Imperfect Information

Every game that does not have perfect information is called a game with imperfect information. Games with imperfect information cannot be solved using backwards induction alone, because there are nodes at which the player designated to make her choice cannot observe everything that has transpired in the game. These nodes invariably belong to information sets that contain more than one node. The two examples below provide a tangible demsonstration of this fact, and thus help to explain why knowing the extensive form of an imperfect information game does not necessarily help us solve this class of games.

The strategic form representation is a less comprehensive description of a game than the extensive form, discarding detail about the order in which moves are taken. The strategic form defines a game by the set of strategies available to all the players and the payoffs induced by a strategic profile. We define the strategic form of a game using a constructive approach that derives it from the extensive form. In two player games, a matrix shows the payoffs as a mapping of the strategies of each player. If three players are involved, more than one matrix is involved, and matters become more complicated as the number of players increases. Nevertheless it is straightforward to provide a formal, or symboic definition of the strategic form for any finite game, which is useful in describing solution copncepts analyzed in subsequent chpaters.

As defined in Chapter 3, a game where no player can make a choice that depends on the moves of the other players is called a simultaneous move game. The strategic form of simultaneous move games assumes special significance, for two reasons. The first of these is related to strategic equivalence. Two games are strategically equivalent if they share the same strategic form. First, every extensive form game is strategically equivalent to a simultaneous move game. Thus much can be learned about any game by analyzing its strategically equivalent simultaneous move game. Second, in contrast to all other games, no information is lost when transforming a simultaneous move games from its extensive form to its strategic form. Without loss of generality, we can therefore restrict our analysis of simultaneous move games to their strategic forms. The special significance of simultaneous move games prompts us to devote the last part of the chapter to identifying them, because they are not always easy to recognize immediately.

Expansion

The simplest structure for an imperfect information game is a simultaneous move game for two players, each of whom must choose one of two branches. Figure 6.1 depicts such a game. A new market has just opened up in an industry with only two firms and no prospects of entry. The size and market value of one fim is curently twice the other. Both firms have the orppotunity to increase their respective capacites to serve this new market, or ignore it, but each makes its own decision without knowing what branch the other firm has chosen. If the larger firm expands and the smaller retains its existing capacity, the value of the larger firm will increase by 25 percent, some of its increase in value at the expense of teh smaller firm which would decline in value by 30 percent, because of greater scope for scale economies in distribution. On the other hand if the smaller firm seizes this market opportunity and the larger firm ignores it, the value fo the smaller firm will increase by 60 percent while the value fo the larger one will decline by 25 percent. Notice that in both cases the combined market value of both firms increases, though a little more if the large firm expands than if the small one does, because its larger initial scale places it a better position to make the upward adjustment. However the consequences of both firms expanding are price cutting and excess capacity in the industry, thus halfing the value of each firm.

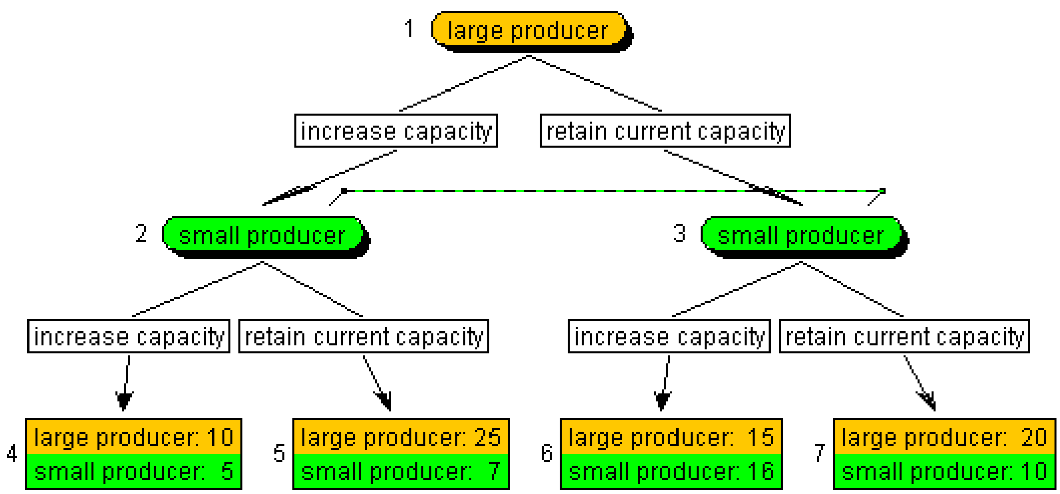


Figure 6.1
Expansion

If the large firm believed the small one was increasing its capacity, it would maintain its current capacity, (since $15 billlion is 50 percent more than $10 billion) and if the small firm believed the large firm had decided not to expand, then the small frim would increase its own capacity (as $16 billion is 60 percent greater then its current value). But what if the roles were reversed? In that case the value of the small producer would be $16 billion (from exapnding) instead of $10 (and the large firm would be worth $15 billion instead of $10 billion. Morover since the game is not repeated, there does not seem to be any decisive reason to explain why a firm's conjecture about its rival should be correct. For example if both firms believe their rival will expand, then neither will expand, only to discover their conjecture is incorrect after the game is over. Similarly if neither firm belileves the other will expand, then both will increase their capacity.

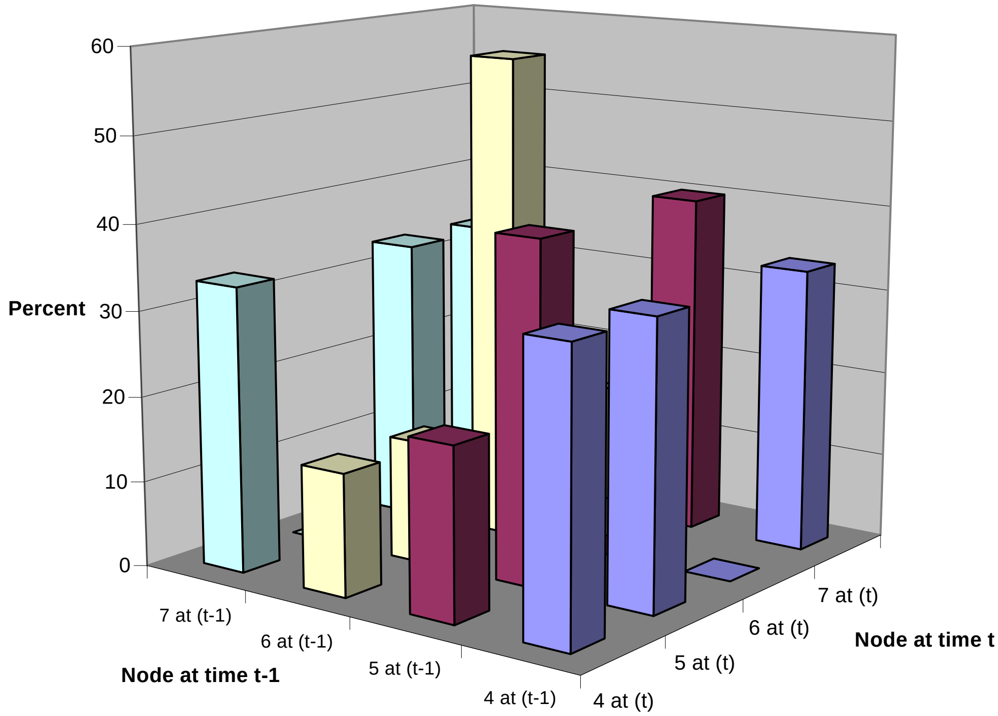
Experiment

Twenty four subjects participated for three round in the expansion game. The overall data in Table 6.1 show that each terminal node was visited on average one four of the time, and that half of the time both subjects chose to increase capacity or retain capacity and were not able to coordinate or predict what the opponent's choice would be.

		Small Producer	
	Large producer\small producer	***Increase capacity***	***Retain capacity***
Large producer	***Increase Capacity***	23.81 % (node 4)	16.67% (node 5)
	Retain capacity	28.57% (node 6)	30.95% (node 7)

Table 6.1: Results by terminal nodes

Although the game is modeled as one period game it was played for four rounds. Table 6.2. reports how subjects responded to the choices they made in previous round. For example if both subjects selected to increase the capacity (node 4) in the previous round, they selected the same choice in the next round 33.33 percent of the time. Similarly if both selected to increase capacity in the previous round the small producer will change the decision to retain the capacity in the next round 33.33 percent of the time or both of the players will react and select retain capacity. If node 5 or node 6 was selected in previous rounds then subject will more likely selected the same choice in the next round (40 percent if the game ended in node 5 in previous rounds and 57.14 percent of the time if game ended in node 6). However if both selected retain capacity in the previous round the reaction is exactly the same as when both of the players chose to increase capacity.



