonterminal nodes are decision nodes. In perfect foresight games every node is a terminal node or a decision node, and there are as many decision nodes as information sets.

Market Saturation

Consider the following perfect foresight game. At the beginning of the game a potential entrant decides whether to enter a whale watching tour boat industry in an isolated vacation resort which is currently served by a single operator. There is a fixed entry cost of $1 million for the new entrant for plant, and constant variable costs. If the new firm enters, the incumbent firm decides whether to discount its product or collude with the new entrant. The monopoly rent from this industry is $3 million. Figure 3.14 depicts the extensive form of the game. At the initial node, the first potential entrant (Entrant 1) has a choice to either enter or not enter tour boat industry. If he enters he might be facing Entrant 2 if he enters the market. Each time another competitor enters the profit is decreased by $1 million to the incumbents.

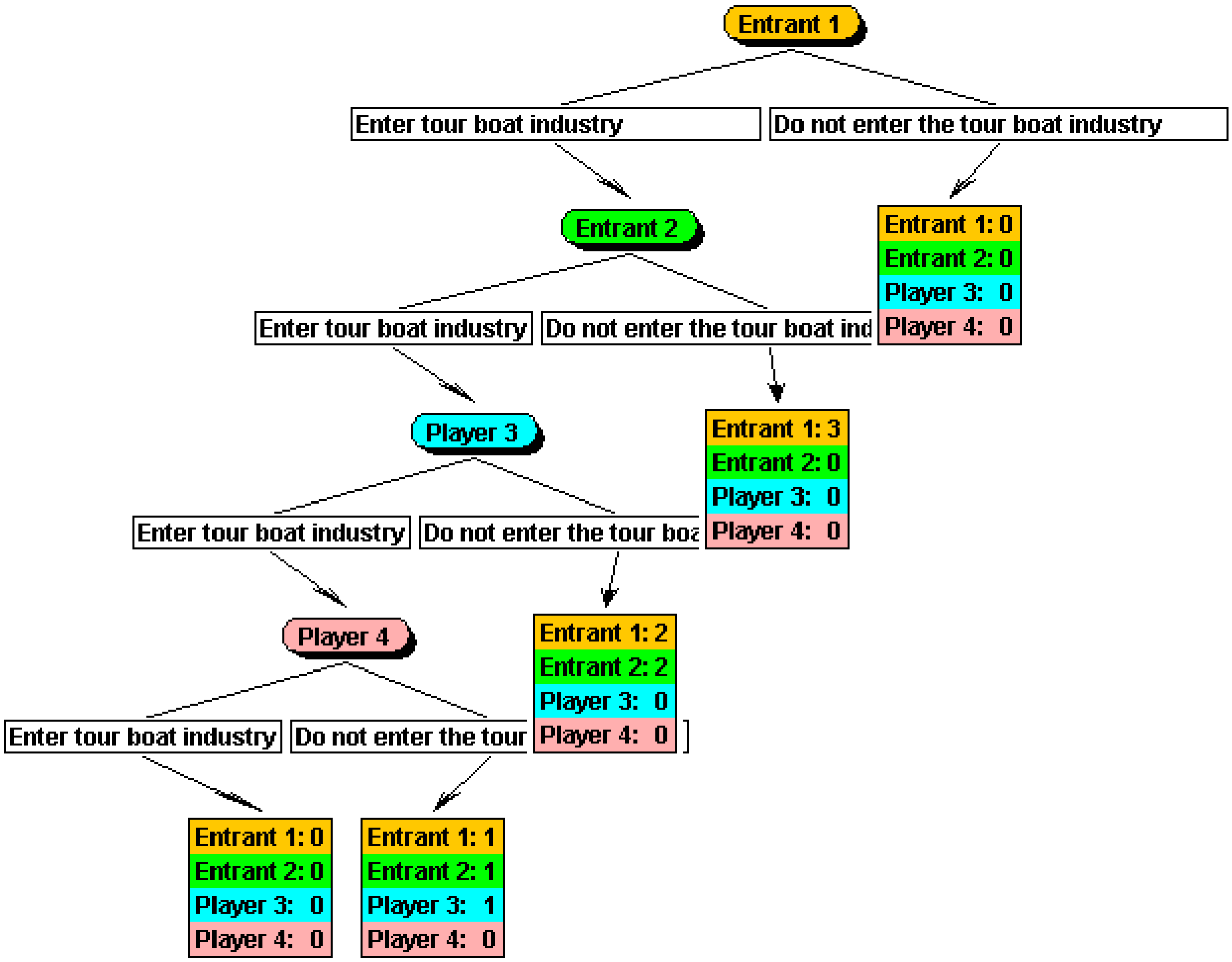

Figure 3.14
Market saturation

The payoffs for the game depicted in Figure 3.14 were derived with the demand schedule: p=5-q, and fixed costs of entry equal to $1 million.

