

Introduction

In games of perfect information players take turns making their moves, knowing what the players who have moved before them have chosen. Every perfect information game has an extensive form in which all the information sets are singletons; there are no dotted lines joining nodes. Thus all the strategic interactions are sequential. The purpose of this chapter is to teach you how to recognize, play, solve and design perfect information games.

The core of this chapter provides a solution technique for games of perfect information. It is based on two analytical tools, the principle of backwards induction, and the expected utility hypothesis. We first develop the principle of backwards induction by analyzing several games of perfect foresight, games with perfect information in which nature or chance does not play any role. This leads to an enunciation of the principle of backwards induction. A pithy statement of it is that players should look forward and reason back. In other words each player should anticipate how the other players will react, and also how she will later react, to decisions made now.

The latter parts of the chapter concentrate on perfect information games in which nature plays a role. Games with uncertainty do not have a unique extensive form, so with the aid of several applications we develop heuristics for recognizing perfect games. Another complication that uncertainty introduces to game theory relates to how the players rank choices that have uncertain outcomes. The expected utility hypothesis is one widely used metric that we shall also adopt throughout much of this book. To evaluate a choice it weights the utility of every outcome that might be reached by the probability of its occurrence and then sums over the weighted outcomes. We show how this hypothesis can be applied to solve one player games, to place a value on information. Finally we turn to multiplayer perfect information games with uncertainty to demonstrate how, subject to computational feasibility, the assumption of the expected utility hypothesis and the principle of backwards induction together suffice to solve all (finite) games of perfect information.

Games with Perfect Foresight

Games of perfect foresight are defined by two features. Every node in the game is a decision node or a terminal node, and every information set contains only one decision node. These features are easy to recognize in the extensive form of the game. All the nodes are assigned to a player, or end the game, and there are no dotted lines joining any of the decision nodes. This simple structure is an ideal setting to introduce the principle of backwards induction. Roughly stated, this principle asserts that players should anticipate what future players will do by putting themselves in their shoes and working out the implications for their own payoffs. We elaborate this principle with several examples and then give a general prescription.

Adjustment Costs

Figure 6.1 illustrates a perfect foresight game of costly adjustment between two competitors of different size. Because of its high adjustment costs, the market value of the large firm is higher from stagnating if the small firm does too. The capitalized value of the small firm however increases from modernizing regardless of what the large firm does. Moreover if the small firm modernizes but the large one stagnates, the small firm gains a significant proportion of the business that previously belonged to the large firm. Finally we assume that the small firm is able to react to the plans of the large firm because it has lower lead times. Thus the large firm must choose between stagnating and modernizing before the small firm. Notice that this process is modeled as a game of perfect foresight because, nature does not play a role, and because the small player knows exactly what the large player has done before making its own decision.

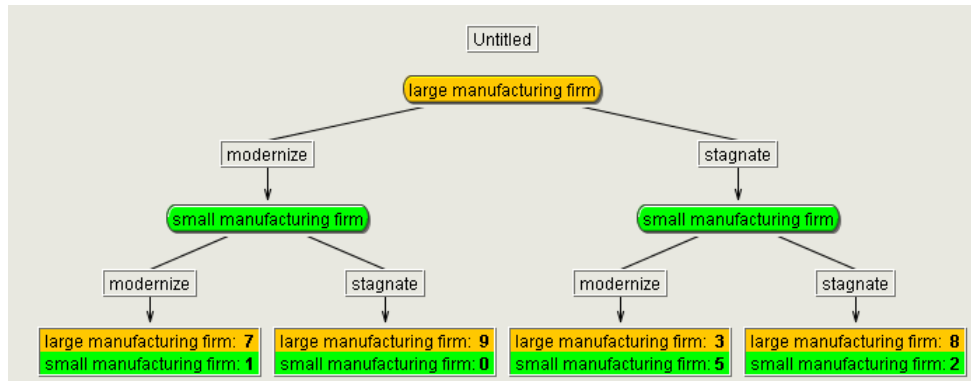


Figure 6.1: Adjustment costs

This is an easy game to solve because the optimal decision of the small firm does not depend on the choice of the large firm; it should modernize. Suppose the large firm believes that the small firm will maximize its own value and accordingly modernize. Then the we can fold back the terminal nodes that can be reached, conditional on the small firm modernizing to obtain a decision tree for the large firm, reproduced in Figure 6.2. I

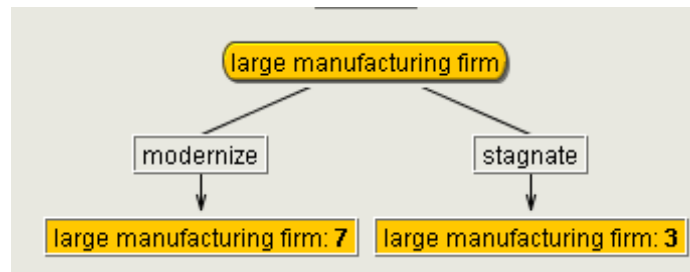


Figure 6.2: Reduced Game of Adjustment Costs

From the figure we deduce that the large firm's value is higher by modernizing than

by stagnating, even though profits to both firms fall as a consequence of this technological advance from 9 and 2 respectively to 7 and 1. It might be reasonable to ask how this situation could be avoided. One possibility that is sometimes adopted by declining industries is through merger and consolidation.

Experiment

Nineteen subjects played the adjustment cost game once before the concept of backwards induction was introduced. If subjects were using backwards induction principle then we should observe both players selecting "modernize" all the time. The results are presented in Figure 4.3. In seventeen out of nineteen cases subjects selected the predicted outcome. Only twice the subjects deviated from the predicted outcomes. The hypothesis that (modernize, modernize) should be selected all the time was not rejected at five percent significance level ($\chi^2 =$)

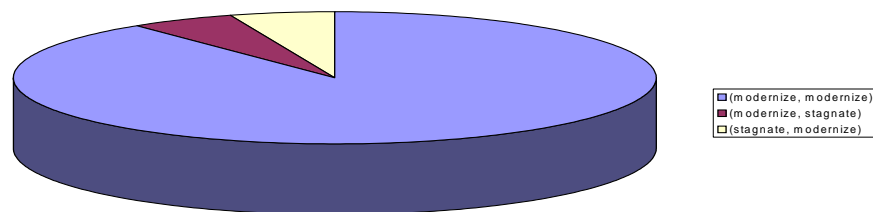


Figure 6.3: Frequency of choices for adjustment game played

Exercise *Regional Competition*

1. Solve the game depicted in Figure 3.1.
2. Test the hypothesis that the outcomes of the classroom experiment previously conducted on this game are not significantly different from the solution found above.

Congestion

Congestion is another phenomenon that is easy to predict but hard to avoid, at least without making changes to how property rights are assigned. Whether one is talking about overfishing an ocean, traffic jams at rush hour, or competition for towel space on a public beach, the basic problem is the same. Each additional fisherman,

car driver or bather does not take into account the costs imposed on others when making her own decision.

Figure 6.4 depicts a stereotypical situation. Three sun bathers sequentially arrive at a beach, which is a mixture of whom have the same preferences over sand and rock. There is an abundance of places that differ only marginally in their desirability, the most desirable rocky spot yields 9 utils for the day, the second most desirable rocky spot 8 utils and the third most desirable rocky spot yields 7 utils. Sand, however, is scarce but common property. If only one person lies on the sand then her utility is 20, but if two lie there the utility of each person is only half that. It falls even further if three crowd on the same patch.

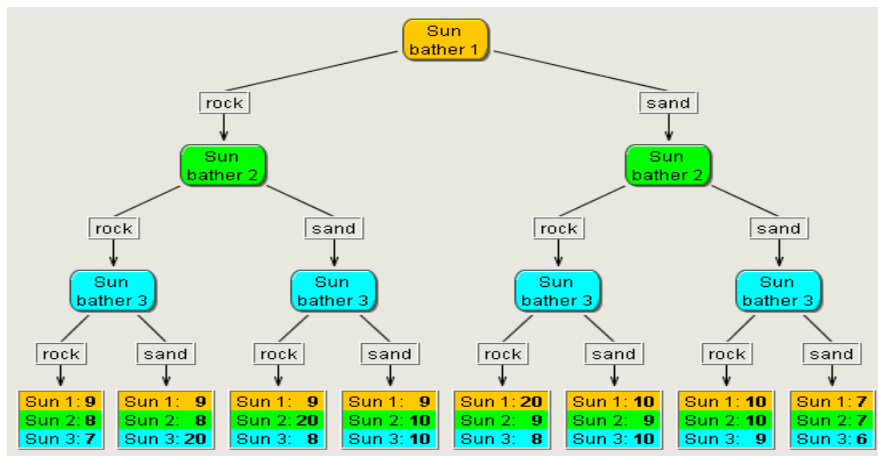
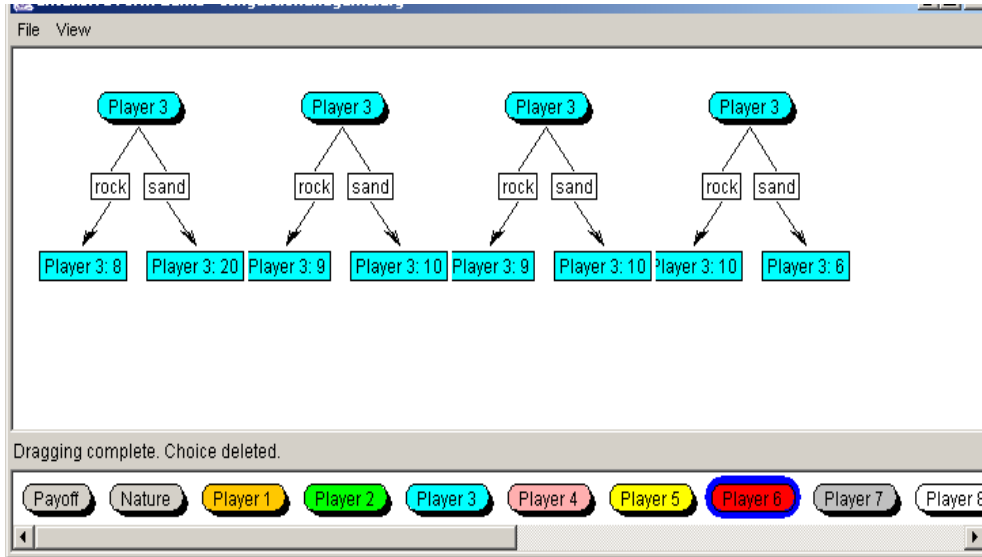


Figure 6.4: Congestion

Again it is easy to see that from the extensive form that this is a perfect foresight game. Each decision node is singleton which means that each player observes the actions of all other players. The solution to the game of beach congestion can be found using the same principle that guided us to the solution of the costly adjustment game. Turning to the decisions which confront the last sunbather, we note that if at least one of the other two both settled on the rock, she will choose the sand, but if the first two took the sand she will choose a spot on the rock. Therefore her payoff is 20 if the sand is completely unoccupied when she arrives, and 10 otherwise, as you can quickly deduce from Figures 4.3 through 4.6.



