Overview

The previous chapter focused on eliminating from consideration those strategies that were unattractive to a player regardless of the strategy profile collectively chosen by the other players, and also discarding those strategies that could only be justified if the other players behaved in an irrational manner. However as our examples at the beginning of this chapter will establish, there are many games in which these dominance principles in their various guises do not yield a unique solution, or even refine the set of strategy profiles in contention. Additional rules are required to yield tighter predictions about game outcomes. These rules are somewhat less plausible, and harder to defend as behavioral norms, which is why we have postponed their introduction until now.

This chapter analyzes the concept of equilibrium, the notion that each player’s behavior can be viewed as the outcome of him optimizing an individual objective function that is partly defined by the solutions to optimization problems of the other players. In order to develop this concept we first define the notion of a best reply, which means a pure or mixed strategy that maximizes the player’s payoff given a strategic profile of choices made by the other players in the game. Whether choosing a best reply is an attractive strategy or not depends on the conjecture the player is making about the strategies of the other players. For example thinking the other players will choose strictly dominated strategies, and forming a best reply based on that conjecture doesn’t seem a very sensible course of action!

This leads to a discussion of a concept that is weaker than equilibrium, individual strategies that are rationalizable. Intuitively, a rationalizable strategy is a best response to some strategy profile of the other players, the elements of this profile are strategies that are also best responses, and so on. In two player games the set of rationalizable strategies exactly corresponds to those strategies that survive the iterative removal of strongly dominated strategies. However rationalizable strategies do further refine the the set of strategy profiles in games for more than two players. One of the reasons why rationalizable strategies are regarded as important is because they also fully exploit what players can achieve without some form of coordination. Further refinements of the solution strategy, found by applying an equilibrium concept for example, are less robust predictors of game outcomes.

Succinctly stated, a Nash equilibrium is a strategy profile for all the players, in which every player’s strategy is a best reply to the strategy profile formed from the other players’ actual choices. Having defined the Nash equilibrium concept, we explore its properties, again mainly by way of example. It is useful to distinguish between a Nash equilibrium where all the players choose pure strategies from one in which some or all the players choose mixed strategies. A pure Nash equilibrium can be interpreted as a self enforcing contract. If all the players in the game were agents told by a principal which pure strategy to choose, in a Nash equilibrium no player would have an incentive to unilaterally deviate from the principal's instructions. This theme is taken up in Part 6 of the book in Chapters 13 and 14, on mechanism design and contracts.

Not every game has an equilibrium in pure strategies, but some games have more than one pure strategy equilibrium. In games with multiple equilibria, one or more of them may violate the principle of weak dominance, and thus be disqualified as solution candidates. When these equilibria occur in perfect information games, they can sometimes be interpreted as an attempt to write a self enforcing contract based on a noncredible threat by one of players about how he will respond to another player's action later in the game. Nevertheless there are many games with multiple equilibrium and no convincing criteria for ranking their likelihood in games. Accordingly we investigate games with multiple equilibria played a repeated number of times in Chapter 11, to analyze how learning and coordination between the players might take place in those cases.

The last section of this chapter concentrates on mixed strategies. Strategies in which individual players randomize their choices, or belong to a group whose homogeneity is compromised only by the fact that its members make different choices, occur quite frequently in the real world. We provide examples of games lacking a pure strategy equilibrium. It has, however, been proved that every finite game has an equilibrium in mixed strategies. At the end of our discussion on this topic, we remark that in games with a unique pure strategy equilibrium the probability distribution describing the conditional choice probabilities is degenerate, whereas in a mixed strategy equilibrium, the probability distribution of choices is proper. Nonetheless through the use of examples, we show how the predictions of mixed strategy equilibrium can be tested in the same way that the predictions of a pure strategy equilibrium.

Best Replies

We have argued that the principle of dominance is uncontraversial, and should be applied as a first step towards selecting the strategy of choice. But after eliminating those strategies which seem implausible because they are dominated, or iteratively dominated, we are sometimes still left with many possible strategic profiles, none of which is weakly dominant . In these cases we must impose more stringent assumptions on how players behave in order to reach a sharper prediction about the outcome of a game. One approach is to argue that each player forms a conjecture about the other players' strategies, and then maximizes his payoffs subject to this conjecture.