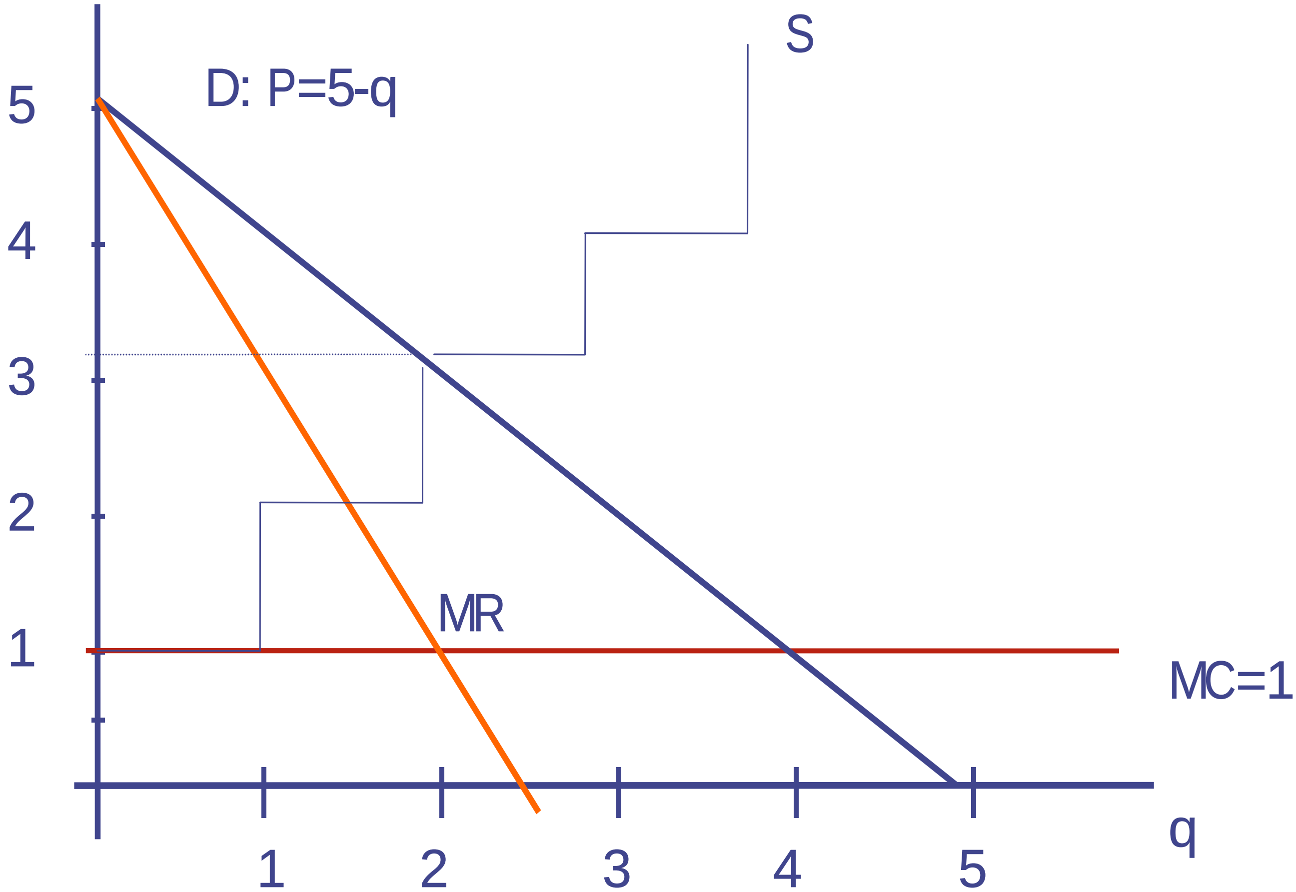

Figure 3.15
Market saturation with demand and supply schedule

Exercise **1.** *Run the game illustrated in Figure 3.4 and tabulate the results, showing which fishermen take up the tourboat investment opportunities.*

2. *Suppose the investor approaches the second entrepreneur and then the first, before approaching the other three. Write down the extensive form, run the game and tabulate the results.*

3. *No suppose the fishermen do not know how has been approached before them. Write down the extensive form of this game, run it in class and tabulate the results.*

4. *How do the game outcomes compare with each other.*

5. *Comment on the value of knowing who has been approached and what their decisions are.*

Climber

Not all perfect information games have the perfect foresight property. Indeed perfect information games are often used to model situations where uncertainty plays an important role. In the next example the payoffs mountaineers receive from reaching a summit depends on whether they are first or not. these expedition teams determine their goals interdependantly with other rival teams. Easier conquests have a higher probability of success but lower payoffs, Similarly later conquests have the benefit of more advanced technology and overall knowledge about local conditions and lower payoffs too.

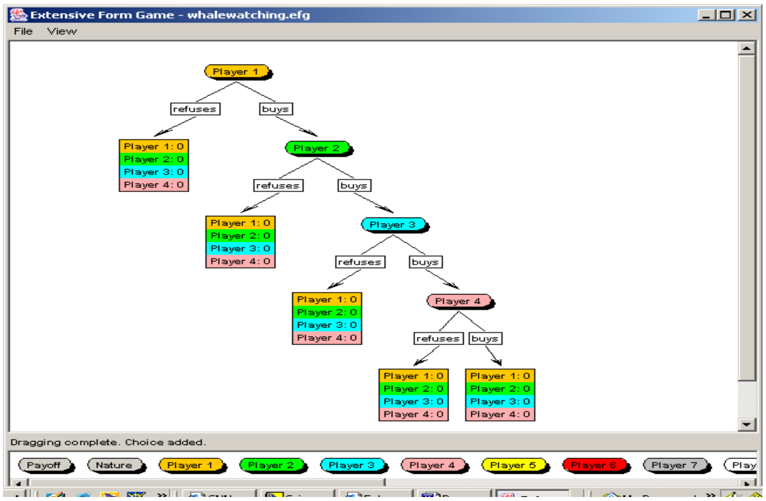

Figure 3.15
Climber

In Figure we model mountaineer as a two player game between rival teams.

Simultaneous Move Games

The defining feature of a simultaneous move games is that no player has any information about the other players’ moves when he makes his own choice. Thus everyone is equally informed about the moves of all the other players, or perhaps we should say, equally ignorant. Because everyone acts in ignorance the possibilities for coordination between players are very limited indeed. This situation starkly contrasts with perfect information games, where no two players are equally informed, thus allowing players who move first to direct the course of play away from outcomes they both detest.