Figures 4.4 through 4.7
Choices the late riser faces

Having deduced how the late riser will choose, we now turn our attention to the bather who keeps regular hours. If the first two sunbathers account for the optimal decision of the late riser, the payoff structure they face can be represented by Figure 4.7. This figure shows the payoffs that each of the first two receive if the late riser optimally chooses when she takes her turn. For example if the first two both choose rocky spots, Figure 4.3 shows the optimal choice of the late riser is to choose sand, which implies the payoffs to the first two and respectively 10 and 9. This appears as the far left payoff in Figure 4.7

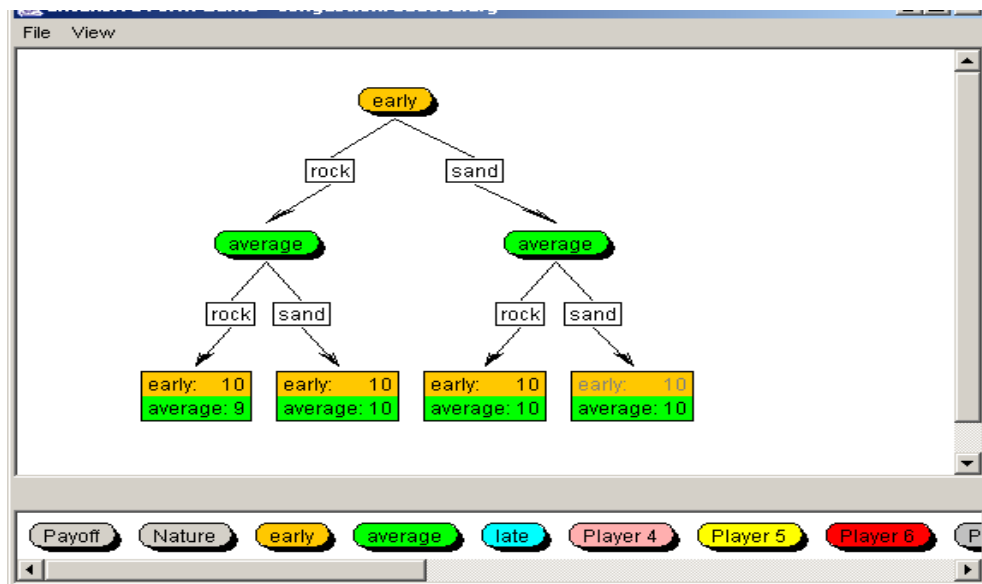


Figure 4.7

Reduce game for first two sunbathers

Suppose the first sunbather chooses the best rocky spot. From Figure 4.7 we see that the reduced problem facing the bather who keeps regular hours is to choose between the rocks, which yields utility of 9 versus the sand, which after accounting for the optimal behavior of the late riser yields a utility of 10. Her subproblem is depicted in Figure 4.8.

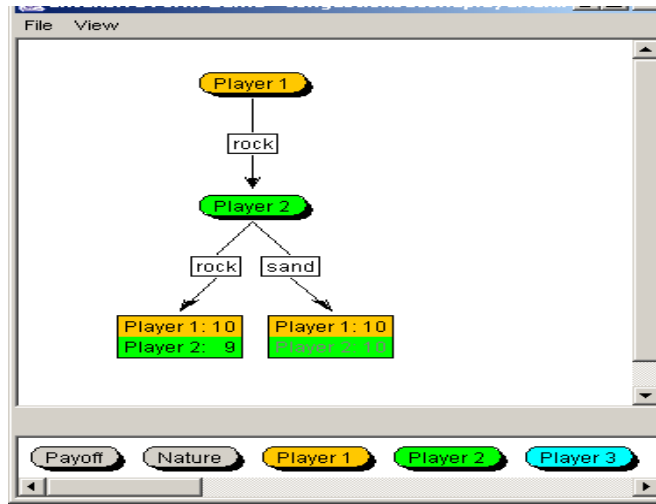


Figure 4.8

Problem of bather who keeps regular hours if early riser chooses a rocky spot
 If the early riser chooses a rocky spot, then the sunbather keeping regular hours chooses the sand. Similarly we can see from Figures 4.7 and 4.9 that she is indifferent between sand and rock.

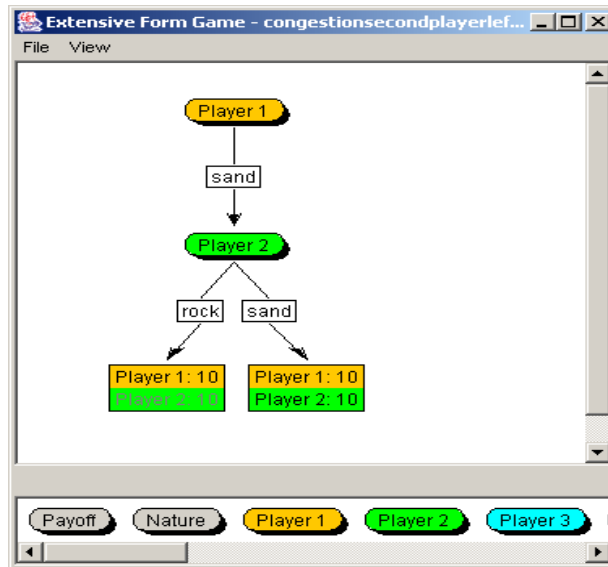


Figure 4.9

Problem of bather who keeps regular hours if early riser chooses a sandy spot

Following the same reasoning as before, the early riser can now see her problem more clearly. As Figure 4.10 shows the early riser exhibits indifference between picking a rocky versus a sandy spot. Therefore if the first player picks rock or sand

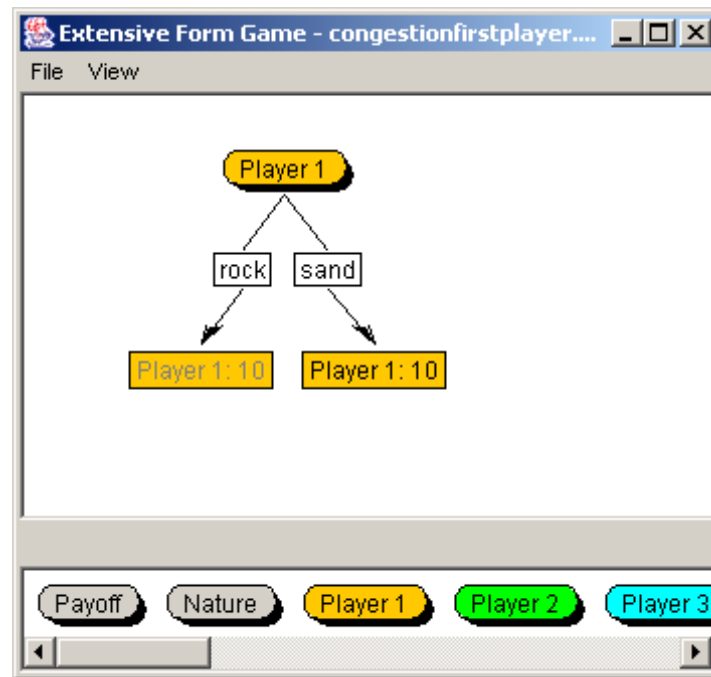


Figure 4.10

The early riser chooses the rocky spot

This is not because she does not care. Like the other players she would be willing to pay the equivalent of 10 utils to have exclusive rights to the sand, and if that could somehow be arranged the total gains for the sunbathers and the entrepreneur would be 39 a surplus of 9 units over the equilibrium outcome.

More generally the presence of congestion creates costs that are barely visible as players expend resources seeking to avoid the congestion. For example in morning rush hour we rarely hear about how much early and late traffic is as commuters choose sleep deprivation in order to avoid the waiting. What about businesses that relocate to smaller cities and forgo the benefits of networking in order to avoid congestion?

Form the point of view of a business seeking to create a market by selling rights to companies that , the benefits are greater than immediately obvious for the same reason.

Exercise *Market Saturation*

1. *Solve the game depicted in Figure 3.14.*
2. *solve the modified game in which players have the opportunity to invest in broad band cable in the reverse order to the one depicted.*
3. *Prove the solution to the game is independent of the order in which*

players move.

Alliance

Elaborating on a stopping game provides one more example to demonstrate how the backwards induction rule should be applied. This is a game for 5 players, each of whom take turns to stop play from progressing further. Regardless of when play stops, the sum of the payoffs to all players is 15, one player receiving 1, another 2, and so forth up to 5. Payouts at the terminal nodes are only distinguished by how the payouts of 1 through 5 are assigned. Because all terminal nodes yield the same total payout, this is called a constant sum game, where each player's objective is diametrically opposed to the collective interest of all the others. Figure 4.13 depicts the extensive form of this stopping game.