Product Differentiation

Depicted below is the strategic form of a game between two producers of a hiking boots. Approaching the new season, each firm faces the question of whether to introduce a new product, undertake minor modifications to its existing line, or run a sale. The high quality producer is so named because it introduced a new line of boots the previous season. Since the introduction of new product lines affect future decisions of the firm, the payoffs in the bimatrix should represent the respective value of the firms for a given strategic profile, not just the net profits for next summer.

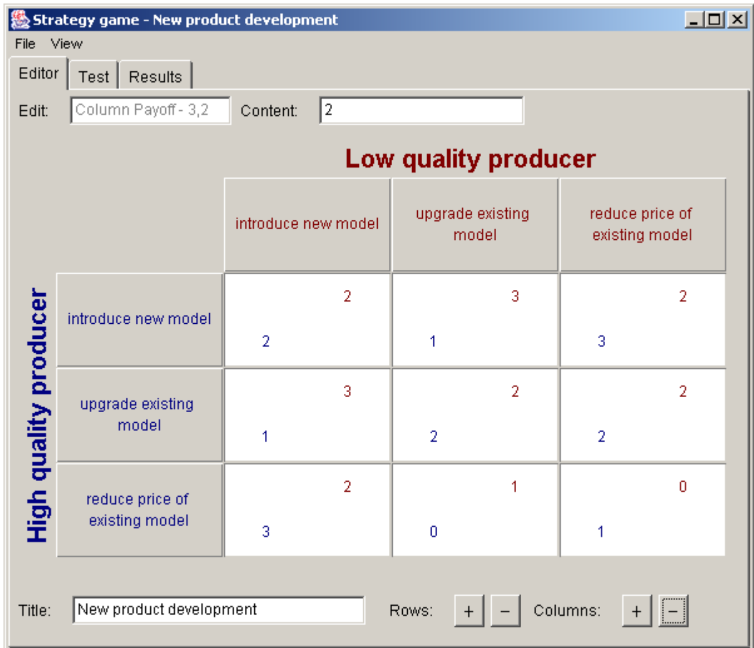


Figure 8.1
Product differentiation

Applying the principle of the previous chapter, we eliminate the low quality producer's sale option, because it is strictly dominated by mixing in any proportion the other two strategies. This leaves the strategic form for the reduced game depicted in Figure 8.2.

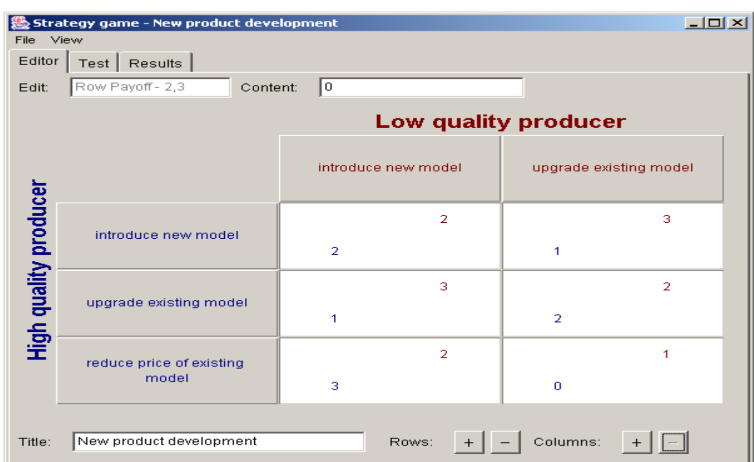


Figure 8.2
Reduced form of Product Differentiation

By inspection we can see that neither firm has a weakly dominant strategy in the reduced game. For example, if the low quality producer introduces a new model the high quality producer should hold a sale, but if the low quality producer upgrades its existing model the high quality producer should do the same thing. Therefore the high quality producer does not have a weakly dominant strategy.

Furthermore none of the strategies in the reduced game are dominated. In the case of the high quality producer this is easy to see. There are only two strategies, neither of which is dominant, neither can be dominated either. With regards the low quality producer, only then middle strategy of upgrading the existing model needs checking, because the strategy of holding a sale yields the highest payoff to the high quality producer if the low quality producer introduces a new product, while the strategy of upgrading the existing model gives the highest payoff if the low quality producer pursues the same strategy. If the low quality producer introduces a new product a mixture of p between upgrading and holding a sale yields $3 - 2p.$ If introducing a new model was a dominated strategy, then $3 - 2p < 2$, implying $2p > 1.$ Similarly if the low quality producer upgrades the existing product a mixture of p between upgrading and holding a sale yields $2p$, implying $2p < 1.$ Because these two inequalities cannot be satisfied simultaneously by any $p,$ we conclude that introducing a new product is not a dominated strategy.