Marriage

Consider the following scenario: both husband and wife lunch together but for business reasons cannot be interrupted so that they can coordinate their plans. Unfortunately there are two places which brings each of equal satisfaction to each partner in this extraordinarily well matched couple. Lack of coordination will lead to a shorter lunch and a more harried time together.

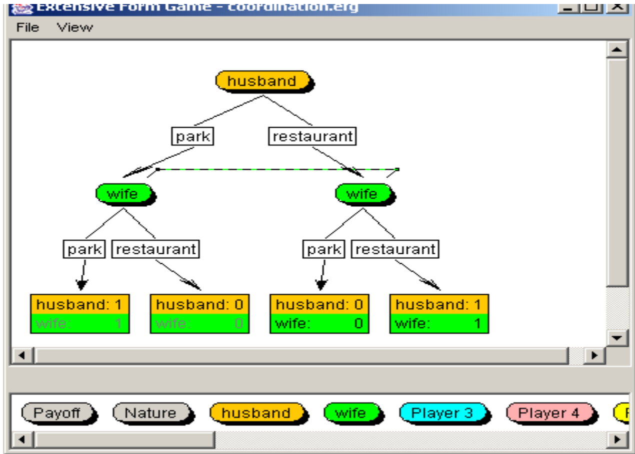

Figure 3.16
Marriage

Figure 3.16 depicts the Executive Marriage game in extensive form. Who reaches for their cell phone first is immaterial, since one of them has been misplaced and there are no convenient public phone boxes nearby. Note that their is no conflict between partners in this problem, just a communication breakdown that threatens to ruin a good lunch and/or a solid marriage.

Exercise

1. *Run matching pennies and executive marriage for one round and compare the outcomes.*

2. *What is the relative frequency that a player chooses the left strategy conditional on choosing from one round to the next?*

3. *What are the unconditional probabilities of*

4. *What are the outcomes?*

5. *The phone message game is a variation on the extensive form executive marriage, in which the wife can leave a message for the husband to pick up before he makes his decision. Write down the extensive form of phone message and prove it is a perfect foresight game.*

6. *Run In the modified form of executive marriage*

7. *Run ten rounds of phone message and ten rounds of executive marriage. Are the average payoffs significantly different*

8. *Do your findings in Questions 1,2 and 7 lead you to any hypotheses about how the probability of divorce might change with the length of a marriage?*

9. *Do your findings in Questions 1,5,7 and 8 lead you to any hypotheses about how the level of communication between partners as the years roll on?*

Congress

In simultaneous move games it is not important whether all the players make their choices at exactly the same instant, for example that the marriage partners simultaneously and independently decide where to lunch. All that matters is that each player does not know what the other has done or will do at the time she takes her own decision.

A further example will help clarify this point. There are three congressmen elected to determine next year's expenditures by the government. Their budget is decided by majority vote. Voting is simultaneous. If one item receives two out of three votes then it will be funded, but if no item receives a plurality the congress deadlocks.