	4 at (t)	5 at (t)	6 at (t)	7 at (t)
4 at (t-1)	33.33	33.33	0	33.33
5 at (t-1)	20	40	0	40
6 at (t-1)	14.29	14.29	57.14	14.29
7 at (t-1)	33.33	0	33.33	33.33

Table 6.2: Transitional probability from round (t-1) to (t)

The difficulties that the firms have in coordinating their response to each other contrasts starkly with the perfect information analogues to this game. Removing the dotted lines in Figure 6.1 creates an asymmetry between the producers' moves. The solution to the game whose extensive form is displayed in Figure 6.2 is for the large producer to increase its capacity and for the small producer to retain its capacity.

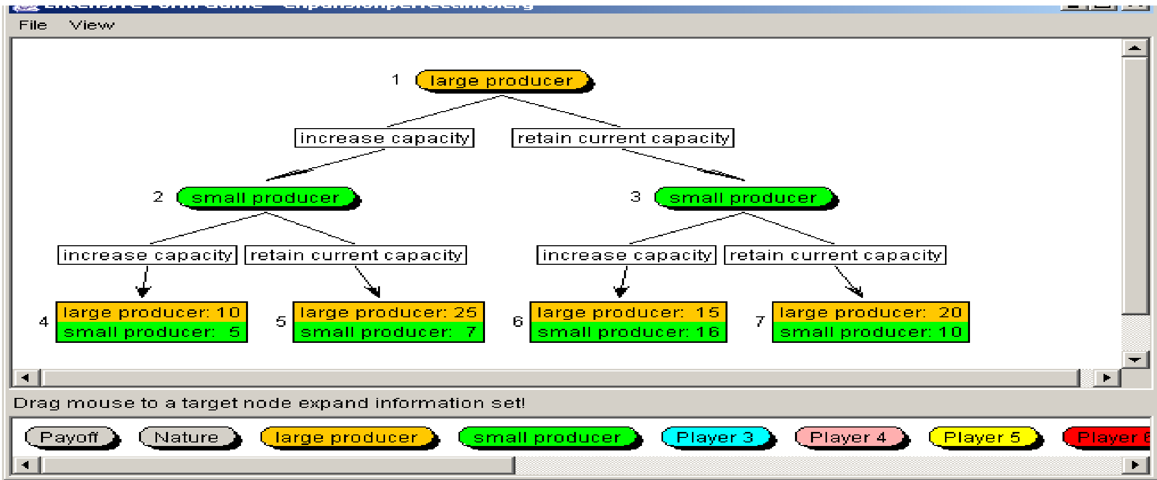


Figure 6.2
Large producer moves first

Alternatively if the small producer moves before the large producer and the large producer can observe what the small producer has done before it makes its own move, then Figure 6.3 applies. The solution to this game is for the small producer, but not the large producer to increase its capacity.

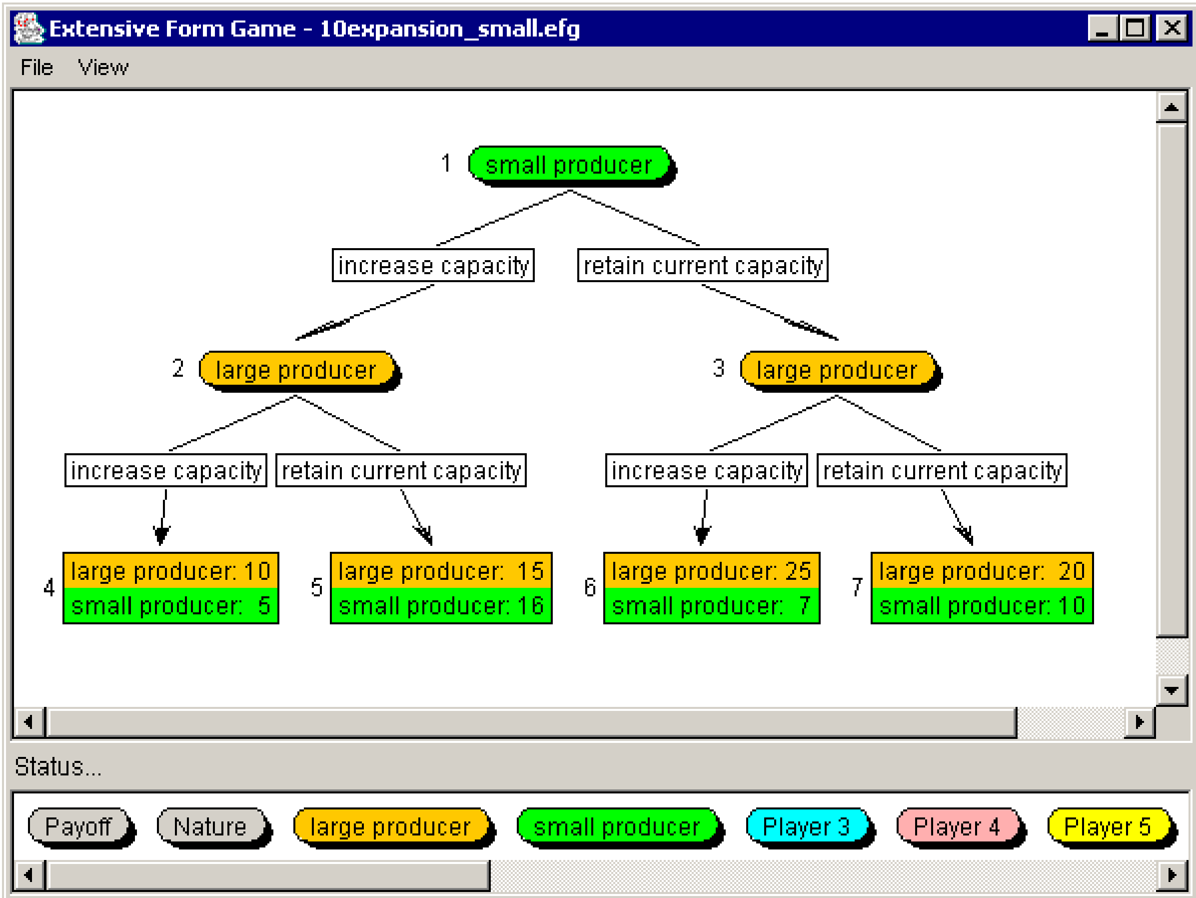


Figure 6.3

Small producer moves first

We remark that in situations like this, the firm which can seize the market opportunity the fastest can benefit from an increase in demand providing it can communicate this to its rivals. If the firm is unable to signal this capability, then the game morphs into a simultaneous move game by virtue of the fact that neither firm can condition on how its rival moved when making its own choices.

Experiment

The two games with perfect information, large producer moves first (Figure 6.2) and the small producer moves first (Figure 6.3) were run in the class for four rounds. When a large producer moves first the game ended in node 5 92.86 percent of the time and when the small producer moved first the game ended in node 5 92.31 percent of the time. When the game was changed to the perfect information game subjects seized the market opportunity if they were the first movers and they used it.

Exercise *Expansion*

1. *Conduct three experiments by running each variation on expansion illustrated in Figures 6.1 through 6.3.Tabulate the proportions of outcomes in each cell for each game.*

2. *Prove that in the solution to the perfect information games displayed in Figures 6.2 and 6.3, the firm moving first increases its capacity, and the other one retains its current capacity.*

3. *Test the hypothesis that the distribution of outcomes to the perfect information games follows the predictions of Question 2.*

4. *Test the hypothesis that the distribution of outcomes in the simultaneous expansion game is the same as one of the distributions of outcomes of the perfect information games.*

5. *Test the hypothesis that the distribution of the outcome from the simultaneous game is an average of the two solutions to the perfect information games, meaning that both players are equally likely to increase their capacity versus retain their current capacity.*

6. *Is one player more likely to increase its capacity than the other, and if so which one? Is the difference statistically significant?*

7. *Play congressional committee with and without the dotted lines. Conduct a similar analysis which compares outcomes of the perfect information games with the outcomes of the simultaneous move games.*

Traffic

That more than one outcome is plausible might lead us to suspect that random strategies are also possible. This following example on drug traffic suggests that the randomized violence associated with this business is easy to rationalize give the nature of the incentives at stake. The extensive form in Figure 6.4 begins with a drug dealer who has an opportunity to enter a new market. He can use this opportunity to facilitate a sting against his overlord by the FDA, or develop the market by establishing a reliable supply chain. If the dealer approaches the government, the FDA has the opportunity to simply arrest and incarcerate the dealer in the war on drugs. By cooperating they can break the ring if the sting is successful. when the dealer approaches the overlord about the new market, the overlord cannot tell whether the dealer is part of a sting or is really interested in developing the new market’s potential.