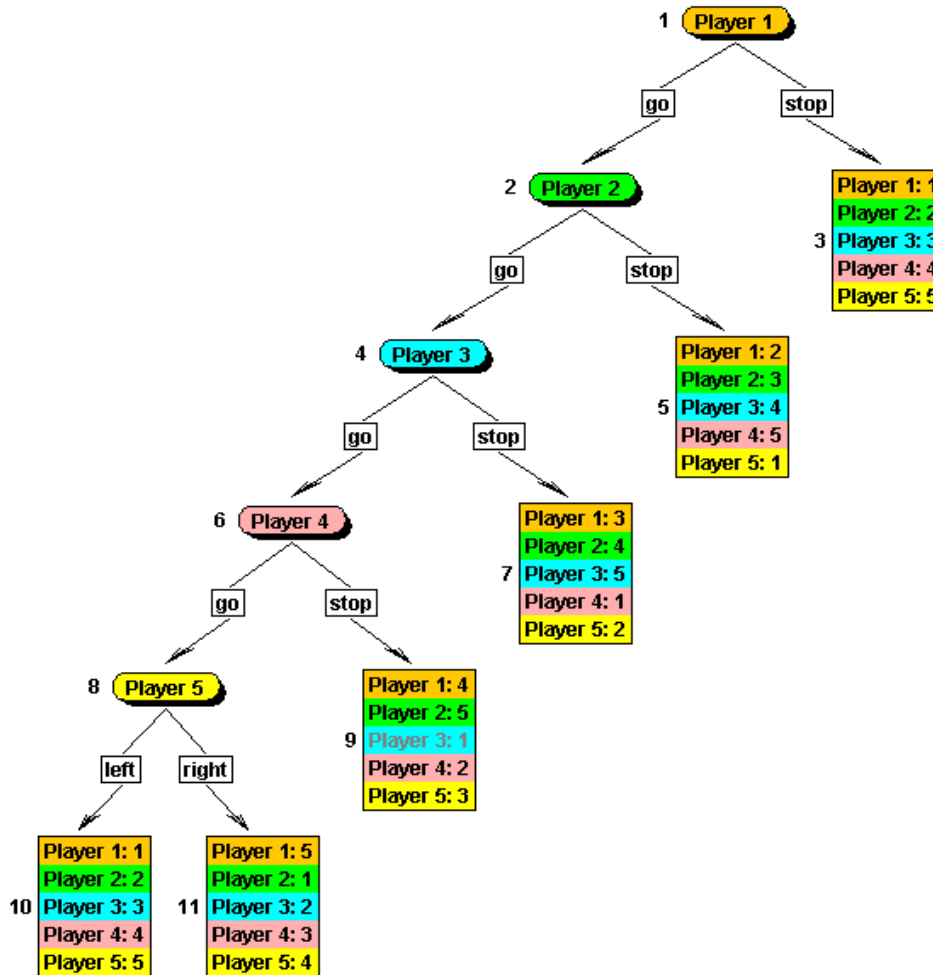


Figure 4.13

Alliance

Following the principle of backwards induction Player 5 would move left if given the opportunity, and hence Player 4 would go. Anticipating these moves Player 3 stops if the game proceeds that far. Since the payoffs of Player 1 and 2 increase with the length of the game at least up until 5 moves, the solution is for Players 1 and 2 to go, and Player 3 to stop. Notice that although this game is a constant sum game in which every extra unit awarded to one player is taken from somebody else, there is an alliance of sorts between the first three players and the last two. Both Players 4 and 5 hope that Player 3 misses her opportunity to stop the game, while Players 1 and 2 rely on Player 3 to act in her own best interests.

Experiment

In this experiment 91 % of subjects selected node 7 (i.e. purple color) that resulted from the decision of Player 1 and Player 2 to go and Player 3 to stop the game. Only in 9% cases were subjects ended in a different decision node, more specifically, the only other node that was visited was node 5 where Player 2 stopped the game too early.

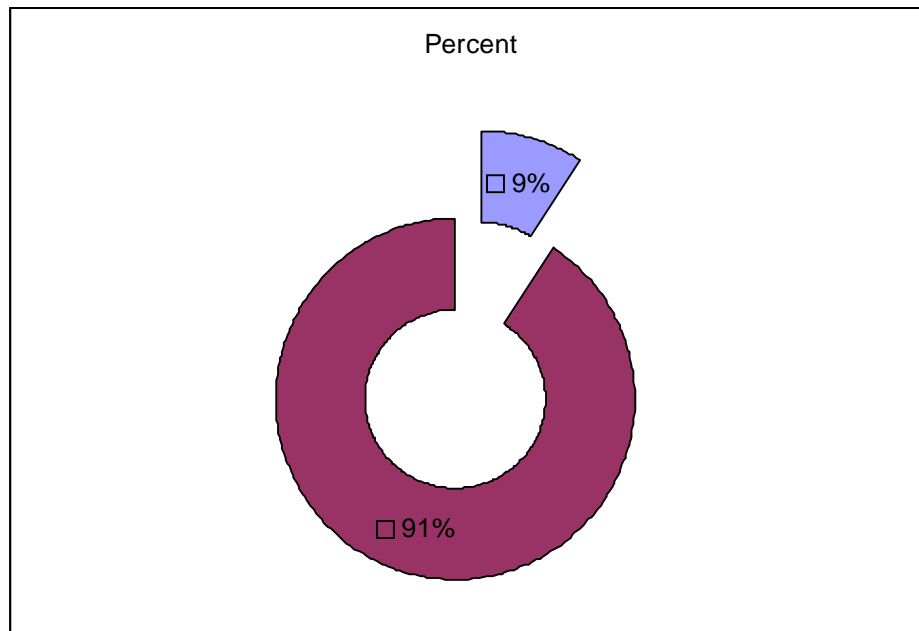


Figure 4.14: Proportion of subjects selecting different nodes

Exercise *Permutations on Alliance*

1. Consider the following modification to the coordinated stopping game depicted in Figure 4.13. Player 4 moves first, followed by Player 5, 1, 2 and 3 in that order. Relabel the "right" choice of Player 5 as "stop", and the "left" choice as "go". Also relabel the "stop" choice of Player 3 as "right" and the "go" choice as "left". Finally assign the payoffs under the "left" choice of

Player 5 in the original game to the "left" choice of Player 5 in the modified game.

2. Derive the solution.
3. Do the alliances remain intact?
4. Permute the payoffs and run the game, allowing precommunication between pairs.
5. Tabulate the number of messages between senders and receivers of each type and relate those to the alliances.

First or Last

You are often confronted with strategic situations in which you have the choice of moving first, second and so forth. For example:

- making a presentation, for your company or your class
- adopting a new technology before or after your rivals

The principle of backwards induction assures us that there is no easy answer to this question, and that each case must be decided on its merits.

Two firms compete for build extra capacity which can only accommodate 1 more plant. both players want to commit first.

When it doesn't matter entry pricing credible threats to punish with lower prices not possible.

Being second to get at the flaws of your competitor ,having the last word against an opponent.

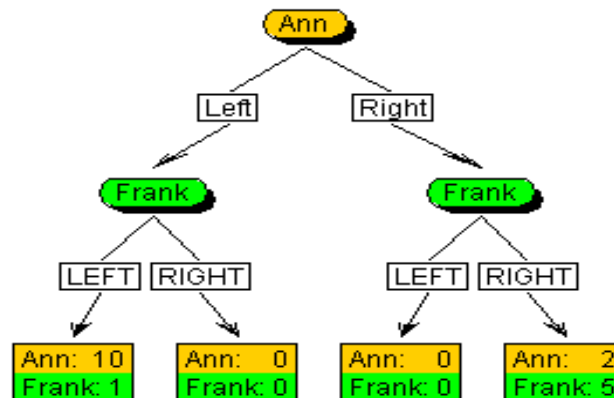


Figure 3.16
Frank and Ann

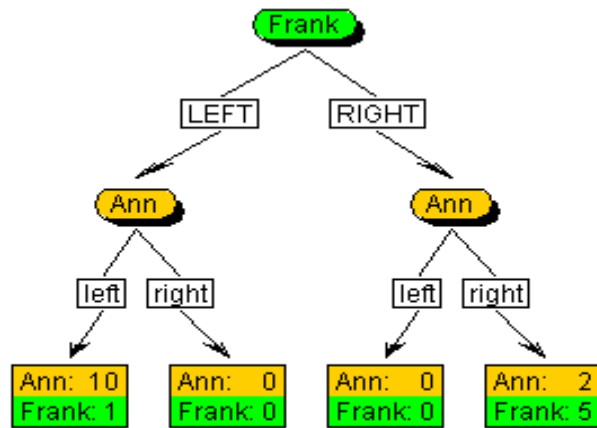


Figure 3.17
Ann and Frank

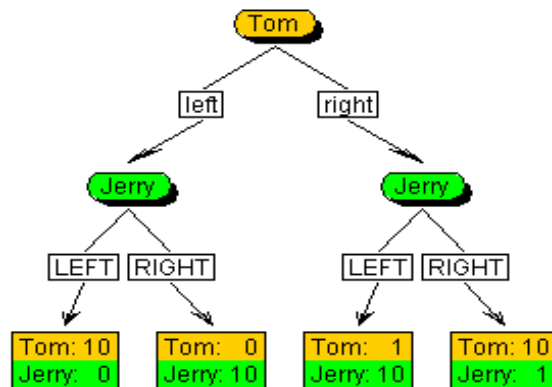


Figure 3.16
Tom and Jerry

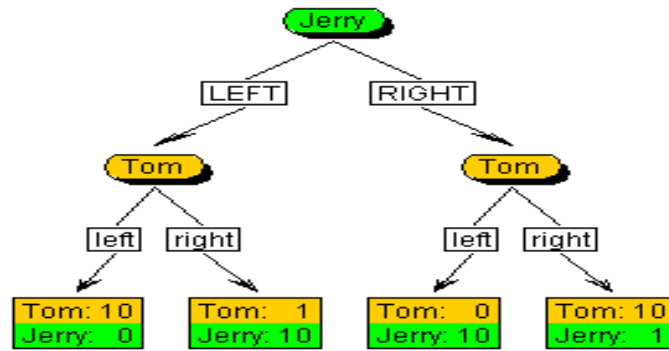


Figure 3.18
Jerry and Tom

Exercise *Derive the solutions to the four games in this section*

Recognizing Games of Perfect Information

The extensive form for perfect foresight games are uniquely defined. If we interchanged any two decision nodes it would be necessary for the first player to move in the original extensive form game to have the information about the second player's move hidden from him. This can only be done by joining the nodes coming from the second player. Moreover the second player cannot distinguish between the actions for the first player. Therefore the second could not have been a game of perfect foresight.