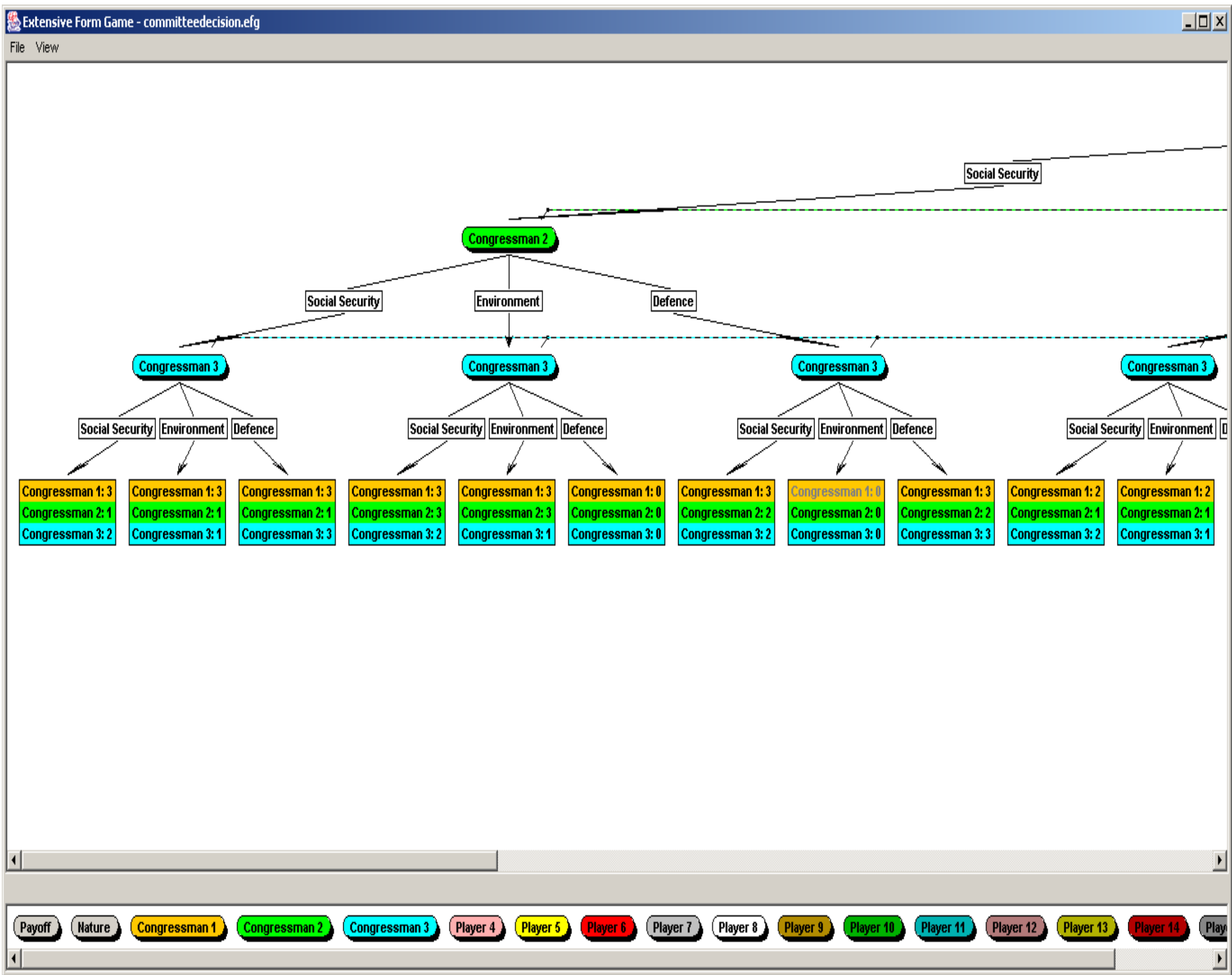

Figure 3.17
Congress

In this example the first Congressman favors bolstering social security over spending more on the environment with more funds for defence coming in third. The priorities of the second congressman are to the environment, defence and social security in that order, while the third congressman places defence at the top of his list and environmental concerns at the bottom. If the committee deadlocks at one vote each everybody looks bad and fact the prospect of loosing their position on the committee. If two or three members vote for the same proposal it is implemented. In that case each congressman is awarded 3 points for his first choice being picked by the committee, two points for his second choice and 1 point for his third.

Exercise **1.** *In the Committee Decision Making game allow each player two minutes of confidential but nonbinding email exchange with each other about the importance of each project (or whatever committee members discuss amongst each other) before voting.*

2. *Compare the outcomes of the game with the preplay communication. Discuss whether there is any conflict between acquiring a reputation for honesty and legislation by majority rule in this game.*

Complete Information Games

Perfect information and simultaneous move games are all part of a larger class called complete information games, games in which each player knows at least as much as the player who has moved before him.

Pioneer versus Imitator

The race to develop and market new products can also be modeled as a complete information game. To differentiate rivals. Firms which specialize in the

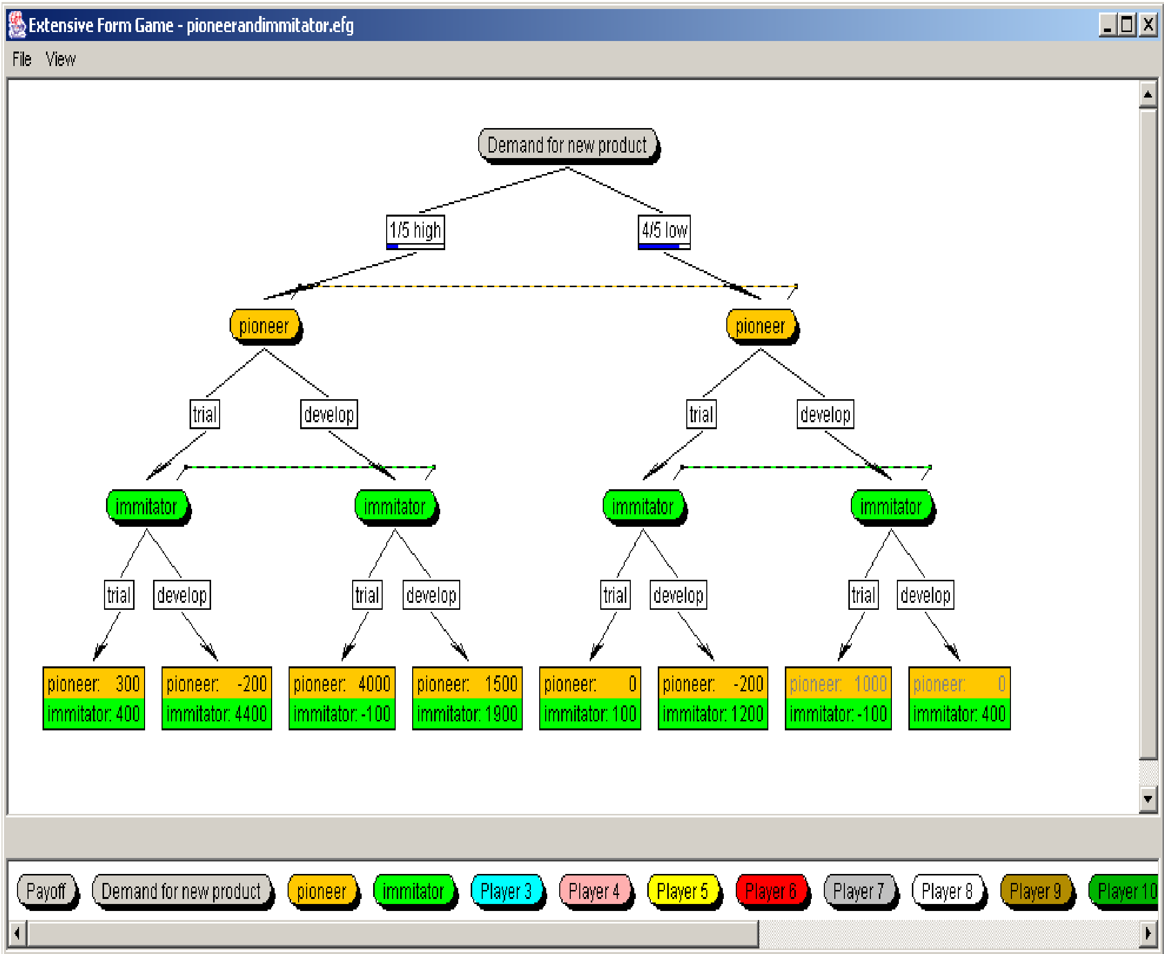

Figure 3.18

Pioneer versus Imitator

This game has imperfect information because there are six decision nodes but only three information sets. It is not a simultaneous game because the imitator has some knowledge about how the game has progressed before choosing between a trial market and fully developing the new product. Finally this game has complete information note that the imitator can observe the demand for the now product but the pioneer, who moves first cannot observe anything about the game history when making his choice.