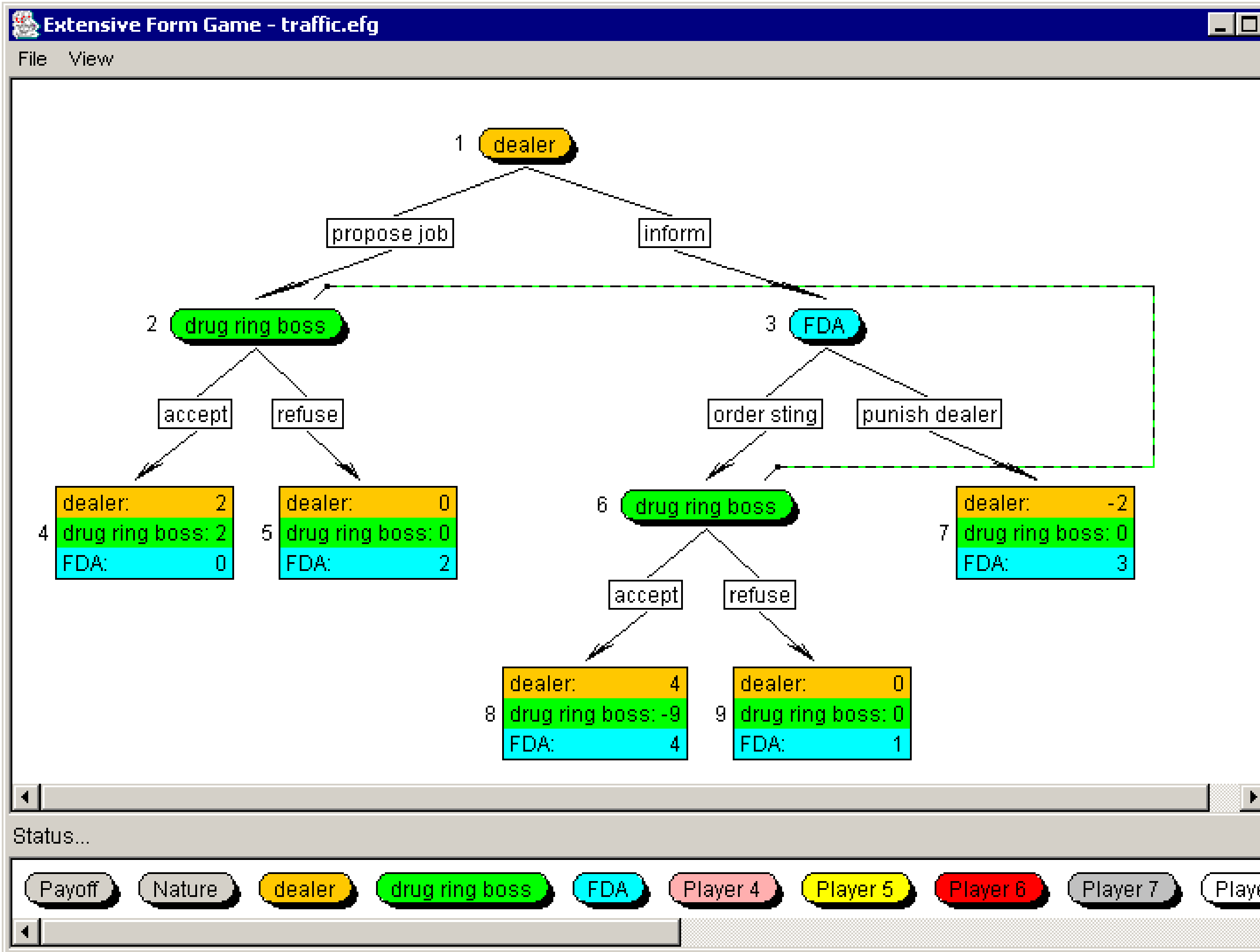


Figure 6.4
Traffic

Suppose the FDA always arrests the dealer. Using the backwards induction argument, the dealer can thus deduce that proposing a deal to the boss exceeds the prospect of certain incarceration. On the other hand if the FDA invariably ordered a sting, and the dealer also player a pure strategy then either the boss would always refuse or the FDA would never have the opportunity to create a sting. In this case not everyone will play a pure strategy. It is therefore incredible to believe that all the players will choose pure strategies. But if a random outcomes was possible, how would the probabilities of observing the different outcomes be determined?

Exercise *Traffic*

1. *Conduct an experiment in which subjects several games of Traffic, depicted in Figure 6.2, preferably matched with different subjects each game.*

2. *Over all the games what proportion of the time does the dealer inform?*

What proportion of the time does the boss accept the deal? What proportion of the time does the FDA order a sting?

3. *Given the proportion of the time does that the dealer informs, and the proportion of the time does the FDA orders a sting, calculated in the previous question, what is the expected utility to the boss from accepting the deal, and what is the expected utility from the boss rejecting it? Which is greater? Is the difference statistically significant?*

4. *Given the proportion of the time does that the dealer informs, and the proportion of the time does the boss accepts the deal, calculated in the Question 2, what is the expected utility to the FDA from ordering a sting, and what is the expected utility from the FDA arresting the dealer? Which is greater? Is the difference statistically significant?*

5. *Given the proportion of the time does that the FDA orders a sting, and the proportion of the time does the boss accepts the deal, calculated in the Question 2, what is the expected utility to the dealer from informing, and what is the expected utility to the dealer from proposing a job? Which is greater? Is the difference statistically significant?*

6. *Having answered questions above, make the public your finding with your experimental group and replay the game, redo the analysis, and discuss your findings. Have you any thoughts about which probabilities might replicate themselves?*

Strategies

Participating as a player in experiments based on these two examples doubtless raises many questions about how players form conjectures about the actions taken by others in games of imperfect information. Clearly the principle of backwards induction cannot be applied in simultaneous move games, and is of limited use in solving games of incomplete information. For these reasons we introduce another representation of a game that does not exploit the order of play at all, called the strategic form representation, and we will seek solutions to imperfect information games using this alternative representation.

The foundation of the strategic form representation is a strategy. Like the words tree and branch, using the word strategy in game theory and experimental economics is related to, but does not coincide with, its everyday usage. In this book, unless stated otherwise, a strategy is a full set of instructions to a player, telling her how to move at all the decision nodes assigned to her. Strategies respect information sets, in that the instructions at two or more decision nodes within the same information set must be identical. Strategies are exhaustive. They include directions about moves the player should make in the event each of her assigned nodes being reached. The set of strategies available to a player is called the strategy space.

Strategies for Expansion

In simultaneous games strategies for each player correspond to the moves themselves. That is because each player has only one information set. In the Expansion game, for example, there are only two strategies for each player, and in this game the set of strategies for the other player in this game is the same. One strategy is to expand, and the other strategy is to maintain the current capacity.

In games of perfect information there are often more than one information set for some of the players, and defining a strategy means issuing instructions about what to do at each information set. For example removing the dotted lines in Figure 6.1 has no effect on the strategy space of the large producer, but increases the strategy space of the small producer, because now its decision may hinge on what the large producer has already decided, as Figure 6.3 shows.

Now there are four strategies for the small producer to pick from:

1. Increase capacity regardless of what the large producer decides

2. Retain current capacity regardless of what the large producer decides

3. Increase capacity if the large producer increases capacity, and retain current capacity if the large producer retains current capacity

4. Increase capacity if the large producer retains current capacity, and retain current capacity if the large producer increases capacity

Note that the first two strategies are independent of the decision taken by the large manufacturer, but that the second two strategies require the small firm to take actions that depend on what the large manufacturer has already done. Thus the third strategy requires the small manufacturer to take the same action as the large manufacturer while the fourth strategy requires the manufacturer to do the opposite.

The differential adjustment cost game shown in Figure 4.1 has a similar game tree structure. There is one information set assigned to the large manufacturing firm, but two information sets assigned to the small manufacturing firm, which waits to see what the large firm has done before taking its own decision. The strategies for the large firm are modernize and stagnate, but there are four strategies the small firm can pick from.