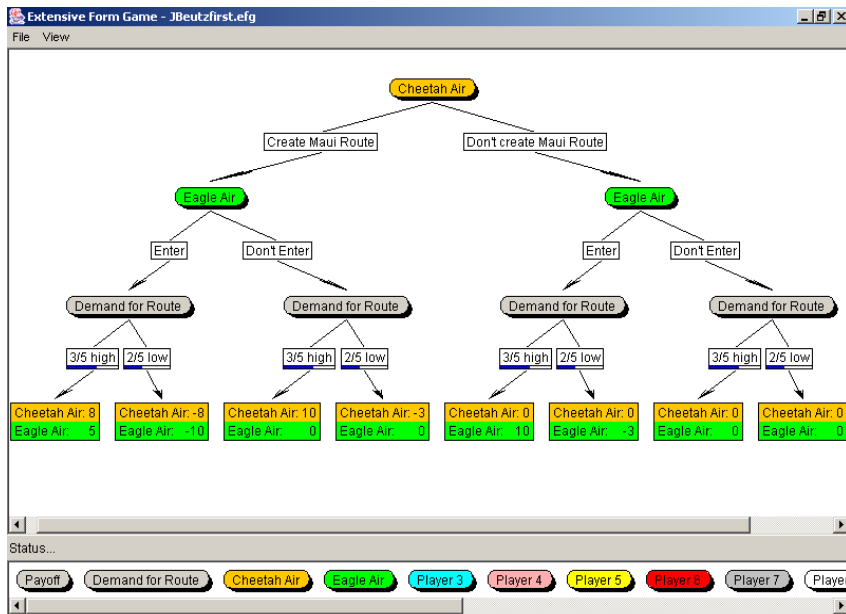
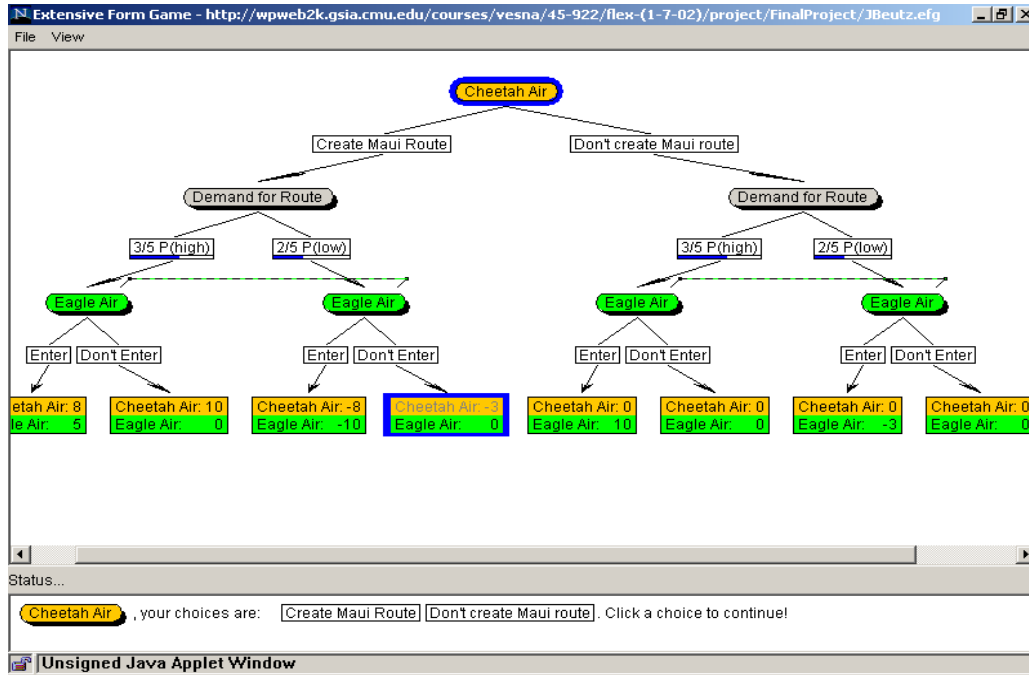
At every turn in a perfect information game, the player about to move has more information than the person who has just moved. A perfect information game can be represented by an extensive form in which there is only one choice node for every information set.

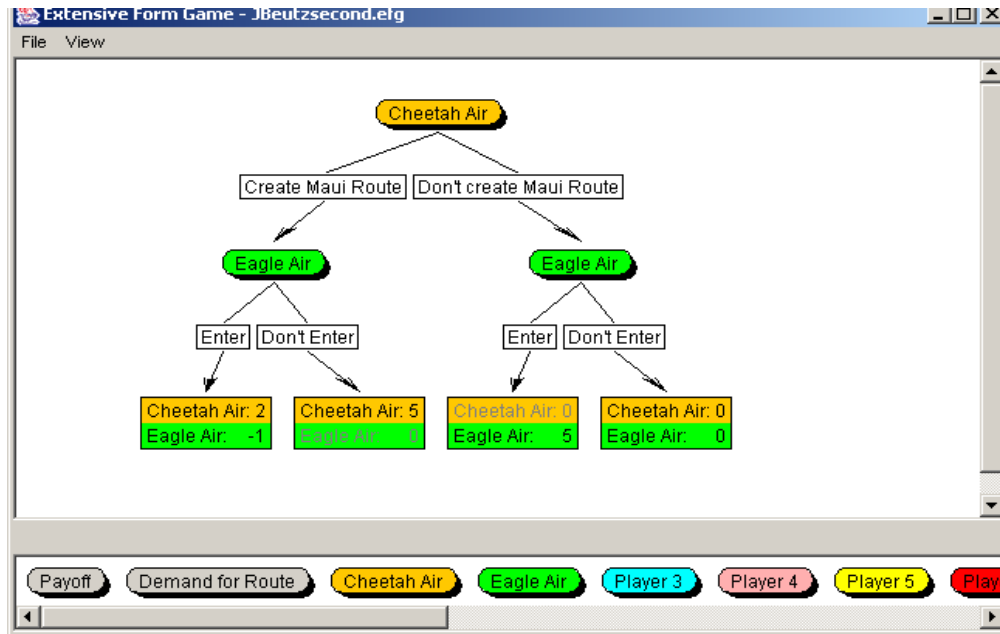
The first is that the extensive form DOES NOT impose an order on the moves, even in simultaneous move games. The sense in which moves are simultaneous is through the structure of the information sets. By joining all nodes for the second player so that he cannot distinguish between them, the player is in effect forced to make a choice without knowing what the first player has done. Comparing Figures and , notice that in the first representation the U.S. but in the Figure below, Japan is the first mover.

It might be tempting to think that a fast way of establishing whether a game has perfect information or not would be to draw its extensive form, and compare the number of decision nodes with the number of information sets. The argument is that every information set would contain one decision node. This is a sufficient condition

but it is not necessary. If a game has an extensive form in which there is an equality, then it follows from the definition that it has perfect information. Recall that games do not always have a unique extensive form, and some extensive form representations of a perfect game.

Air service





Venture Capital

Consider an innovator seeking funds for a project. Neither the innovator nor the venture capitalist know if the project has a commercial.

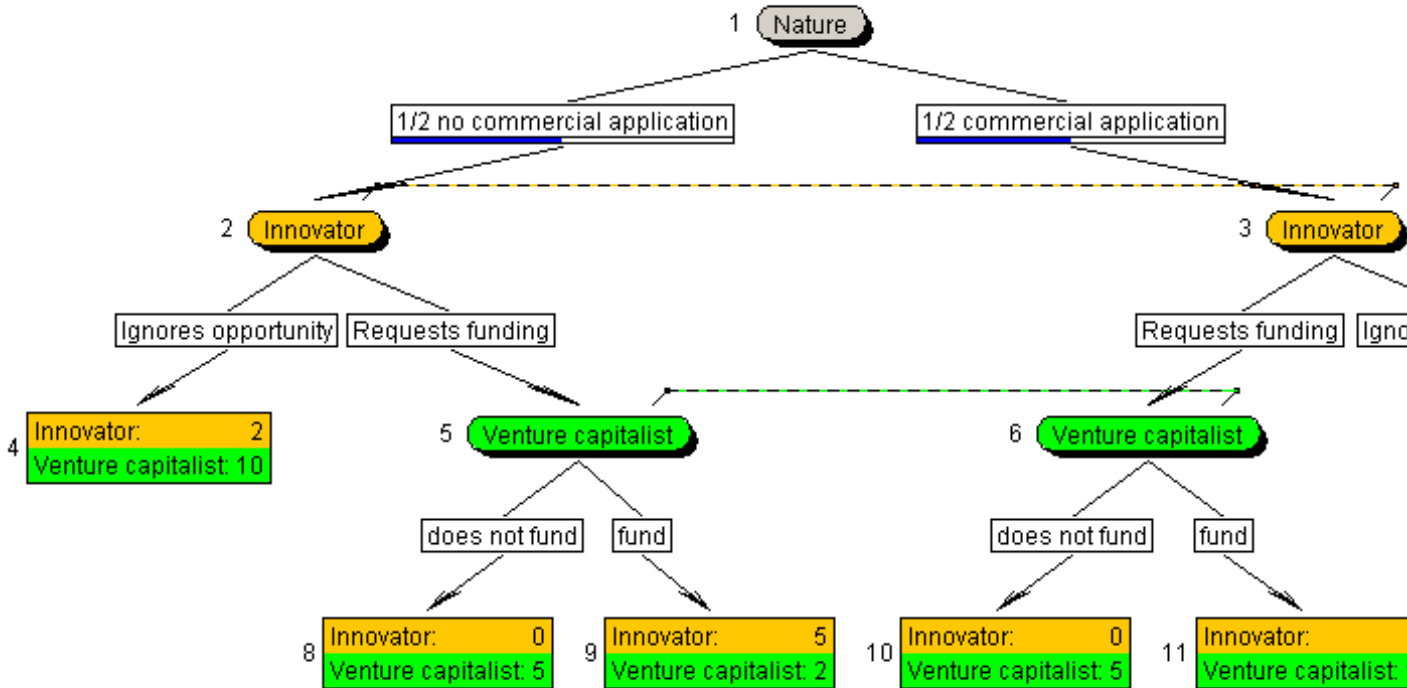


Figure 4.4

Venture Capital

Can this game be redrawn as a game of perfect information? For pedagogical reasons we do this in two steps. Notice that the venture capitalist has the same information as the innovator.