Exercise *Games that are repeated with the same subject assignments are called repeated games. We discuss these at length in Chapter show that repeating The rerun both games with ten rounds and compare the outcomes. Can you explain how the roles of information and incentives in these games explain the similarities and differences?*

Games with incomplete information

All games are have either complete or incomplete information.

Industrial espionage

Consider the following example of industrial espionage. A product development team of a rival company is infiltrated by a spy who passes information back to his own company about the directions the rival is likely to take.

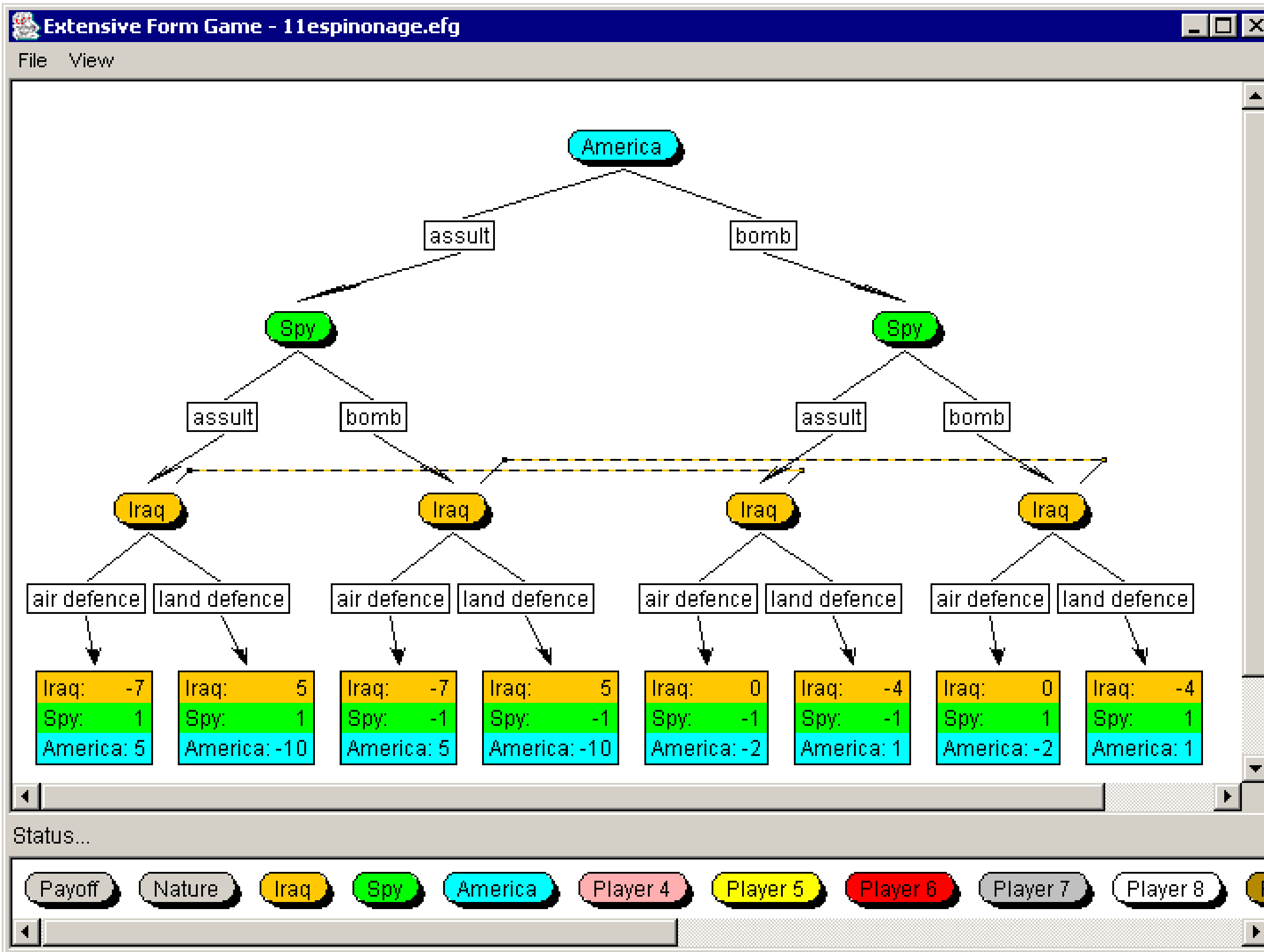

Figure 3.14
Industrial espionage

Discount Retailing

For example discount outlets may think twice before returning goods they know are defective to the factory, if their