Exercise *Strategy spaces in perfect information and simultaneous move games*

1. *In the game of perfect information depicted in Figure 6.3 where the small producer moves first, what are the strategies for each player?*

2. *What are the four strategies of the small firm in Figure 4.1?*

3. *Define the strategy space for the regular riser in the congestion game illustrated in Figure 4.3.*

4. *What are the strategies for Eagle Airlines in the Air service game depicted in Figure 4.14 and 4.15.*

5. *Write down the set of strategies for each of the three players in the* *committee decision making game depicted in Figure 3.17.*

Alliance

The two variations on the expansion game displayed in Figures 6.1 through 6.3 illustrate two extremes about how much information the second mover has relative to the first one. If the first player has more than two choices, then intermediate cases are also possible. The next example considers the extent to which firms might be able to capitalize on mutual

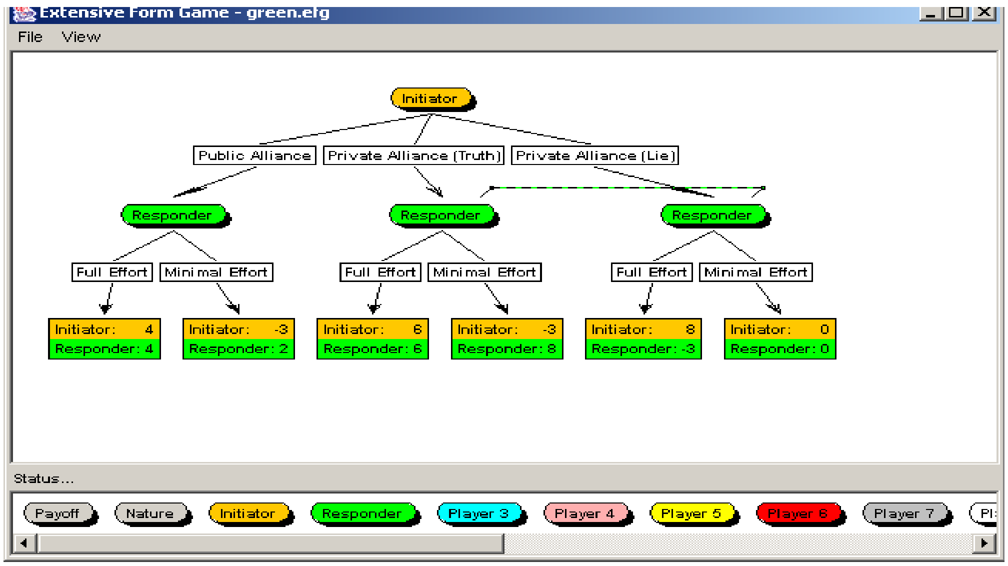


Figure 6.5

1. form public alliance merger

2. go through the motions of forming a private alliance but do not commit any resources to building it

3. devote administrative resources and technical support to forming private alliance

The responder can condition its response to the proposer on whether the alliance is public or private, but in the latter case cannot discern whether the proposer has committed resources to follow through. They can accept the proposal or reject it. In the latter case this simply involves putting no resources into the alliance either. There are therefore four strategies the responder

Exercise *Strategies in Sequential Games of Imperfect Information*

1. *Oil extraction is similar in Figure3.6*

2. *What is the strategy space of the boss and his worker in the labor contracting game depicted in Figure 3.12?*

3. *What is the strategy space for the boss, dealer and FDA in Traffic, illustrated in Figure 6.4?*

4. *What is the strategy space for the players in Industrial Espionage, illustrated in Figure 3.14?*

5. *Pioneer and Imitator in Product Development of Figure 3.18*

6. *Discount retailing in Figure 3.11*

Subcontracting

We have argued above that a strategy must give directions about how to act at every decision node in the game, and that the directions must condition on all the information that the player has. At first sight this might seem a little too demanding since many nodes will not be visited during play. However it is worth bearing in mind that the reason why many decision nodes are not visited is because of the decisions that would be taken if they were. Recall the sun bather example, the first bather chose the best rock spot even though she preferred sand, because she anticipated that another player would spoil her solitude if she chose the sandy spot.

This argument is strongest when applied to situations where the actions of one player rule out the decision nodes of another player. It is much less convincing when we are contemplating decision nodes that have already been ruled our of play by previous actions taken by the same player. For example consider the following situation in which two firms have competed for a defence contract that only one firm can win. In this example the winner is called the contractor and the loser is called the subcontractor. Having been awarded the contract the winner has the opportunity to build everything itself, or subcontract part of the contract to the rival bidder. Since the rival has an established plant to meet the contract, it may be in a position to supply parts of the contract relatively cheaply, and one might think that the contractor's winning bid reflected the fact that ti might be able to draw upon the expertise of the rival after the contract award.

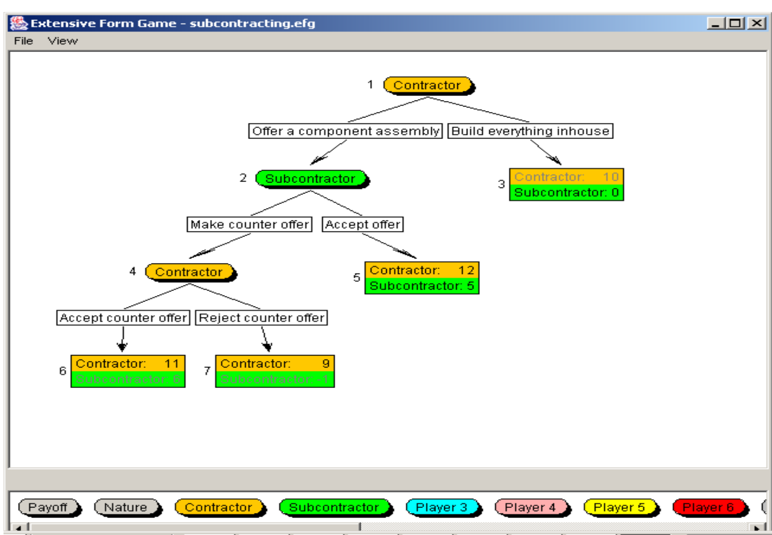


Figure 6.6
Subcontracting

There are two decision nodes for the contractor, Nodes 1 and 4, two corresponding information sets, and two branches stemming from each node. Therefore there are four strategies, namely:

1. Subcontract and accept counter offer
2. Subcontract and reject counter offer
3. Build everything and accept counter offer
4. Build everything and reject counter offer

This example highlights a small ambiguity about how a strategy is defined. In principle a complete set of instructions to the contractor, requires directions about how to proceed if the he has ignored his previous instructions. For example, we might instruct the contractor to build everything inhouse at the top node, and the provide further directions about whether to accept the counter offer or not if he does not build everything inhouse as instructed. Less plausibly we might provide instructions to him about the counter offer conditional on the subcontractor accepting the original offer at the second node in the extensive form. For this reason we call this the unabridged strategy space.

The problem with Strategies 3 and 4 is that if followed they are indistinguishable from each other, because Node 4 can only be reached if the contractor chooses one of the first two strategies. Only if player deviate from that strategy could one observe a difference between them, and in that case it would not be possible to know if the player accidently deviated from or had simply chosen one of the first two strategies. The only reason for differentiating between the last two strategies is to predict how play would proceed if there was some probability that players following instructions accidently deviate from them, a subject that has been extensively explored in game theory. Since our analysis of games will not venture in those directions, we will not treat the third and second strategies as the same in violation of practically every graduate textbook on equilibrium refinements. Hence the abridged strategy space for the first player becomes:

1. Subcontract and accept counter offer
2. Subcontract and reject counter offer
3. Build everything inhouse

In this text we will conduct all our analysis with the abridged strategy space. Thus instruction must be complete, but they are of no use to a player seeking guidance if that player has mistakenly deviated from them.

Exercise *Strategies*

1. *Write down the unabridged and the abridged strategy space for the*

moon race

2. *Alliance*
3. *Congestion*
4. *Speed to market*

Pure versus Mixed Strategies

We will argue in later chapters to this book that strategic behavior sometimes involves making moves in a random way, and several of the example games we have already suggest that sticking to a deterministic strategy invites disaster. Consider the matching pennies game discussed in Chapter 3, whose its extensive form is displayed in Figure 3.4. In this game there are two available strategy to each player, playing a head or a tail. Now suppose the first players always plays the same strategy. If this became public information, then the second player could guarantee a win for herself by always playing the opposite strategy. If the first player was relatively sophisticated he might realize .

Traffic has elements similar to Matching Pennies. The strategies for the For example suppose one was to argue that the FDA should pursue a consistent policy with respect to its informants, treating the same ways. In the game depicted in Figure The matching penny game provides another example If the first player say alswas plays heads

These examples suggest that limiting players will not confine themselves to deterministic strategies, henceforth called pure strategies, and it is therefore counterproductive to ignore this fact in seeking to explain strategic behavior.

in the case of the matching pennies example randomized behavour. Suppose that the second players before the the strategy space for the large firm is For this reason we often refer to a deterministic strategy as a pure strategy.

the difference between a randomized strategy and a mixed strategy. Interesting from the player's point of view not so interesting from the moderators point of view. Is there convergence?