One approach a player might consider to selecting a strategy is to determine which strategy maximizes her payoff given the strategies selected by the other players. This is called her best reply. With regards the high quality producer in the product differentiation game:

1. The best reply to the low quality producer introducing a new product is to hold a sale

2. The best reply to the low quality producer upgrading an existing model is to do the same thing.

With regards the low quality producer:

1. The best reply to the high quality producer introducing a new product is to upgrading the existing model

2. The best reply to the high quality producer upgrading the existing model is to introduce a new model

3. The best reply to the high quality producer holding a sale is to introduce a new model

Best replies can be depicted by arrows drawn in the strategic from of two player games, or matrix games, horizontal arrows indicating the best replies of the column player, and vertical arrows representing the best replies of the the row player. Figure 8.3 shows how this is done in the reduced product differentiation game. The long arrow pointing down indicates that 3 exceeds both 1 and 2, the bottom arrow pointing left indicates 2 exceeds 1, and so forth.

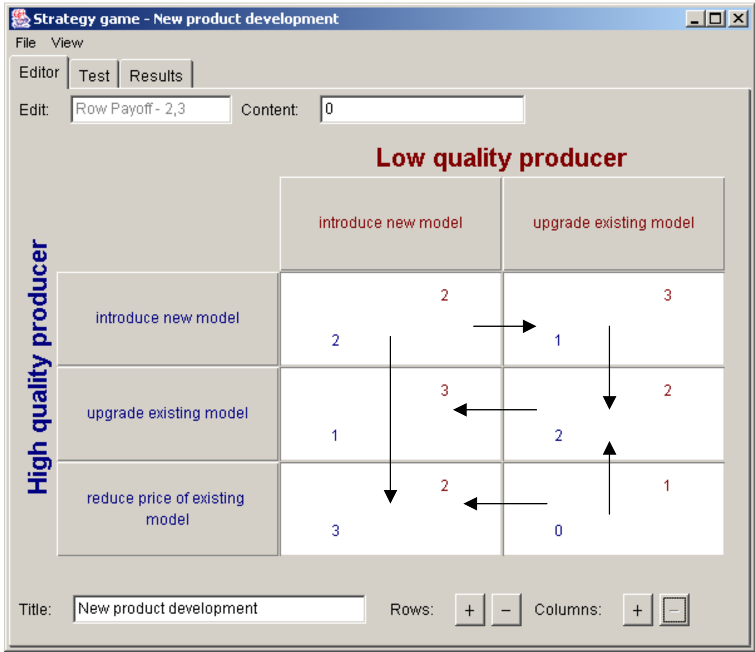
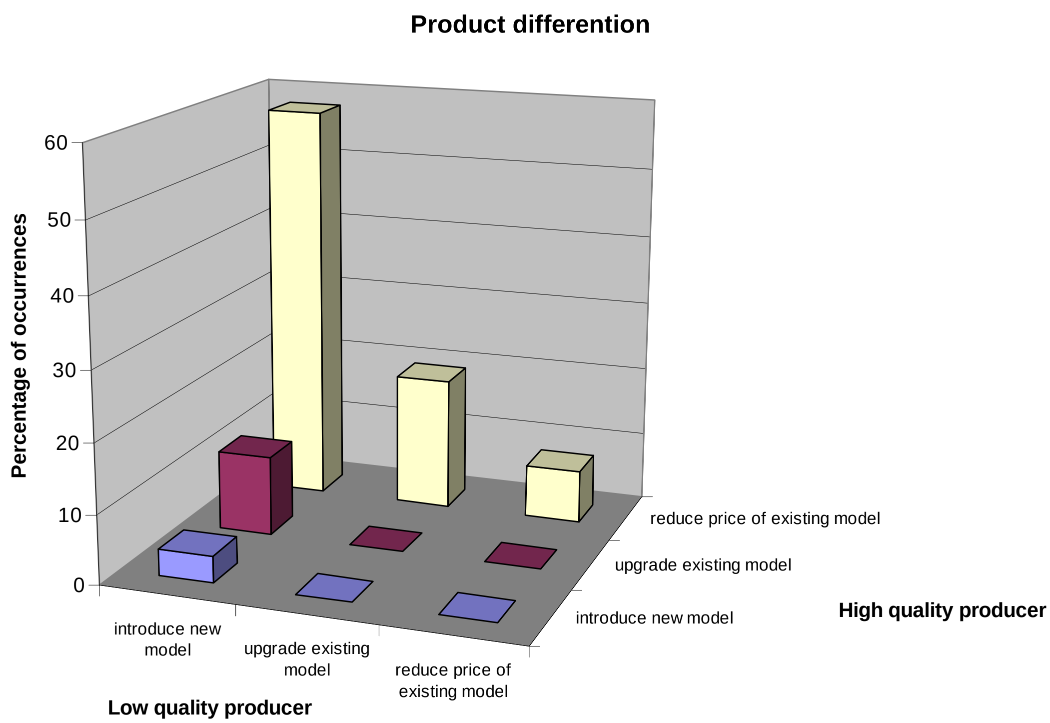