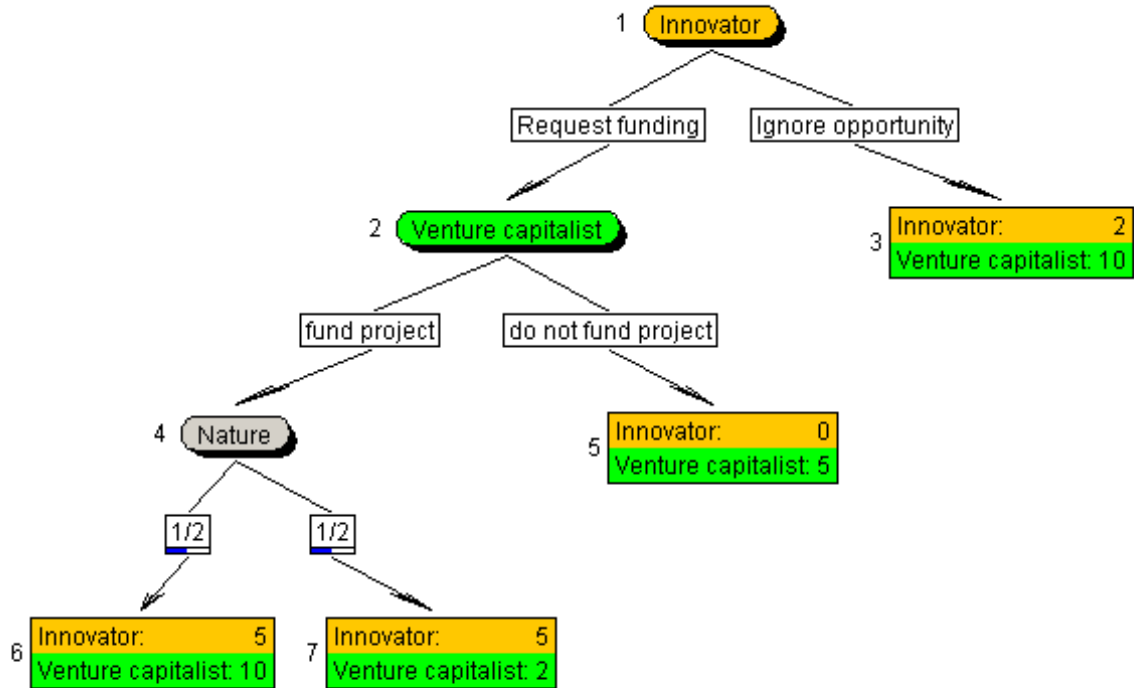


Figure 4.5
Perfect information revealed

Upon redrawing the extensive form, it becomes evident that the venture capital game underlying Figure 4.5 is indeed a game of perfect information.

Experiment

Subjects played the game in both representations. In perfect information presentation there were no deviations from the backwards induction principle that innovator requests funding and venture capitalist accepts it (Node 6 and 7). In the original game subjects were making the predicted decision 80 percent of the time

(Node 9 and 11).

	Node 6 and 7	Node 7	Node 8	Node 9 and 11	Node 11
Innovator game (%)	-	5	5	80	10
Innovator game – perfect information (%)	100	-	-	-	-

Table 4.5: Data for the original innovator game and the perfect information representation

Exercise Program the three extensive forms, and run them. Can you reject the null hypothesis that the distribution of experimental outcomes the same?

Demand for CD players

Uncertain Listenership: The case of CDs

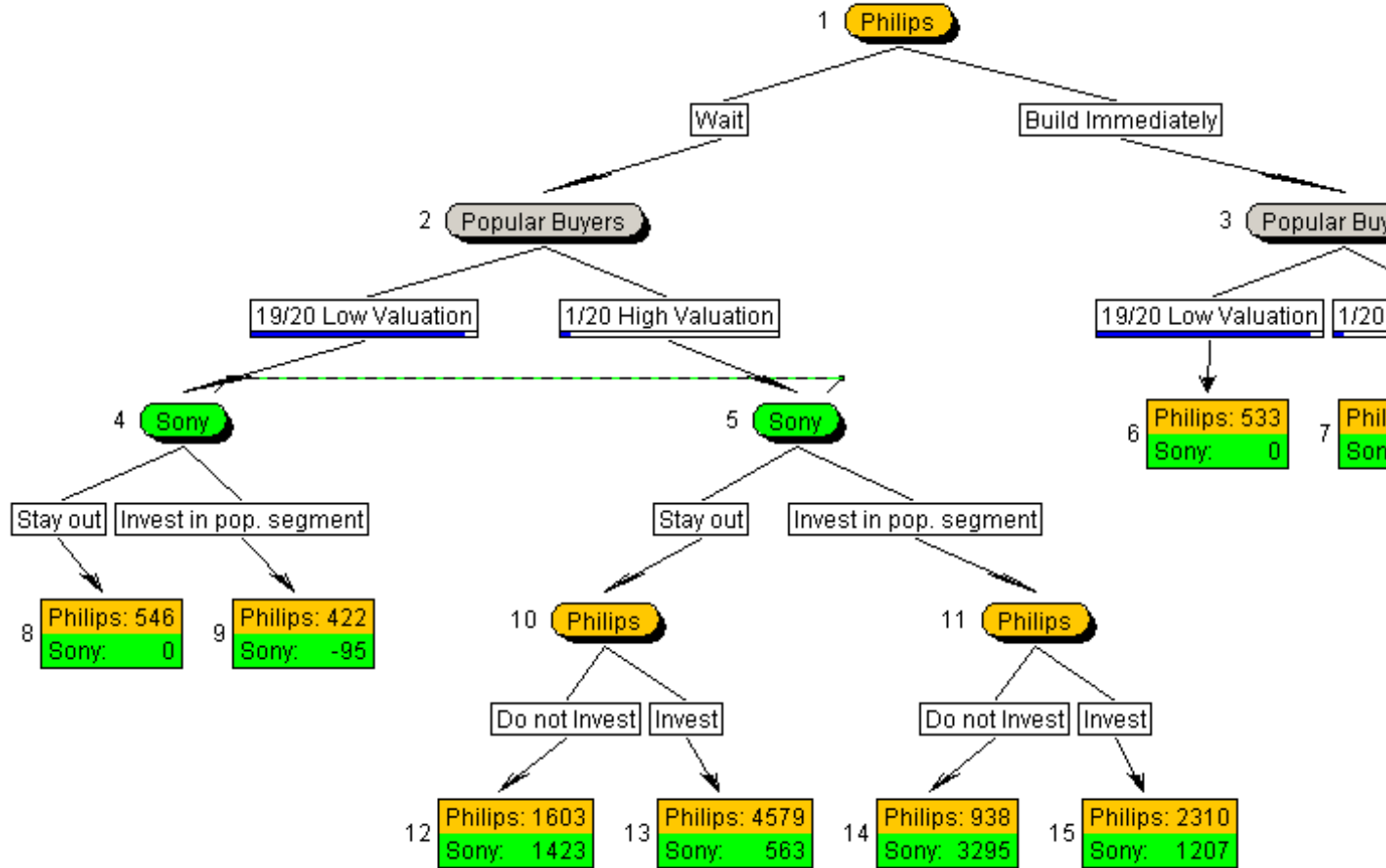


Figure 4.7
Investing in CD players

At first glance this does not look like a game of perfect information. Solving the game backward, if Phillips has not already invested, it will do so at stage 3 if buyers of popular music are revealed to have a high valuation for CDs but will not if they have a low valuation. At stage 2 Sony must condition its choice on its assessment of the probability p that popular buyers place a high valuation on CDs. It will invest as soon as it can if $1207p - 95(1-p) > 563p$. According to information, Phillips “put the probability of quick acceptance in the popular segment at about 5 percent. And there is no reason to believe that Sony’s managers would not make the same assessment. With $p=0.05$ (even $p=0.10$), the inequality given above is violated, implying that Sony should not invest. Armed with this conclusion, we can now work through Phillips’s choice in stage 1. Phillips should build immediately if and only if $4585p + 533(1-p) > 4579p + 546(1-p)$. With $p=0.05$, the left-hand side of the inequality comes to 735.6 and the right-hand side to 747.7, suggesting that Phillips stands to gain an extra \$12

million by waiting in stage 1 rather than building immediately.

An alternative representation of the extensive form. But this is a game of perfect information.