Notation for the general case

To summarize the discussion on strategy spaces, we now introduce some algebraic notation to describe these concepts, which we will repeated draw upon in this chapter and the two which follow it. Consider an N player game where there are a finite number of information sets. We suppose the n^{th} player has d_n pure strategies to pick from. We label a typical strategy for Player n by $s_n^{(d)}$ where $d \in \{1, 2, \ldots, d_n\}$ and denote her strategy space by

$$S_n \equiv \left\{ s_n^{(1)}, s_n^{(2)}, \ldots, s_n^{(d_n)} \right\}$$

A pure strategy profile is a strategy by each of the players, which we may write as

$$s \equiv (s_1, s_2, \ldots, s_N) \in S \equiv S_1 \times S_2 \times \ldots \times S_N$$

Using this terminology, a mixed strategy for the n^{th} player is a d_n dimensional probability vector of the form

$$\pi_n \equiv (\pi_{n1}, \pi_{n2}, \ldots, \pi_{nd_n})$$

such that $\pi_{nd} \geq 0$ for all $d \in \{1, 2, \ldots, d_n\}$ and

$$\sum_{d=1}^{d_n} \pi_{nd} = 1$$

where π_{nd} is interpreted as the probability that player n will pick strategy $s_n^{(d)}$. Analogous to our definition of a pure strategy profile, a mixed strategy profile, denoted $\pi,$ consists of a mixed strategy for each player, defined as:

$$\pi \equiv (\pi_1, \pi_2, \ldots, \pi_N) \in \Pi \equiv \Pi_1 \times \Pi_2 \times \ldots \times \Pi_N$$

The Strategic Form

Having defined the concept of a strategy space and illustrated it in many examples, it now time to explain and elaborate on the strategic form of a game. Rather than describe a game by the elements comprising the extensive form, the strategic form of the game is a list of all the possible pure strategies for each of the players and the (expected) utilities resulting from them. Suppose every player chooses a pure strategy, and that nature does not play any role all the decision nodes in the extensive form. In that case the strategy profile would inexorably yield a unique terminal node and thus map into payoffs. For those games we can define the strategic form as the set of all strategic profiles formed from pure strategies and their associated payoffs.

Games for two players

A simultaneous move game for two players can be depicted by a matrix. The number of strategies available to each player respectively define the dimension of the matrix representation of the strategic form. A Matrix depiction of the Strategic form In two player games, a matrix shows the payoffs as a mapping of the strategies of each player. If three players are involved, more than one matrix is involved, and things get more complicated from there. The possible strategies of one player are listed as row entries, the strategies for the other player are listed as column entries, and the payoffs are listed as bi matrix elements.

In the simultaneous move expansion game we can list the strategies for the small producer down the left side of an array, and the strategies for the large producer across the top. Listing the strategies determines the what values the elements take in the array. Payoffs in the lower left corner of each matrix refer to the small producer and payoffs to the large one are in the top right corner of each elements. From Figure 6.1 we can see that if both producers increase capacity, then the payoffs are 5 and 10 respectively. This strategy profile yields payoffs the top left bimatrix element in Figure 6.7. We enter 5 in the lower left corner and 10 in the upper right corner. The rest of the bimatrix is completed in similar fashion.

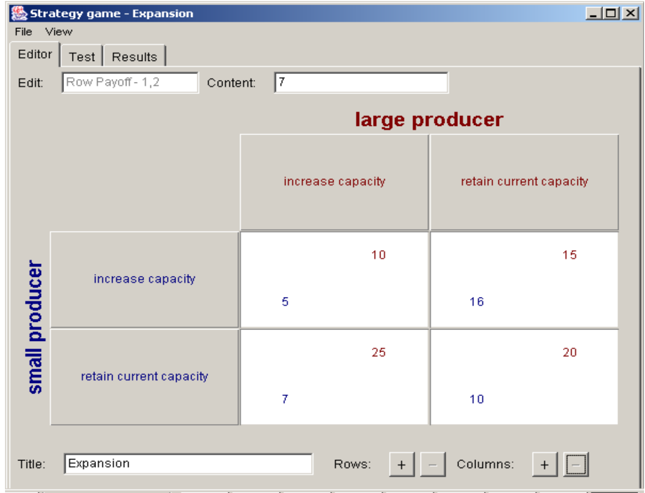


Figure 6.7
Expansion in strategic form

The the strategic form of the subcontracting game is also easy to derive providing one follows the procedure of first writing down the strategies for each player (either in a bridged or unabridged form), and then completing the bimatrix by simply recording the outcomes of each strategic profile. In the abridged version of this game, the contractor has three strategies and the subcontractor has two. figure 6.8 displays the matrix representation of the game in its strategic form.

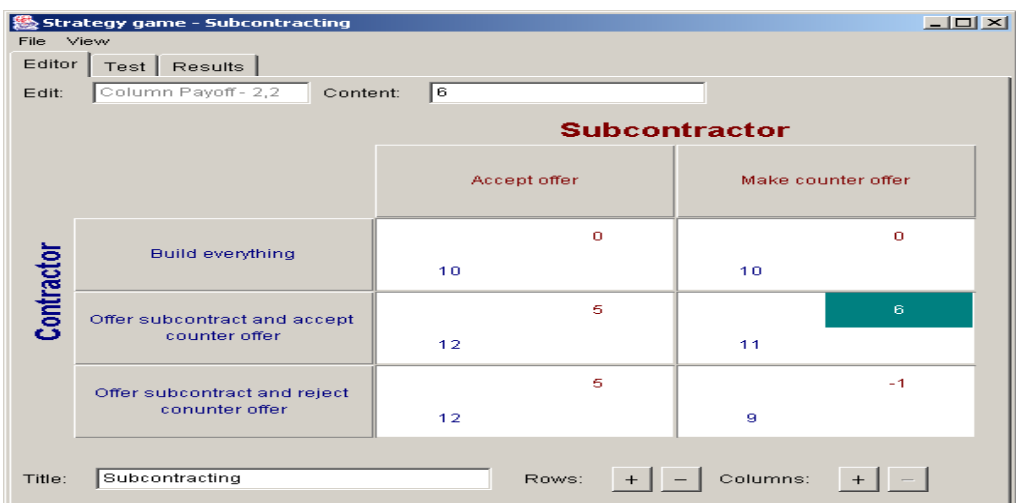


Figure 6.8
Subcontracting in strategic form

In this example, the number of cells in the matrix representing the strategic form exceeds the number of terminal nodes in the extensive form by 2. Consequently 4 matrix cells in Figure 6.5 contain payoffs that are not unique to that cell. For example if the contractor builds everything inhouse, then the payoffs are 10 and 0 regardless of whether the subcontractor would accept the contractor’s offer or make a counteroffer.

This implies the two elements comprising the top row of the bimatrix representing the strategic form for this game are identical. The redundancy is an unavoidable consequence of using the strategic form to represent this particular game.

Exercise *Strategic Form of Games for two players*

1. *Matching Pennies*
2. *Retail discounting*
3. *Adjustment*

Two player games with uncertainty

With one eye on the entrepreneurial innovation required to market applications process to college, the next game concerns the degree to which human capital should be utilized in non profit organizations. A backpacker has the choice of inviting a friend to join him on an expedition, or hiring a mule. the extensive form is depicted in Figure 4.6.

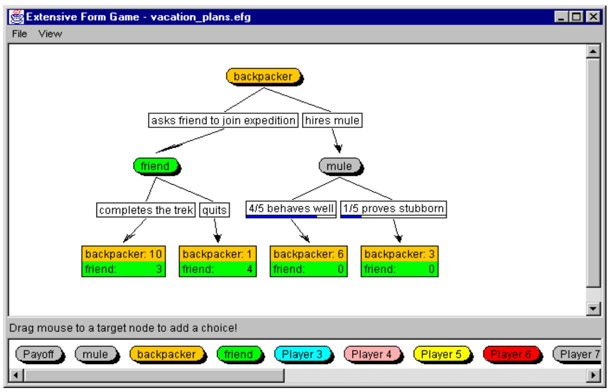


Figure 6.6

Factor utilization in nonprofit organizations

In this application the backpacker and his friend have two strategies each. The backpacker's strategies are to ask his **friend** to join expedition or hire a **mule** instead, while if asked, the friend chooses to **complete** the trek or to **quit**. Supposing the backpacker chooses to hire a mule, them the outcome for the friend is 0 for sure, but the outcome to the backpacker is a lottery which depends on the temperament of the mule. This is shown in figure 6.7.