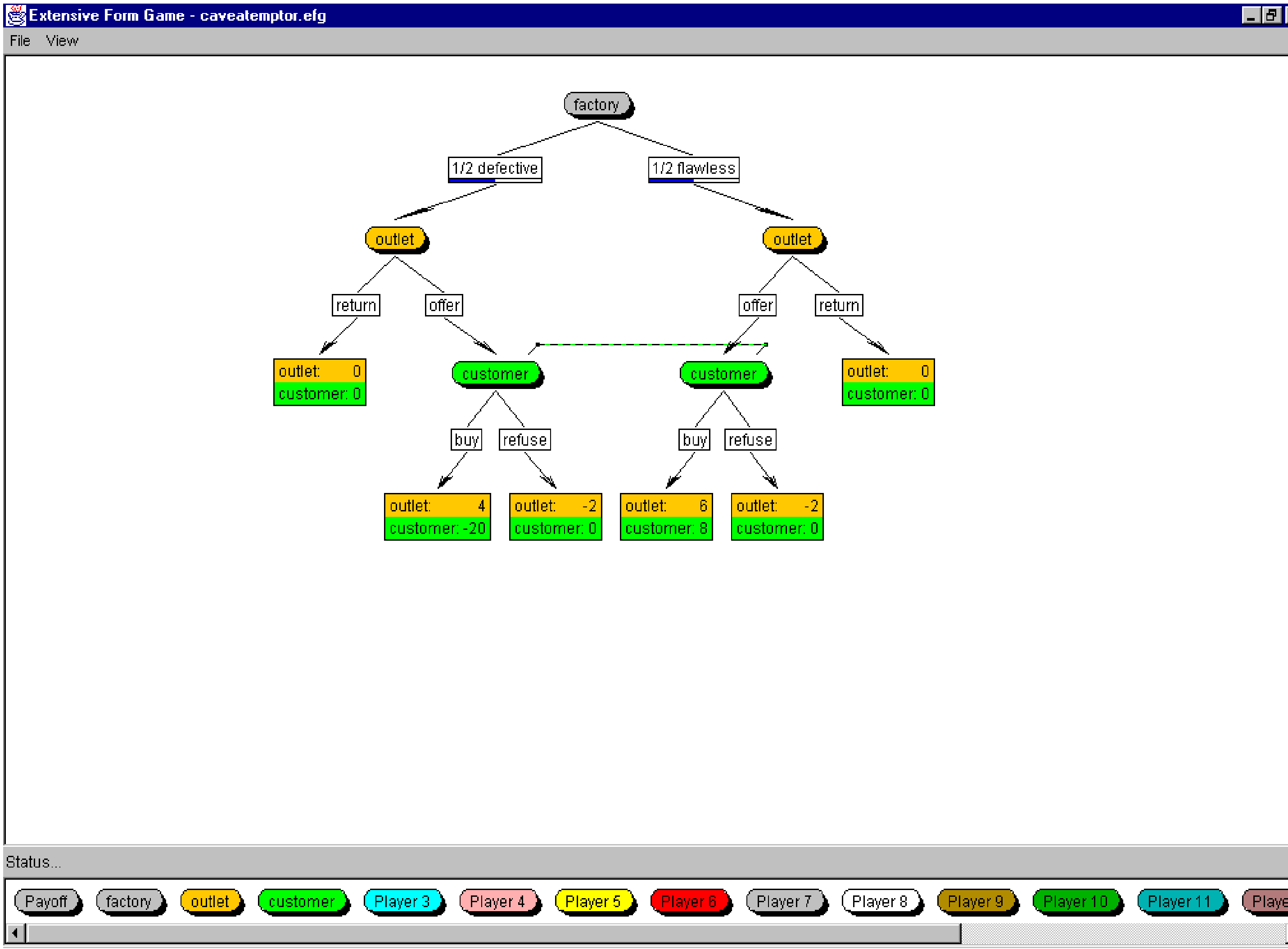

Figure 3.11
Discount Retailing

Exercise *The company accountant must decide between disclosing a loss on the income statement of the firm versus hiding it. If the statement is disclosed. Write down the extensive form of this game. Which player is assigned the initial node, label the branches.*

Equivalence in extensive forms

The discussion up until now has shown how the extensive form representation gives rise to a game, but little has been said about the reverse operation. More specifically does every game have a unique extensive form? The answer to this question obviously hinges on how the concept of a game is defined.

US versus Japan

To help motivate the discussion consider Figures 3.10 and 3.11 which depict different extensive forms. In Figure 3.11 America moves first choosing between big and small. Then, without knowing what America has chosen, it is Japan's turn to move, choosing between High and Low. In Figure 3.11 the moves of each player are the same, but the order is reversed, so some might argue that on this criteria alone the games differ. However our view is that these extensive forms are equivalent representations of the same game. In both cases the second player in the game does not receive any information about the course of play before taking his turn. There is therefore no apparent reason why either player should prefer one game over the other.