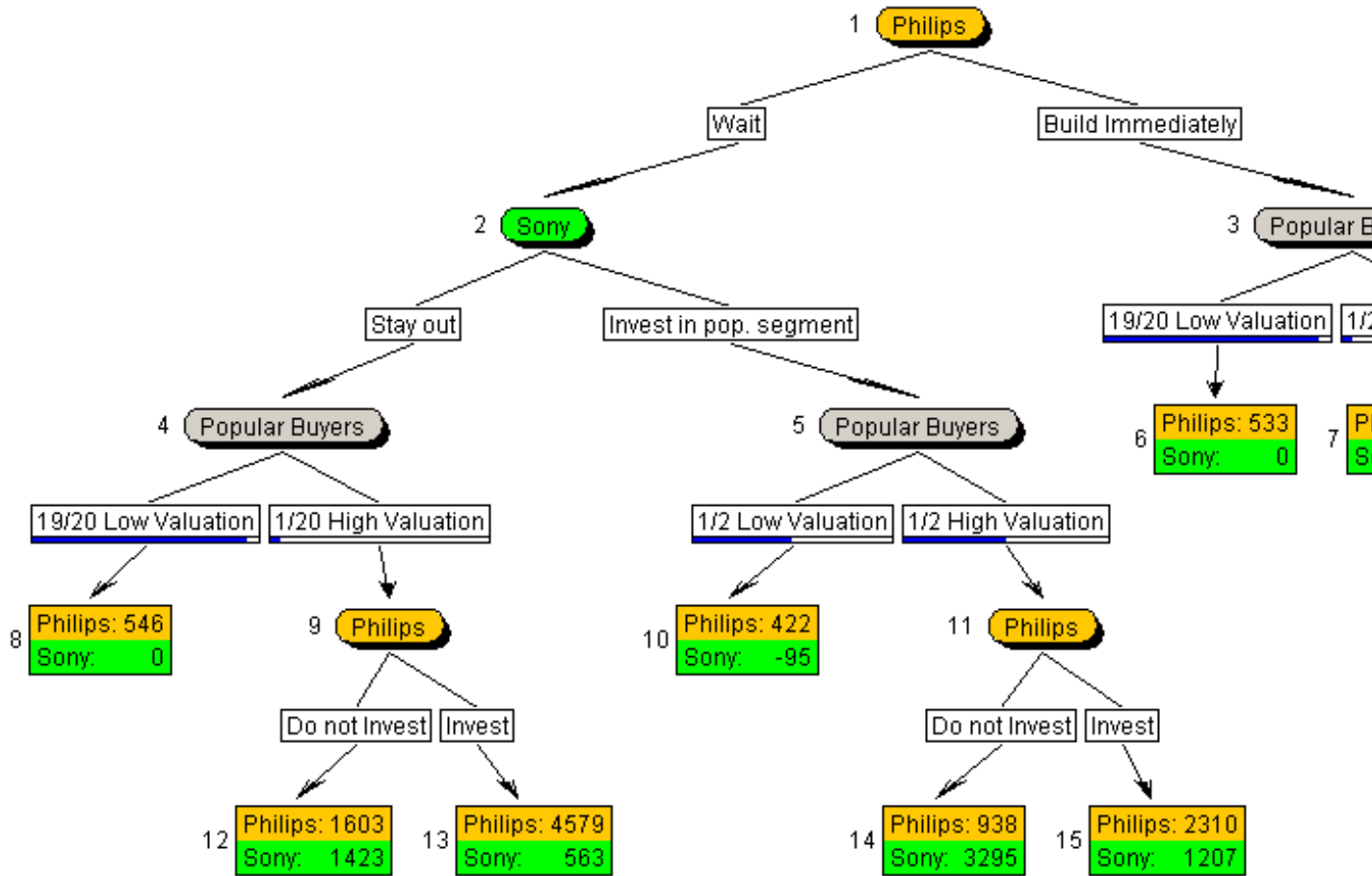


Figure 4.8
Perfect information revealed

So, abstracting from risk aversion, our first Rule implies that Philips should wait to build in stage 1 because it can count on Sony waiting to build in stage 2.

Experiment

Subjects again played the game in two different representations. In the original game subjects should have ended in Node 8 and 13 all the time. Instead these two nodes were visited only 37.93 percent. The data suggest that subjects had much more difficult time which choice to select. One explanation is that it was harder to calculate

the expected payoffs with the four digit numbers. The results for the perfect information did not differ.

	Node 6	Node 8 and 13	Node 9	Node 10
Phillips and Sony (%)	41.38	37.93	6.90	-
Phillips and Sony – perfect information (%)	40.74	40.74	-	18.52

Table 4.8: Data for the original Phillips and Sony game and the perfect information representation

The inventor’s dream

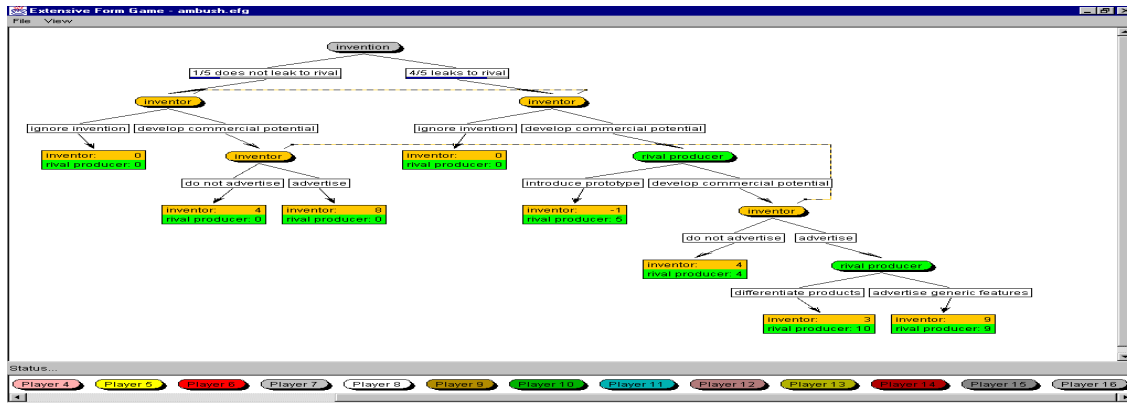


Figure 4.9
The inventor’s dream

At first sight, the extensive form depicted in figure 3.20 does not look as if is associated with a game of perfect information. A research scientist has just discovered a product of commercial value. In his excitement he has mentioned this to several of his colleagues. Consequently there is an 80 percent chance that his invention leaks to a rival producer. Recognizing this indiscretion, the scientist first chooses whether to ignore the invention or begin the arduous process of bringing it to fruition.

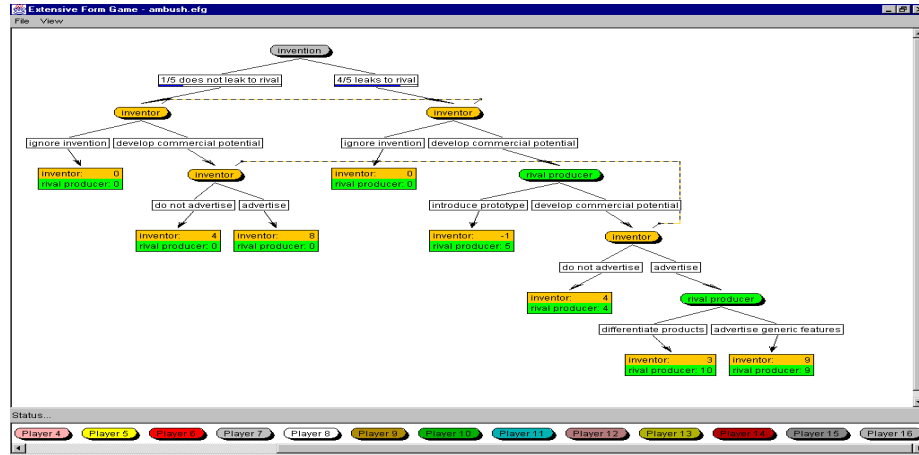


Figure 4.10

Despite the scope for intrigue and hidden activity, the game supporting this extensive form is supported by a perfect information game. First note that inverting the order of the nature and the inventor's first move has no effect on the inventor's decision, because at that time he cannot . With this modification, the revised extensive form now appears as Figure

Solving perfect information games

The final section in this chapter combines the tools we have developed in the previous parts to solve perfect information games where there is uncertainty and players obey the expected utility hypothesis.

Moon Race

This definition does not preclude uncertainty from playing some role in the analysis. In Figure 3.1 the only uncertainty that enters the game comes from the actions of Player 2. If player 1 cannot anticipate what player 2 will do, then this is the only source of uncertainty. But now consider the following modification to the game. After Player 1 moves, there is some probability that a technological progress occurs. Here the number of decision nodes assigned to players equals the number of information sets, and so the game is a game of perfect information. Notice that in this example the total number of nodes is greater than the number of information sets, a consequence of nature's role in this example.

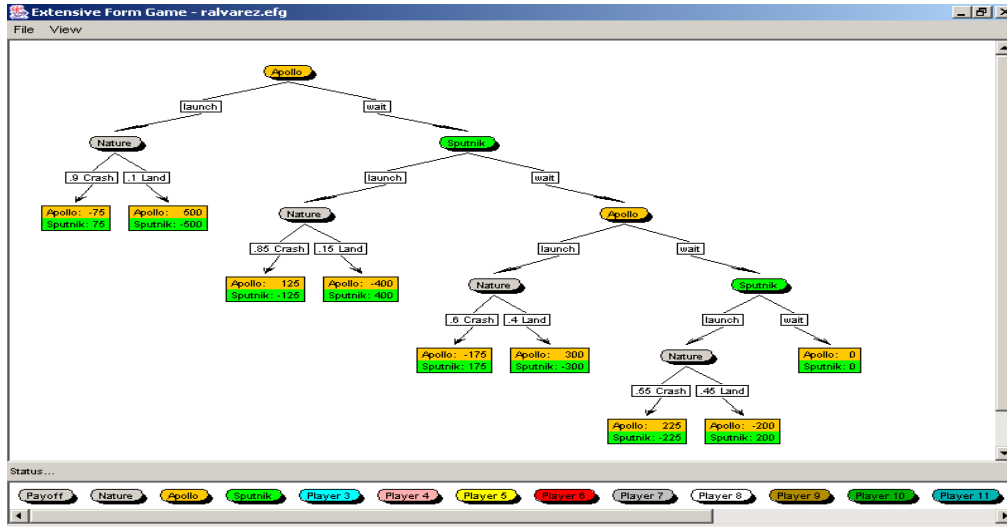


Figure 4.3
Moon race

Harassment

A problem in labor relations:

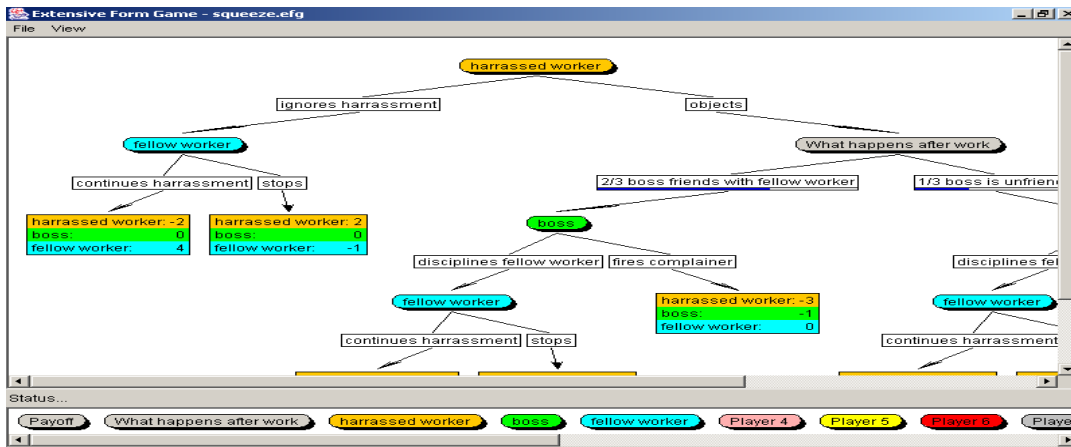
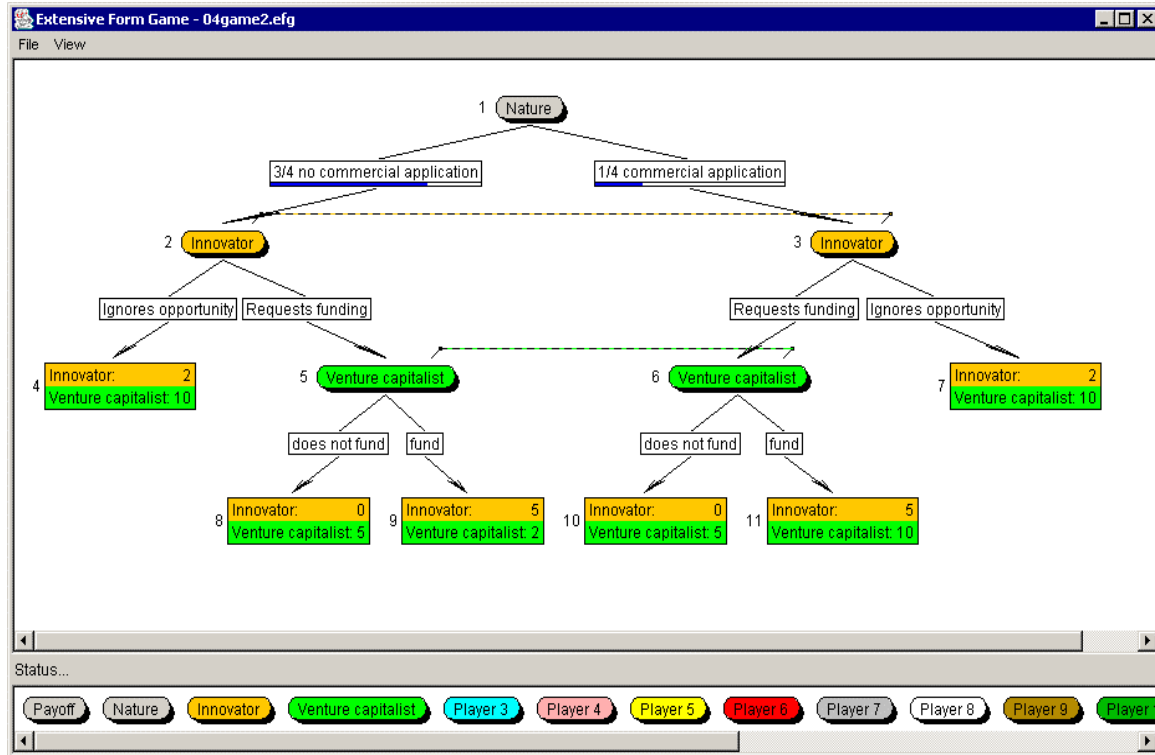


Figure 4.15
Discrimination

Retrenchment

Another example is the staff reductions:



Comments and Solution

This game is similar to the game represented in Figure 4.4. The only difference is that the probabilities on no commercial application were changed from $1/2$ to $3/4$ and for commercial application to $1/4$ respectively.

Expected payoff for the innovator if venture capitalist fund the project is $3/4 \cdot 5 + 1/4 \cdot 5 = 5$. Expected payoff to venture capitalist if he funds the project is $3/4 \cdot 2 + 1/4 \cdot 10 = 4$. Venture capitalist will not fund the project because he can earn 5 by not funding the project (i.e. versus 4 if he funds it). That leads innovator to ignore opportunity (nodes 4 and 7). Innovator receives 2 and venture capitalist 10.

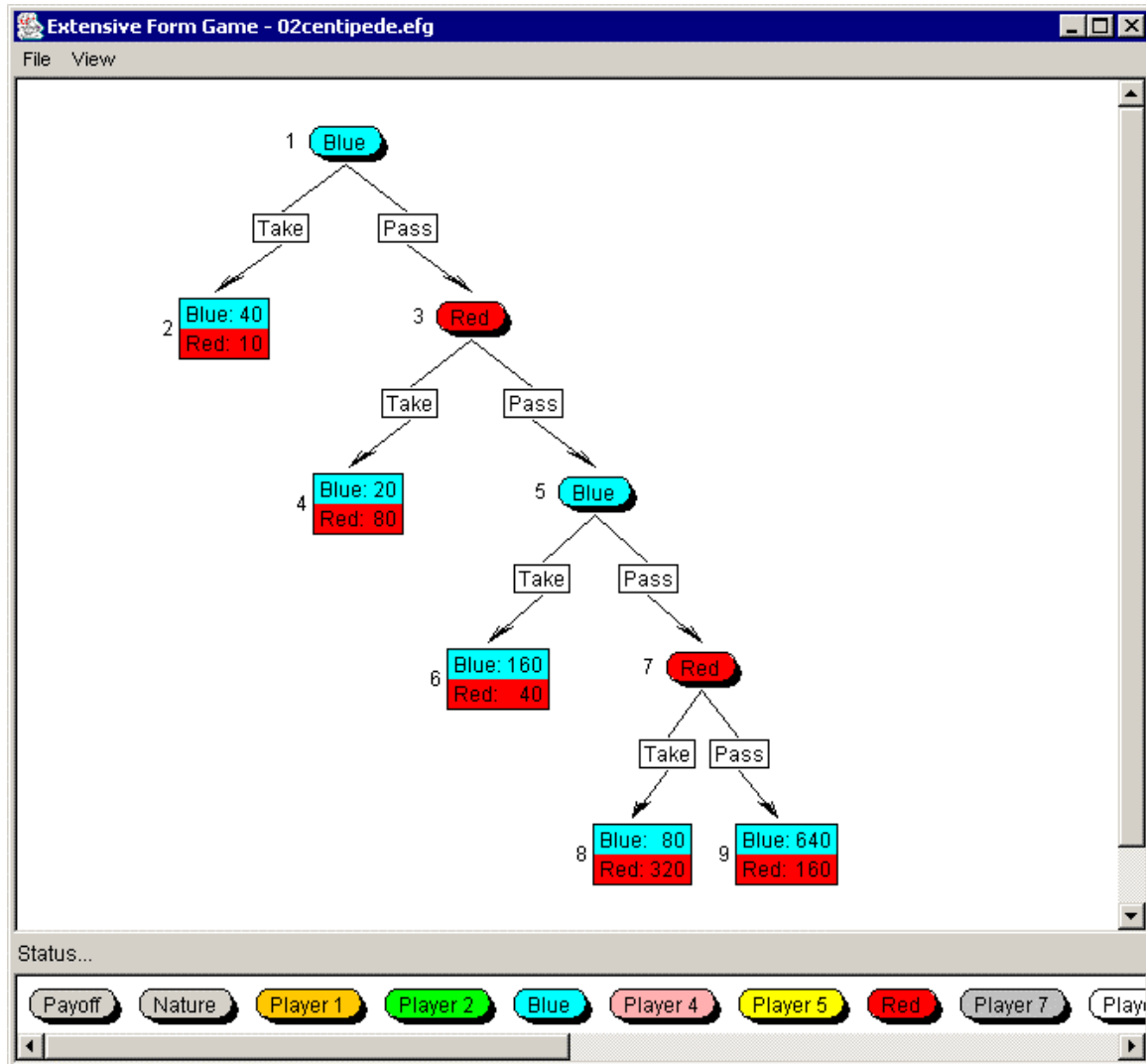
The problem is graded in the following way: If innovator makes a mistake and request funding that he/she receives 0 points. However venture capitalist receives a point if he/she selected does not fund.

Test results

Session Num	Session Name	Term Node	Player1 Innovator	Player1 Payoff	Player 1 Grade	Player2 Venture Cap	Player 2 Grade	Player2 Payoff
2	04game2.efg	4	aa	2	1	hh	1	10
15	04game2.efg	8	bb	0	0	ff	1	5
3	04game2.efg	10	cc	0	0	ii	1	5
6	04game2.efg	4	idan	2	1	jj	1	10
19	04game2.efg	4	aa	2	1	kk	1	10
21	04game2.efg	4	cc	2	1	ii	1	10
16	04game2.efg	4	dd	2	1	ll	1	10
10	04game2.efg	11	ee	5	0	mm	0	10
22	04game2.efg	4	aa	2	1	kk	1	10
20	04game2.efg	4	ff	2	1	jj	1	10
26	04game2.efg	4	cc	2	1	nn	1	10
17	04game2.efg	4	gg	2	1	ee	1	10

When P1	When P2	Total	# Times	Grade
0.0	3.0	3.0	3	1.00
2.0	1.0	3.0	3	1.00
2.0	2.0	4.0	4	1.00
3.0	1.0	4.0	4	1.00
0.0	0.0	0.0	3	0.00
1.0	2.0	3.0	3	1.00
2.0	2.0	4.0	5	0.80
2.0	1.0	3.0	3	1.00
1.0	2.0	3.0	3	1.00
0.0	1.0	1.0	3	0.33
1.0	2.0	3.0	3	1.00
0.0	2.0	2.0	3	0.67
2.0	1.0	3.0	3	1.00
1.0	1.0	2.0	3	0.67
0.0	1.0	1.0	3	0.33
1.0	2.0	3.0	3	1.00
0.0	1.0	1.0	3	0.33
0.0	3.0	3.0	4	0.75
0.0	2.0	2.0	3	0.67
2.0	1.0	3.0	3	1.00
2.0	1.0	3.0	3	1.00
3.0	2.0	5.0	5	1.00
0.0	0.0	0.0	3	0.00
2.0	1.0	3.0	4	0.75
3.0	0.0	3.0	3	1.00
0.0	4.0	4.0	4	1.00
1.0	2.0	3.0	3	1.00
1.0	2.0	3.0	3	1.00
0.0	3.0	3.0	3	1.00
1.0	0.0	1.0	3	0.33
1.0	0.0	1.0	4	0.25
		80.0	103	0.78