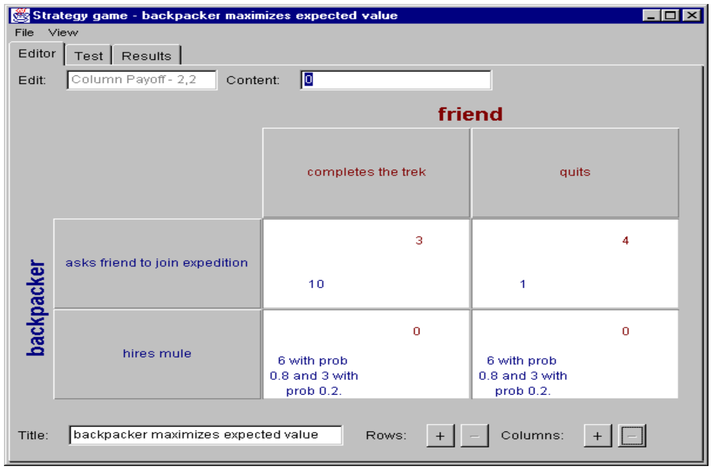


Figure 6.7
Strategic Form for Factor Utilization

This is not the usual form of the strategic representation. To summarize the payoff consequences of hiring a mule with a single number (rather than a probability plus two numbers), we need to know the backpacker's attitude towards uncertainty. Now supposing the backpacker follows the expected utility hypothesis, and the payoffs 6 an 4 represent his respective utilities when the mule behaves well versus badly, his expected utility from hiring the mule is

$$6 \times 0.8 + 3 \times 0.2 = 5.4$$

Under that assumption the strategic form may be simplified to Figure 4.8.

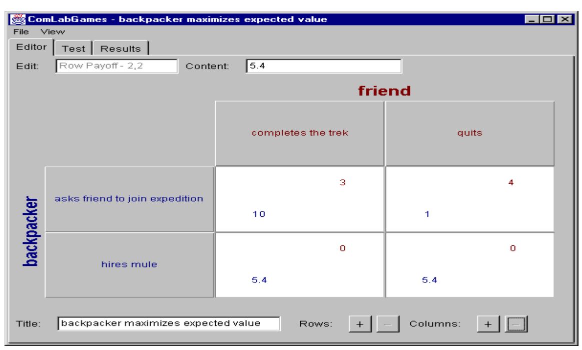


Figure 6.8
Strategic Form assuming Expected Utility

If each player is given a strategy instructing her how to play the game, it would be possible to calculate the expected utility of each player from everyone followed their respective strategies.

In Figure 6.2 the second player receives a payoff of 6 one third of the time and a payoff of 12 the other tow thirds of the time if she plays in, whereas if she plays out she receives a payoff of 9. Her expected utility from playing in is 10 if she is risk neutral, but 1/3log9 + 2/3log10

The role of expected utility in defining mixed strategies.

As the example above illustrated, in contrast to the extensive form representation, this representation of the game embodies a player's attitude towards uncertainty. It implies that if a different subject is substituted for the player, then the representation might change. To avoid confusion, Unless stated otherwise we shall assume that the payoffs in the extensive form represent utility and that players are expected utility maximizers.

However the problem does not end there, since in conducting experiments towards risk, the subjects presumably have their own utilities to consider, a subject that we have discussed in Chapters 2 and 3. If the subjects playing the games do not have similar attitude towards uncertainty as the players they are modelling then the analysis becomes tangled.

Exercise *Strategic Form for Two players games where there is uncertainty*

1. *Air service*
2. *Discount retailing*
3. *Use a new utility function to define the expected payoffs in these examples*

Games for 3 players

Games with three or more players do not have a matrix representation because a three dimensional object is required to represent. To illustrate this point, consider the following game, whose extensive form is depicted in Figure 4.9.

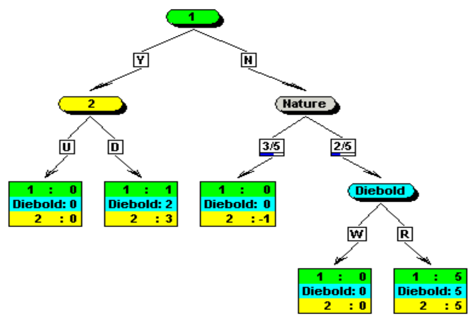


Figure 4.9
A Three Player Game

In this game the strategies for Player 1 are **Y** and **N**, the strategies of Player 2 are **U** and **D**, and those of Diebold are **W** and **R**. Therefore the strategic form could be written in cube. Although we cannot represent this game with one matrix we could slice the cube by the third player's strategy, and depict the strategies and payoffs in a two player game conditional on a strategy of the third player. If diebold picks **W** we obtain

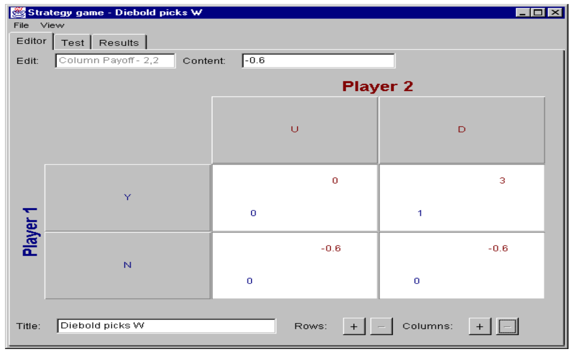


Figure 4.10
Diebold picks W

Alternatively if Diebold picks **R** the following matrix applies to the other two players.

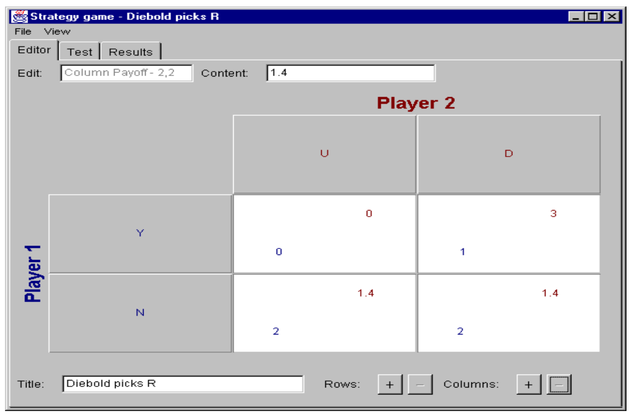


Figure 4.11
Diebold picks R

We shall return to the following example in the Chapter on incomplete information games. Suppose a drug dealer has the choice of proposing a job . . the extensive form of this game, called Traffic after the movie which bears its name is given in Figure 4.12.

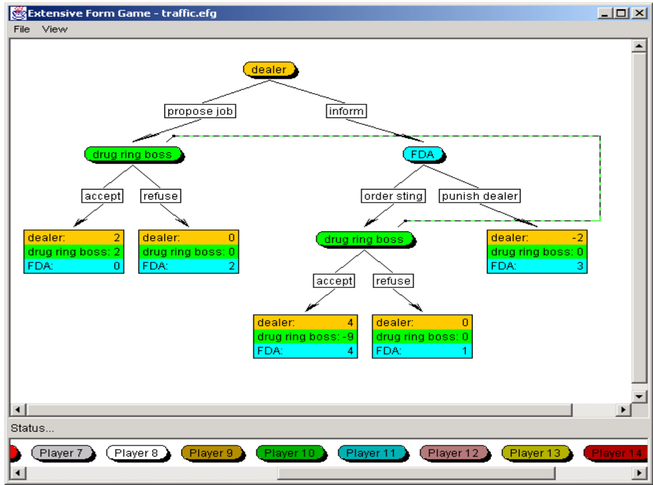


figure 4.12
Extensive form for Traffic

We remark that each player has two strategies. The dealer can **Propose** or **Inform**, the boss can either **Accept** or **Reject**, and the FDA can either **Sting** or **Punish**. To proceed to the strategic form, suppose we define the strategic form for the two player game in which the boss accepts the job. See Figure 4.13.

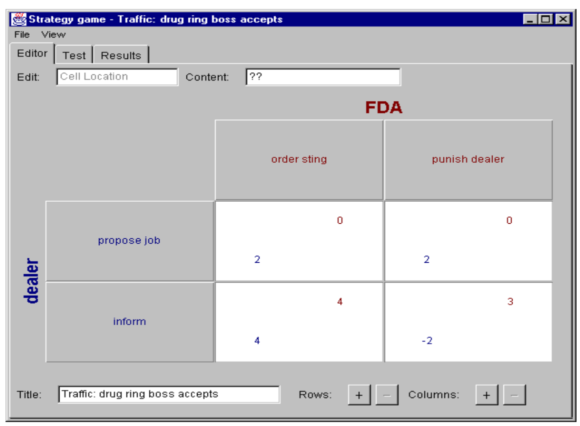


Figure 4.13
The boss accepts

The boss's alternative, to **Reject**, yields the payoff matrix to the other two players depicted in Figure 4.14.