Figure 8.3
Best replies in product differentiation

In the product differentiation game, there are no opportunities for players to communicate. Nevertheless it is instructive to consider what might happen if we permitted an arbitrator to propose before the game a nonbinding agreement or suggestion about which strategy each player should choose. In particular suppose that upon reviewing Figure 8.3 the arbitrator followed the arrows to the point they converged, and suggested that the high quality producer should hold a sale and the low quality producer should introduce a new model. Each producer has no incentive to unilaterally deviate from this proposal. For this reason the proposal is called a self enforcing agreement. Notice that the suggestion that both firms upgrade is not a self enforcing agreement. In fact the exercise below asks you to establish a stronger result, that the (sale, new model) strategy profile is the unique self enforcing agreement for this game.

Experiment in product differentiation

Twenty six subjects participated in this experiment and they played the game twice, each time with the different opponent. The best reply (see Figure 8.3) in this game is for high quality producer to reduce the price of the existing model and for the low quality producer to introduce a new model. According to the theoretical precision the cell:(reduce price of existing model, introduce new model) should be observed with probability 1. The basic frequency table in Figure 8.3. shows that (reduce price of existing model, introduce new model) is chosen most often, 58.69 percent of the time. However it is interesting to observe that there were no occurrences of row players deviating from the equilibrium prediction at the same time as column players. Column players always chose "introduce new model" when row players deviated from their predicted behavior and vice versa.