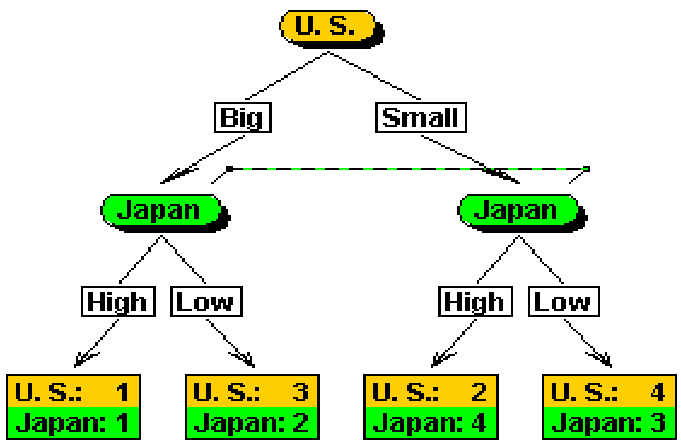

Figure 3.15

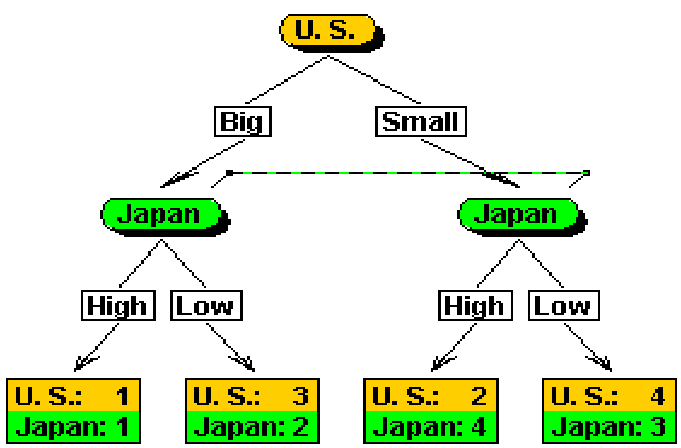

Figure 3.16

The example illustrates the fact that games with the same information sets may support more than one extensive form. There are essentially two ways this can occur. In some games the order in which players move can be interchanged with each other without affecting the ways in which players' choices can depend on information sets.

The extensive form imposes an order on the moves, even in simultaneous move games, studied in depth in Part 3 of this book, and of which the technology race is an example. The sense in which moves are simultaneous is through the structure of the information sets. By joining all nodes for the second player so that he cannot distinguish between them, the player is in effect forced to make a choice without knowing what the first player has done. Comparing Figures and , notice that in the first representation the U.S. but in the Figure below, Japan is the first mover. In both representations, however, neither player can condition on what the other has done when making a choice.

Vacation Plans

The second way distinct extensive form representations arise from the same game is through the role of uncertainty. This can be illustrated in a decision theoretic problem, that is where there is only one player, In Figure 3.12 a holiday must choose between visiting a beach versus a historic city. The main source of his uncertainty revolves around the weather at the beach. If it is fine then he would certainly prefer sun and sand to cultural attractions, but if the weather is inclement, then the city's offerings are preferable. the traveller must finalize his travel plans one month before his trip, at which time the best available long range forecasters predict that the probably of fine whether is one half. the traveler accordingly draws his game tree, which has been reproduced here as Figure 3.12, complete with the relevant payoffs.

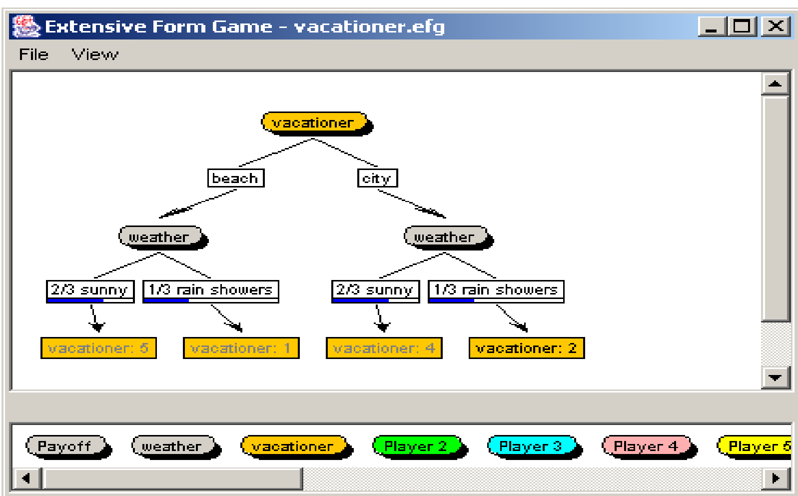

Figure 3.17
Vacation plans

Now suppose a research meteorologist confidently informed the traveller that although we do not yet know the we have established that the weather is established months in advance. Rather than In the second representation there are two decision nodes for the vacationer, but since he does not know whether he will enjoy the experience or not, there is only one information set which implies that his decision cannot be based on the particular node he lands. One might expect that the vacationer would prefer to make a decision after the weather has revealed itself, and in the next chapter we will analyze how much he would be willing to pay for that information. But whether the weather is meterioriallucally determined before or after his decision is made is unfortunately irrelevant until forecasters can provide the information to the vacationeers before they finalize their vacation plans.

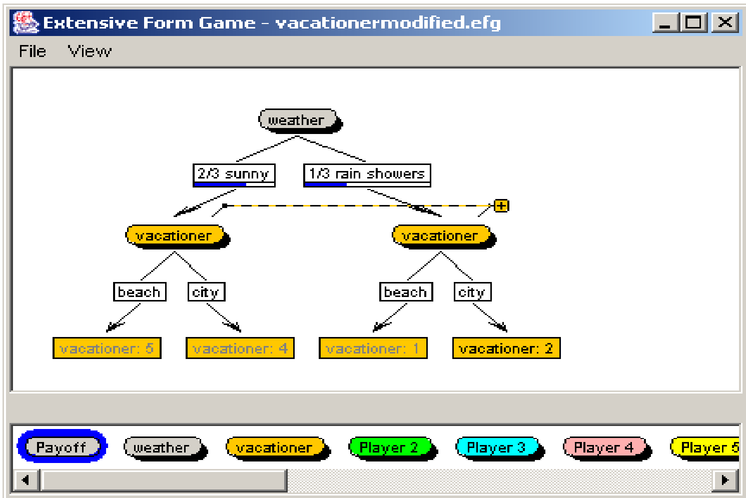

Figure 3.18
Vacation plans redrawn

Summary

An intuitive way to define and characterize a game is to present it in extensive form. Every game has at least one extensive form, although it is not necessarily unique. The extensive form of a game looks like a root system, or an inverted tree, which explains why the extensive form is often described as a game tree. Starting from the trunk, roots fan out and subdivide, never to rejoin. The initial node is at the beginning of the root system, or the bottom of the tree trunk. The point at which a root divides into two or more is called a node, and the length of root between two consecutive nodes is called a branch. Play proceeds from the initial node to one of the terminal nodes never revising direction, rendering it impossible to return to a node that has already been visited. When play reaches a terminal node no further choices are made. at each terminal node every player receives a (possibly negative) payoff. Tracing a path from the initial node to one of the terminal nodes is called a history of play.