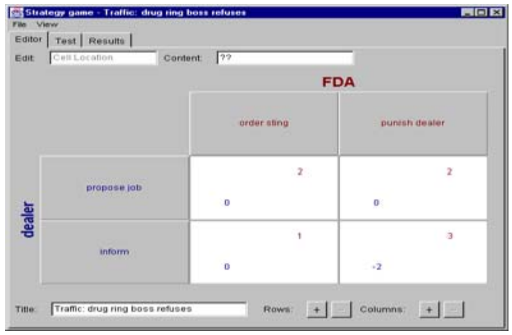


Figure 4.14
The boss rejects

Although the matrix representation is less appealing when there are more than two players, we will show that in many applications of games involving three players, this format is nevertheless quite useful.

Exercise *Strategic Form for Three Player Games*

1. *congressional committee*
2. *congestion*
3. *industrial espionage*

Games for N players

The strategic form of a game for N players consists of a strategy space denoted by $S_n \equiv \left\{s_n^{(1)}, s_n^{(2)}, \ldots, s_n^{(d_n)}\right\}$ for each player $n \in \{1,2,\ldots,N\}$, that evaluates each pure strategy profile $s \equiv (s_1, s_2, \ldots, s_N) \in S \equiv S_1 \times S_2 \times \ldots \times S_N$, a utility function $u_n(s)$ for each player $n \in \{1,2,\ldots,N\}$. Formally $u_n(s)$ is a mapping from S to R, the real line. When players use mixed strategies $\pi_n \equiv (\pi_{n1}, \pi_{n2}, \ldots, \pi_{nd_n})$, their expected utility is thus

$$\sum_{d=1}^{d_1} \sum_{d=1}^{d_2} \pi_{n1} \pi_{nd} u(\quad)$$

For example if each person was asked to contribute to a public good, then s_n might denote an integer number of dollars, and the utility each person gets could depend on the total expenditure:

$$u_n\left(\sum_{m=1}^{N} s_m\right)$$

Strategic Equivalence

Comparing extensive forms with the same strategic form

Delegation

comparing extensive form with a strategic form

Exercise *The first part focuses on deriving the strategic form representation of* *a game from its extensive form. The reverse exercise is also instructive: writing down several extensive forms that give rise to specified strategic form.*

Exercise *A question to be addressed is whether players pick different strategies if confronted with the strategic form versus the extensive form.*

Exercise *A related question is whether players behave differently when confronted with distinct extensive forms that share the same strategic form. These questions can be addressed in perfect information games, where students have already learned the solution from Chapters 4 and 5, and in games of imperfect information, where they have not.*

Is play affected by the representation of the game?

Recognizing Simultaneous move games

The benefit from analyzing the strategic form of the game is that the principles we shall develop in subsequent chapters apply universally. These principles contrast with the backwards induction principle, which can only be applied in limited, albeit useful, ways to games of imperfect information. A limitation of the strategic form of a game is that it hides the order of the moves. Indeed the last section showed there are many different games that are strategically equivalent to each other. Game theorists debate how much is given up by ignoring the sequential aspects of play. There is one class of games, though, where nothing is lost from ignoring the extensive form altogether and focusing exclusively on its strategic form, simultaneous move games. Recall that within this class of games, each strategy uniquely corresponds to a move. It is therefore useful to be able to recognize the extensive form of a simultaneous move game, and distinguish it from a game of imperfect information.

The defining feature of a simultaneous game is that it supports an extensive form in which only one information set is assigned to each player. The Executive Marriage and the Committee Decision-making games depicted in Figures 3.16 and 3.17 illustrate this structure in games for two and three players respectively. However as we have remarked, many games have more than one extensive form, so it sometimes requires some thought and manipulation to check for the simultaneous move property.

Fellon

In the first example we consider in this section an escaped convict in being sought by the police. The fellon has the opportunity to flee the county or remain there. The FBI decides how much resources to devote the case without knowing the felon’s decision. Figure 6.15 depicts the extensive form in a way that captures the sequential nature of play.

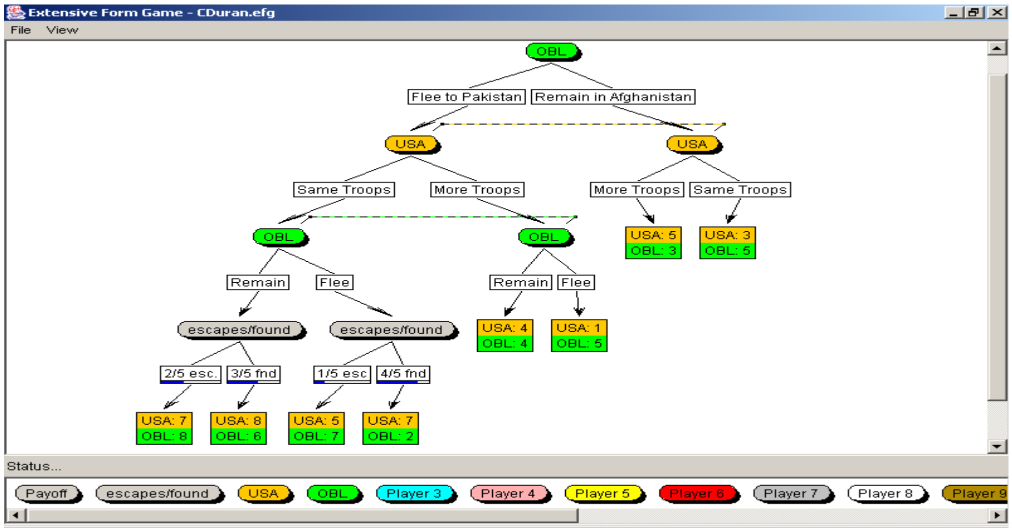


Figure 6.15

Escaped Felon

It is evident from this Figure that no decisions are taken by any player after chance determines whether the felon’s attempt to escape is ultimately successful or not. Assuming the expected utility hypothesis applies to both players, we can fold back the game from its terminal nodes to obtain Figure 6.16.

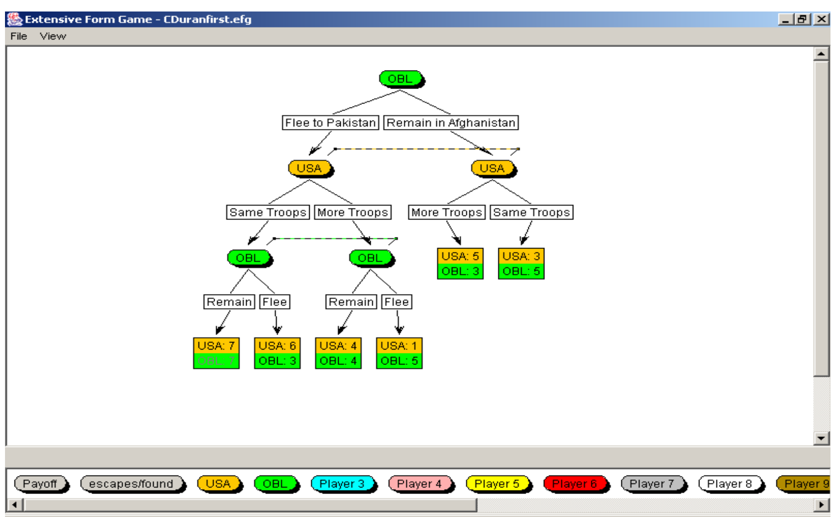


Figure 6.16

Applying the expected utility hypothesis

Anther feature of this game is top note that the felon does not know how many resources the police have devoted to recapturing him at the time he has a second opportunity to flee the county. From his point of view, nothing is lost from committing to a decision at the initial node of the game. Similarly the police have no knowledge of the fellon’ whereabouts at the time they make their decision about the number of resources to devote to the case. For this reason, we shall treat this game as a simultaneous move game, because it supports an extensive form in which both players have only one information set

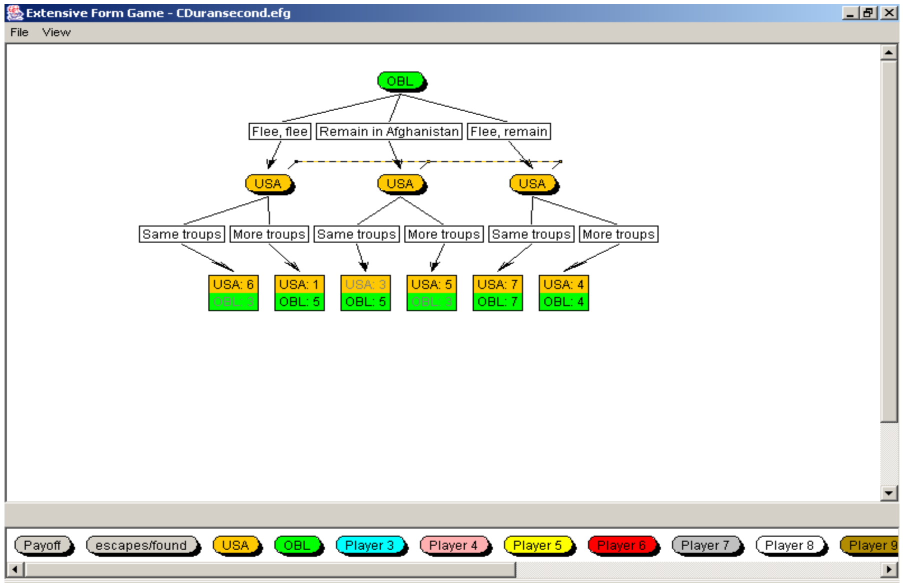


Figure 6.17
A simultaneous game revealed

The strategic form for the game is shown in Figure 6.18.