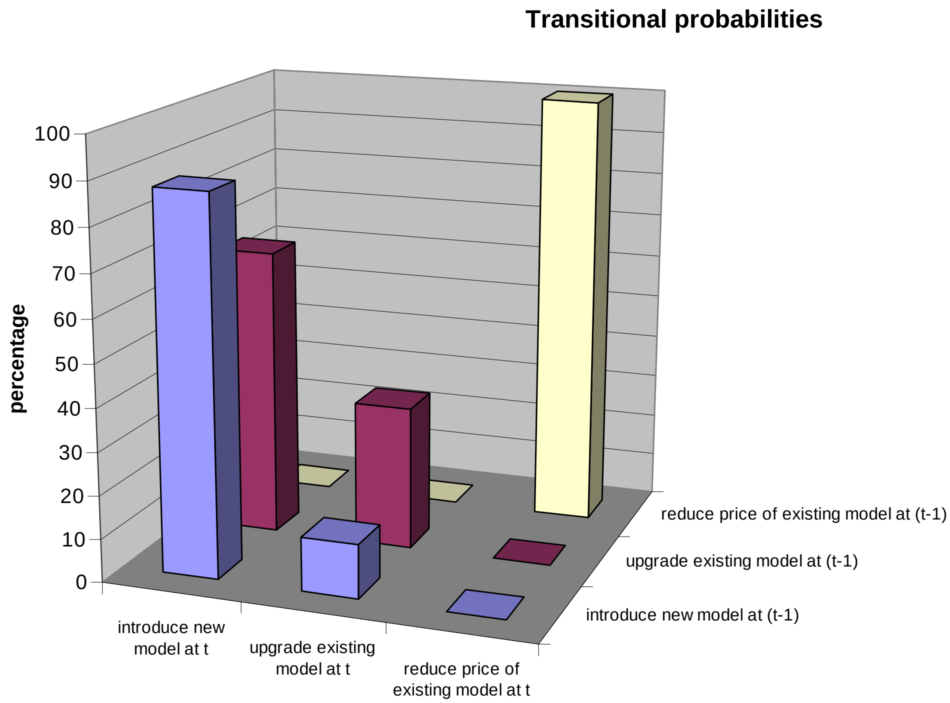


	introduce new model	upgrade existing model	reduce price of existing model
introduce new model	3.85	0	0
upgrade existing model	11.54	0	0
reduce price of existing model	57.69	19.23	7.69

introduce new model upgrade existing model reduce price of existing model

Figure 8.1 a: Actual responses in product differentiation

When we compare if subjects changed their behavior from the first round to the next round we observe that subjects who picked reduced price of the existing model would select the same choice in the next round. In general subject did not change the choice from previous round to the next round.



	introduce new model at t	upgrade existing model at t	reduce price of existing model at t
introduce new model at (t-1)	87.5	12.5	0
upgrade existing model at (t-1)	66.67	33.33	0
reduce price of existing model at (t-1)	0	0	100

The formal test of the null hypothesis that subjects acted according to the theoretical prediction is:

Exercise *Product differentiation*

1. *Drawing upon the payoffs in the matrix representation of the product differentiation game shown in Figure 8.2, prove that the low quality producer does not have a weakly dominant strategy.*

2. *Run the strategic form of the grocery marketing game and tabulate the results by cell. Consider cells that were visited once by both players. Are they more likely to be visited again?*

3. *Show that in this particular game, only one payoff cell in the matrix supports a self enforcing agreement.*

Blind date

The second example we consider in this chapter concerns blind dating. Meeting a new person in the city poses special challenges, ranging from identification to the logistics of timing. Thus the means of transportation used by each party may play a critical role in forming first impressions which are so important in shaping any future interplay. In this example we suppose that the female can take a taxi or come by the underground, whereas the male has a range of three options including bus, taxi or driving and parking his own car in a nearby garage. The payoffs depicted in Figure 8.3, the strategic form of this game, obviously depend on the routes and reliability of the public transportation system, how easy and expensive it is to engage a taxi, driving and parking within the city, the point of rendezvous, and the personal tastes of the dating couple.