At each terminal node payoffs are assigned to all the players in the game. Every other node is assigned to a player, or to nature, and labeled accordingly. The players are thus defined as the set of people whose names appear on at least one of the non terminal nodes. The branches are also labeled by actions they represent. One of the nodes a branch connects indicates which player is responsible for the action or in the case of nature what led to it. A player is assigned to each decision node. All nodes are decision nodes, nature’s nodes, or terminal nodes. Thus all nonterminal nodes are designated to a particular player or to nature. Each decision node is assigned to a player, who chooses one of the branches connecting the node consistent with the flow of play to determine the directions the play will flow from that point. Each branch is uniquely labeled by an action that signifies a portion of the history if play proceeds down the branch. In this way the decision nodes show whose turn it is. Not all non terminal nodes need to be assigned to a player. If nature is assigned to a node, then probabilities are attached to each of the possible directions, and play at that node follows a random direction in accordance with the law of motion defined by the assigned probabilities.

Each non-terminal decision node assigned to a player is associated with an information set. Dotted lines (or their absence) define the information set. Information sets show what paths of play a person making a choice can distinguish between.

Games can be classified by how much each player knows about the history when it is his or her turn to move. Games of perfect information are distinguished by the fact that each information set is a singleton. In practical terms this means that each player can tell the course history has taken up to the node . In simultaneous move games, players have no information about how play has proceeded when they make a move. This implies that each player makes only one move without knowing what anyone else has done or is doing. All perfect information and simultaneous move games are examples of complete information games. In a perfect information game a player knows at least as much as about the preceding history as everyone who has moved before her. All games are either games of complete information or incomplete information. Thus in games of incomplete information, two or more players have overlapping information sets at some point in the game, in which each player’s information is a proper subset of their union. Thus each player knows something about the history of the game that the other player does not know.

The next chapters in this text describe solution techniques that are used to predict how rational players behave in these different kinds of games. We will argue that deriving those solutions is helpful for understanding the models and predicting how play might proceed. In formulating hypotheses about how players choose, we will start with those hypotheses that seem most compelling and sequentially introduce less convincing hypotheses. The argument for this procedure is twofold. More persuasive hypotheses are easier to explain, and therefore should be taught before less persuasive ones. Moreover some games can be fully solved without resorting to less plausible hypotheses, and therefore their solution has more intuitive appeal. when appealing to the solution of a game It behoves us to state which axiomatic principles or behavioral rules have been used to derive it, so that its plausibility can be investigated.

Rather than deriving a set of rules from game theory about how players behave, there is of course a more direct method, which is to investigate the laboratory results directly, and make inferences about behavior from the empirical observations. This short circuits the process of analyzing the problem that motives the game and its experiment. If the principles of behavior derived from game theory apply to the problems that motive the game, one could argue that the main predictions of the experiments could survive some misspecifications in the way the payoffs and other features of the problem are modeled. On the other hand if the principles do not apply but the experiment very accurately portrays the situation under consideration, then the solution to the game would hold less interest than the experimental outcomes themselves

We see little reason to take a strong stand on this issue. The tools that you have acquired in this chapter suffice to take this more direct approach to studying human behavior from an experimentalist's perspective. In summarizing the experimental results we have argued . . .

Exercise *Complete the following table, which classifies the various games discussed in this chapter by the way information about game histories is revealed to the players.*

		Type of the game		
		Perfect Foresight	Perfect Information	Simultaneous
	Regional Competition			
	Recruiting			
	Matching Pennies			
	Oil Extraction			
	Employment Contract			
	Market Saturation			
Name of the game	Mountaineer			
	Executive Marriage			
	Committee Decision			
	Pioneer and Imitator			
	Industrial Espionage			
	Discount Retailing			
	Component supplier			
	Vacation Plans			

Table 1
Classifying different games

Further Reading

Discussions of the extensive form for a game can be found in many books on game theory, such as those we referred in Chapter 1. Raiffa's, introduction to decision analysis, is a classic text on using decision theoretic models to solve problems of the sort encountered in Chapters 2 when there is only one player involved.

Bibliography

Burnham, Terence C., Kevin A. McCabe, and Vernon L. Smith (2000) "Friend-or-Foe: Intentionality Priming in an Extensive Form Trust Game," Journal of Economic Behavior and Organization, forthcoming.

Dufwenberg, Martin, and Georg Kirchsteiger (1998) "A Theory of Sequential Reciprocity," Tilburg University, Discussion Paper, presented at the Summer 1998 ESA Meetings.

Fudenberg, Drew, and David K. Levine (1995) "How Irrational Are Subjects in Extensive-Form Games?," American Economic Review, 85.

McCabe, Kevin A., Stephen J. Rassenti, and Vernon L. Smith (1996) "Game Theory and Reciprocity in Some Extensive Form Experimental Games," Proceedings of the National Academy of Sciences, 9313421-13428.

Schotter, Andrew, Keith Weigelt, and Charles Wilson (1994) "A Laboratory Investigation of Multiperson Rationality and Presentation Effects," Games and Economic Behavior, 6:3 (May), 445-468.