Telecommunications

AOl and AT&T

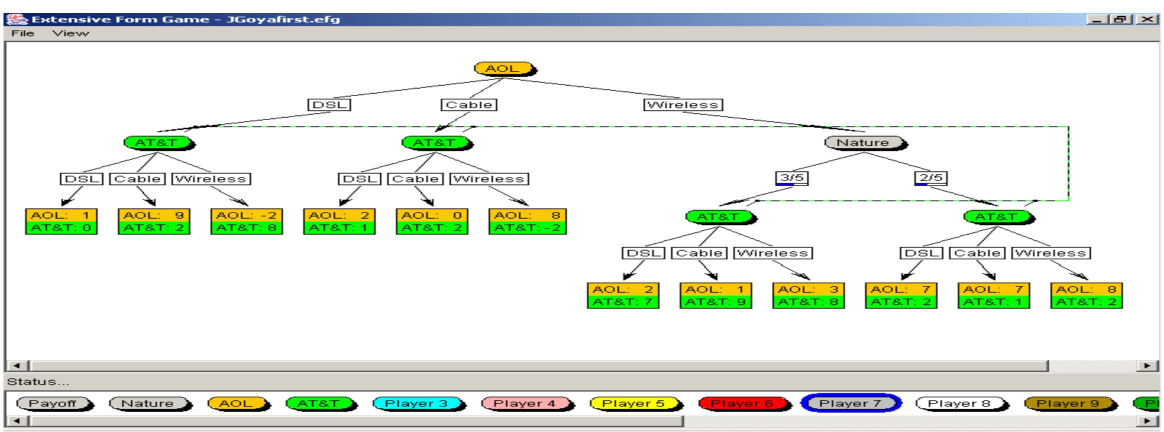


Figure 6.19
Telecommunications

First note that the nodes can be interchanged with

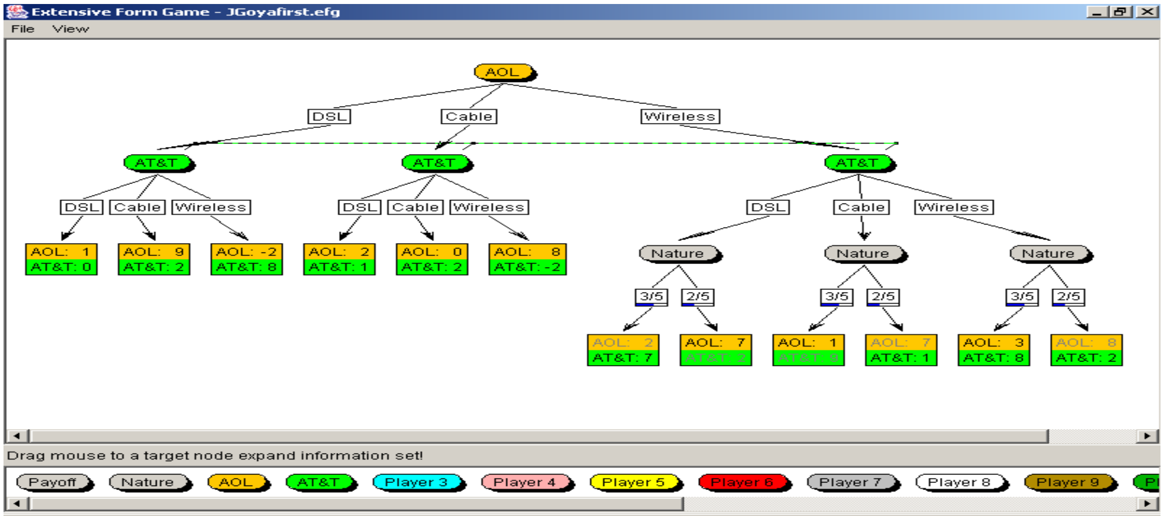


Figure 6.20
Telecommunications

Observe from Figure 6.20 that this is simultaneous move game where each firm has three strategies.

New Product Marketing

The last example we provide focuses on marketing strategies for companies launching a new product when they are uncertain about the appearance of close substitutes at roughly the same time. companies that are confident no one is in a position to provide a similar type of innovation must spend relatively more resources to convince potential demanders of the usefulness of their new product. If however a company is reasonably sure that rivals will also launch a close substitute, generic advertising of the product is a costly investment that increases the demand for both companies, whereas a strategy that differentiates the products, such as price cutting becomes more appealing.

Often companies do not know precisely how much competition they will face before launching a new product:

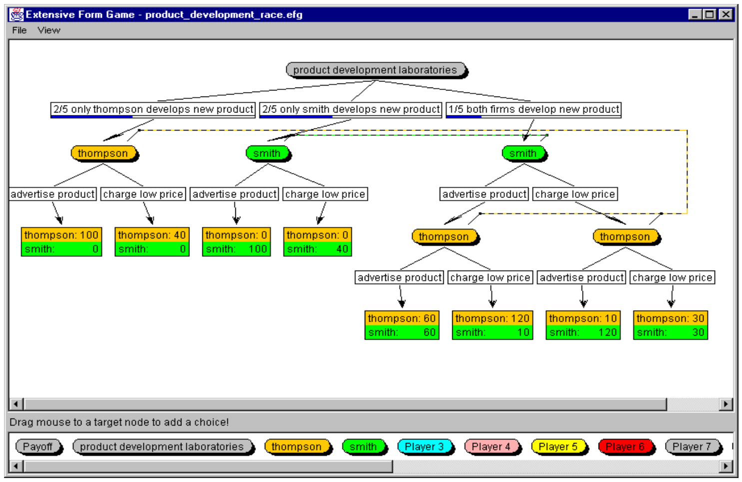


Figure 6.18
Product launch

of a simultaneous game is also easy to spot because each player only makes choices at one information set. In the Product launch game, each company will introduce remote control cooking with probability one half. These events are independent so the probability of them both introducing the system is one quarter. The game begins if and only if at least one firm can make the remote system commercially viable, which occurs with probability one quarter, calculated from the formula

$$\begin{aligned}\Pr\{\text{Game begins}\} &= 1 - \Pr\{\text{Neither firm launches product}\}\\ &= 1 - \Pr\{\text{Both firms launch product}\}\\ &\quad - \Pr\{\text{First firm launches product by itself}\}\\ &\quad - \Pr\{\text{Second firm launches product by itself}\}\end{aligned}$$

Therefore the conditional probability of only one firm introducing the product conditional on the product , and the conditional probability of is The strategies for both players are:

1. advertise new product
2. make low introductory price offer

The expected revenue from c form of the product launch game is found by calculating t

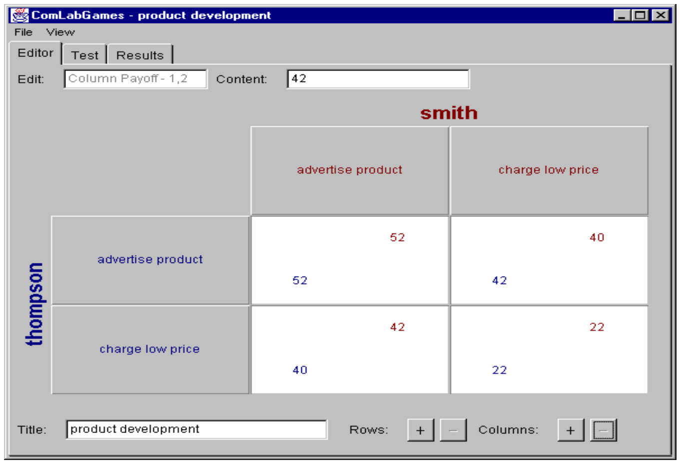


Figure 6.29
Strategic Form of Product Launch Game

Exercise *Simultaneous Move Games*

1. *Write down the matrix representation of telecommunications and Product Launch*

2. *Conduct experiments on*

3. *Test the hypothesis that the distribution of the outcomes does not vary with the from of the game presented to subjects..*

Further Reading

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