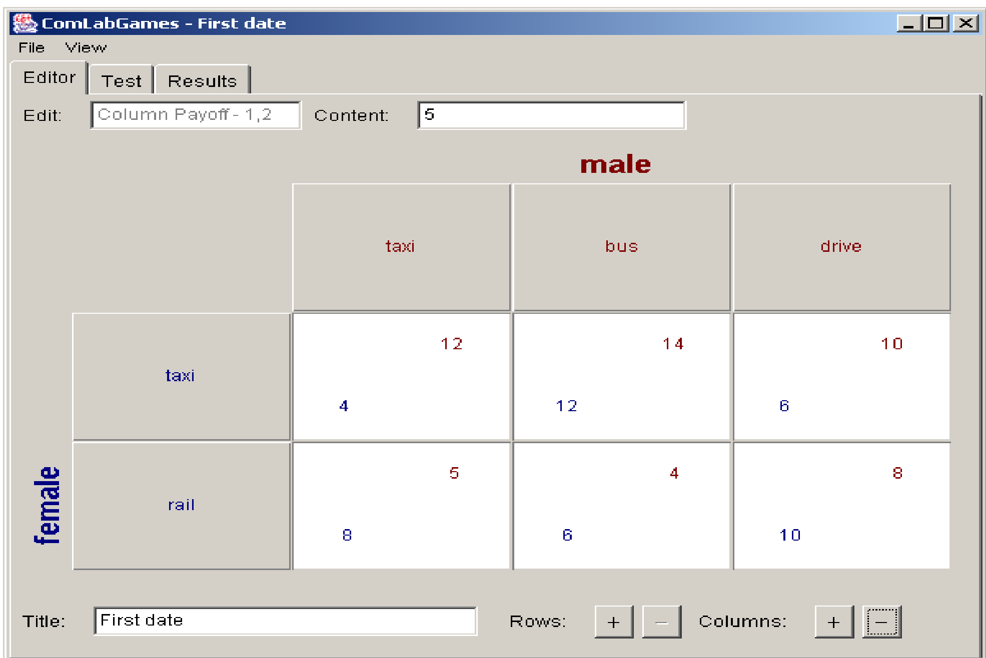


Figure 8.4
Blind date

Although neither the pure strategies of driving nor taking the bus choice dominate taking a taxi, there are mixed strategies for the male that randomize between driving and taking a bus that dominate taking a taxi. Consequently one can apply the dominance principle to eliminate the choice for the male to take a taxi. After eliminating that choice from consideration, we are left with matrix game in which both players have two choices.

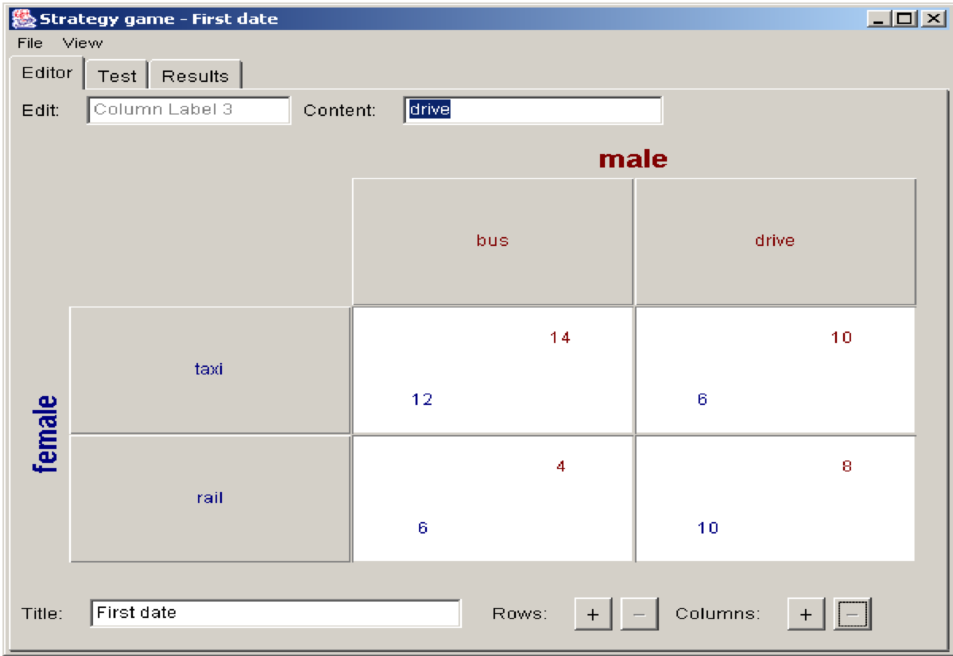


Figure 8.5
First date reduced

Unfortunately the principle of eliminating iteratively dominated strategies is of no help in this example. For example, if the female takes a taxi to her date, the male should take a bus, but if she goes by rail then he should drive. Indeed there is no dominant strategy for either party. Hence the applying the principles of dominance cannot help us to refining the set of strategy profiles beyond the four embedded within the reduced matrix game.

To make further progress we list the best replies of each party. First focusing on the payoffs to the female:

1. If her date takes the bus, she should arrive by taxi
2. If her date drives in from the suburbs, she should come by train

The best replies of the male also depends on what mode of transportation his companion will choose

1. If she comes by taxi, he should take the bus
2. If she comes by train, he should drive.

The best replies are schematically shown in Figure 8.3. The horizontal arrows represent the male's best replies and the vertical arrows represent the female's best replies.