2. Centipede game



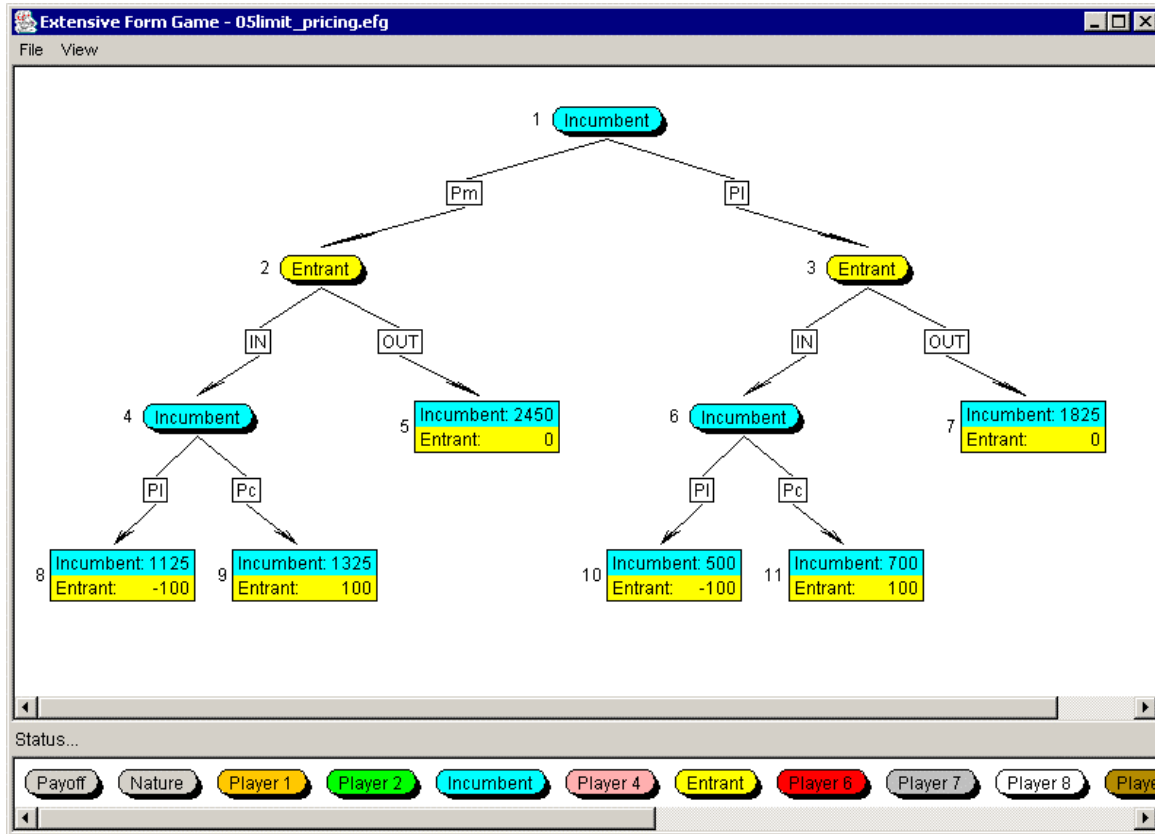
Comments and Solution

Use backwards induction (i.e. rolling back principle) to solve the problem: at node 7 RED will Take and receive 320. Knowing that Blue will take at node 5 because 160 is more than 80 (i.e. the amount he can expect if he passes). At node 3, RED will take because 80 is more than 40. At node 1 Blue will Take because 40 is more than 20. The solution to this game is: Blue takes immediately and the game ends in node 2.

3. Limit pricing

The following game tries to model the decision of a monopolist (i.e. Incumbent) who is facing an entrant and has to decide about the pricing policy. One option that

he has is to go with the monopoly price (P_m) or limited price (P_l). If Entrant decides to enter Incumbent can revise the price option between (P_l) and competitive price (P_c).



Comments and Solution

This is a game of perfect information and we will use backwards induction to solve it.

- At node 4 Incumbent will pick (P_c) and receive 1325. Entrant will receive 100.
- At node 6 incumbent will select (P_c) and receive 700. Entrant will get 100.
- Entrant will enter at node 2 and at node 3.
- Incumbent will select (P_m) at node 1 because 1325 is more than 700.

Solution: terminal node 9.

Test results

Figure 4.1: shows an incert from the output file. Terminal node 9 was the solution to the limit pricing problem and all these players ended in terminal node 9. They received one point for the correct answer.

Session Num	Session Name	Term Node	Player1 Incumbent	Grade P1	Player2 Entrant	Grade P2
5	05limit_pricing.efg	9	aa	1	bb	1
4	05limit_pricing.efg	9	cc	1	kk	1
12	05limit_pricing.efg	9	dd	1	ll	1
7	05limit_pricing.efg	9	ee	1	mm	1
18	05limit_pricing.efg	9	aa	1	bb	1
1	05limit_pricing.efg	9	ff	1	nn	1
2	05limit_pricing.efg	9	gg	1	oo	1
10	05limit_pricing.efg	9	hh	1	pp	1
19	05limit_pricing.efg	9	cc	1	kk	1
16	05limit_pricing.efg	9	ii	1	ss	1
14	05limit_pricing.efg	9	jj	1	zz	1

Figure 4.1: Individual results for Limit pricing

	When P1	When P2	Total	# Times	Grade
1	3	0	3	3	1.00
2	1	2	3	3	1.00
3	3	0	3	3	1.00
4	0	3	3	3	1.00
5	3	0	3	3	1.00
6	1	2	3	3	1.00
7	1	0	1	1	1.00
8	3	0	3	3	1.00
9	0	3	3	3	1.00
10	1	2	3	3	1.00
11	1	2	3	3	1.00
12	2	1	3	3	1.00
13	0	3	3	3	1.00
14	1	2	3	3	1.00
15	1	1	2	2	1.00
16	2	1	3	3	1.00
17	2	1	3	3	1.00
18	1	2	3	3	1.00
19	1	2	3	3	1.00
20	1	2	3	3	1.00
21	2	1	3	3	1.00
22	2	1	3	3	1.00
23	2	1	3	3	1.00
24	1	1	2	2	1.00
25	1	1	2	2	1.00
26	2	2	4	4	1.00
27	2	0	2	3	0.67
28	1	2	3	3	1.00
29	1	2	3	3	1.00
30	2	1	3	3	1.00
31	0	4	4	4	1.00
			89	90	0.99

Bibliography

Andreoni, Janes, Paul Brown, and Lise Vesterlund (1999) "What Produces Fairness? Some Experimental Evidence," Iowa State University, Discussion Paper, presented at the Summer 1999 ESA Meeting. Abstract: The experiment compares behavior in related two-person public goods games: one with simultaneous play and summation-based payoffs, one with sequential play and summation-based payoffs, and one with sequential play and maximum (best shot) payoffs. All three have a subgame perfect equilibrium where one player contributes and the other doesn't. In the experiment, cooperation is highest in the simultaneous game and lowest in the best-shot game. In the sequential games, selfish initial choices are not punished as much in the best-shot game.

Beard, T. Randolph, and Richard O. Beil Jr. (1994) "Do People Rely on the Self-Interested Utility Maximization of Others? An Experimental Test," *Management Science*, 40:252-262. Abstract: The experiments involve two stage games in which one player chooses between a safe (punishment proof) and a risky strategy. The second player sees the decision and decides whether to deliver a costly punishment. The frequency of the subgame perfect equilibrium (risky, not punish) depends on changes in payoff parameters that do not alter the equilibrium.

Belda, Carles Sola (1999) "Punishment in Sequential Games: Experimental Evidence," in , Universitat Autònoma de Barcelona.

Brandts, Jordi, and Carles Sola (1998) "Reference Points and Negative Reciprocity in Simple Sequential Games," University of Barcelona, Discussion Paper, presented at the Summer 1998 ESA Meeting. Abstract: The experiments involves sequential two-person with ultimatum and best-shot structures. Punishments in the experiments seem to be explained in terms of fairness relative to reference points.

Camerer, Colin F., and Keith Weigelt (1988) "Experimental Tests of a Sequential

Equilibrium Reputation Model,” *Econometrica*, 56:1 (January), 1-36.

Capra, C. Monica, Rosario Gómez, and Susana Cabrera-Yeto (1999) “A Bunch of Deviating Choices in Common Pool Resource Games with Sequential Decisions,” University of Málaga, Discussion Paper, presented at the Fall 1999 European Regional ESA Meeting. Abstract: Two individuals make alternating decisions in a common pool resource game. The remaining stock grows at a fixed rate for a finite number of periods. Consistent deviations from the subgame perfect Nash equilibrium are observed.

Cooper, Russell, Douglas V. DeJong, Robert Forsythe, and Thomas W. Ross (1992) “Forward Induction in Coordination Games,” *Economic Letters*, 40:2 (October), 167-172.

Eckel, Catherine C., and Rick K. Wilson (1999) “The Human Face of Game Theory: Trust and Reciprocity in Sequential Games,” Virginia Polytechnic Institute and State University, Discussion Paper, presented at the Fall 1997 ESA Meetings.

Falk, Armin, and Urs Fischbacher (1998) “Kindness is the Parent of Kindness: A Model of Reciprocity,” University of Zurich, Discussion Paper, presented at the Summer 1998 ESA Meetings. Abstract: The framework of psychological game theory is extended to extensive-form games. Reciprocity is used to explain behavior in a variety of sequential games.

Harrison, Glenn W., and Jack Hirshleifer (1989) “An Experimental Evaluation of Weakest Link/Best Shot Models of Public Goods,” *Journal of Political Economy*, 97:1 (February), 201-225.

McCabe, Kevin A., Stephen J. Rassenti, and Vernon L. Smith (1998) “Reciprocity, Trust, and Payoff Privacy in Extensive Form Bargaining,” *Games and Economic Behavior*, 24:1-2 (July-August), 10-24.

McKelvey, Richard D., and Thomas R. Palfrey (1992) “An Experimental Study of the Centipede Game,” *Econometrica*, 60, 803-836.

McKelvey, Richard D., and Thomas R. Palfrey (1998) "Quantal Response Equilibria for Extensive Form Games," *Experimental Economics*, 1:1 9-41.

Mitropoulos, A., Joachim Weimann, and Chun-Lei Yang (1998) "Rent-Seeking Experiments," Magdeburg University, Discussion Paper, presented at the Summer 1998 ESA Meeting. Abstract: The experiments implement a sequential rent-seeking game. The first-mover advantage predicted in theory is not observed in the data, due to the "revealed toughness" of the second mover. Also discussed are best-shot, ultimatum, trust, and dictator games.

Prasnikar, Vesna, and Alvin E. Roth (1992) "Considerations of Fairness and Strategy: Experimental Data From Sequential Games," *Quarterly Journal of Economics*, 107:3 (August), 865-888.