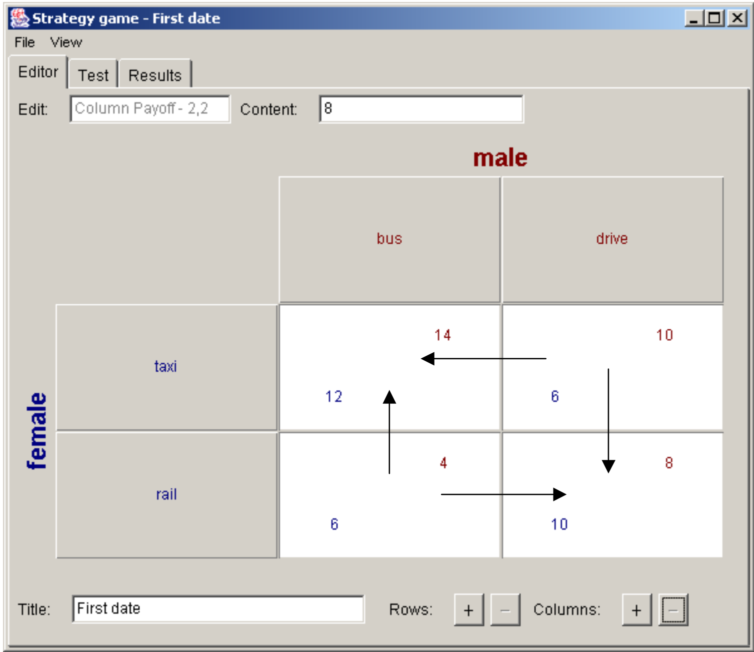


Figure 8.6

Best replies in blind Date

Two cells in figure 8.6 look more promising than the other two. Suppose that in preparing their blind date the couple agree that the male should take the bus and the female should take a taxi. Having made these arrangements there is no incentive for either party to deviate from then unilaterally. There is also no incentive to deviate from an agreement in which the male drives, and the female comes by rail. Thus (taxi,bus) and (rail, drive) are both incentive compatible agreements. Note though that the payoffs to both parties are higher from the (taxi, bus) than the (rail, car) strategy profile, (12,14) versus (10,9) so one would think this is more likely to the final outcome than any other coordinate pair.

Exercise *Blind Date*

1. *What are the mixed strategies available to the male that dominate taking a taxi?*

2. *Prove that the remaining cell does not support a self enforcing agreement.*

3. *Conduct an experiment of the full blind date game base on figure 8.4 and the reduced game figure 8.6*

4. *Test the hypothesis that the outcomes are not affected by eliminating the dominated strategy.*

5. *Test the hypothesis that in each experiment the*

6. *Allow preplay communication between the players and rerun the experiments*

7. *Prove that in every two by two matrix in which is not dominance solvable, that if there is a self enforcing agreements, it is not unique*

Parking Meter

The fact that many metropolitan areas employ parking meter patrols strongly suggests that many drivers would not pay parking meters if there were no deterrent effects. In the next example a shopper consider whether or not to feed the meter. At $1.00 an hour she requires 4 eight quarters to complete her 2 hour excursion, and if she does not pay, and the patrol checks her meter within that hour, a fine of $10.00 is levied. The patrol is rewarded on its success in discovering parking violations. An alternative use of the patrol's time would be to spend less time checking meters and more time in the air conditioned patrol car.

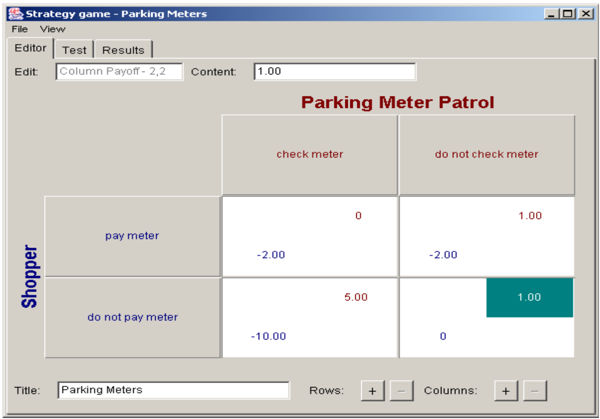


Figure 8.7
Parking Meters

Neither player has a dominant strategy in this game. Furthermore none of the cells support a self enforcing agreement. For example suppose the parking meter patrol is checking meters, the shopper should feed her meter $2.00 rather than pay a $10.00 fine. But if the shopper feeds the meter, the patrol is wasting his own item checking it, and values his best alternative time use engaging in some other activity. Now suppose the patrol does not check the meter: the shopper loses $2.00 by feeding it, so chooses not to. In that case the patrol should check it. The circle of possibilities is complete, Figure 8.8 showing the best replies!

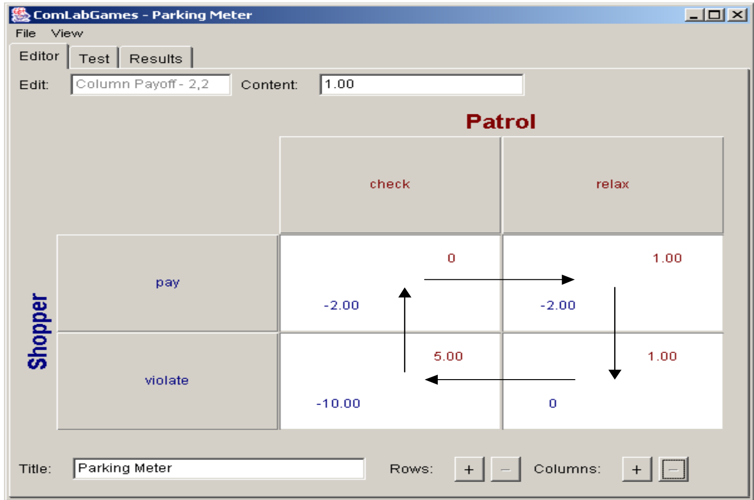


Figure 8.8

Best Replies in Parking Meter

This naturally begs the question of how the shopper and the patrol behave.

Exercise *Suppose the shopper will only take 30 minutes to complete her rounds. There is 8 minutes on the meter. She has a choice of feeding the meter for 15 minutes, 30 minutes or not at all. The parking patrol . Design the game, run it and tabulate the results.*

Defining a best reply

A definition of a best reply can be given for any finite game is strategic form. As before, let $u_n(s_n, s_{-n})$ denote the expected utility to the n^{th} player in a game when he plays strategy s_n and the other players strategy s_{-n}. For example, in the parking meter game, if the shopper is the n^{th} player, s_n means not paying the meter, and s_{-n} represents the patrol checking the meter, it follows that $u_n(s_n, s_{-n}) = -10.00$. Alternatively letting n stand for the male, s_n for taking a bus, and s_{-n} for the female taking the train, it follows that $u_n(s_n, s_{-n}) = 4$. We denote the set of strategies available to the n^{th} player by S_n. Then the strategy $s_n^* \in S_n$ is called a best reply to s_{-n} if

$$u_n(s_n^*, s_{-n}) = \max_{s_n \in S_n} [u_n(s_n, s_{-n})]$$

Following through with the examples above, if the n^{th} player is the shopper and s_{-n} represents the patrol checking the meter in the parking meter game, then s_n^* is paying the meter and $u_n(s_n^*, s_{-n}) = -2.00$. Using the same logic, if the n^{th} player is the male and s_{-n} means the female takes the train, then s_n^* is driving and $u_n(s_n^*, s_{-n}) = 8$. Note that the best reply is not necessarily uniquely defined. For example in the strategic form of the entry game in Figure 8.9, the best reply of the incumbent to the rival staying out is to either fight or accommodate.

Best replies for mixed strategies are defined in an analogous manner, and require us to define . Using this terminology, a mixed strategy for the n^{th} player is a d_n

dimensional probability vector of the form $\pi_n \equiv (\pi_{n1}, \pi_{n2}, \ldots, \pi_{nd_n})$ such that $\pi_{nd} \geq 0$ for all $d \in \{1, 2, \ldots, d_n\}$ and

$$\sum_{d=1}^{d_n} \pi_{nd} = 1$$

where π_{nd} is interpreted as the probability that player n will pick strategy $s_n^{(d)}$.

$$u_n(s_n^*, s_{-n}) = \max_{s_n \in S_n} \left[\sum_{d=1}^{d_n} \sum_{d=1}^{d_n} \pi_{nd} u_n(s_n, s_{-n}) \right]$$

Exercise *Some general results*

1. *Prove that a weakly dominant strategy is a best reply*

2. *Prove that every strategy that survives the principle of iterated dominance is a best reply to some strategy profile by the other players in the reduced game, and in the original game.*

3. *Show that a best response is one or more pure strategy, and that every mixed strategy is a best response only puts weight on those pure strategy best responses.*

Rationalizable strategies

To advise a player to form conjecture about the actions of the other players, and choose a best reply to this conjecture is only as compelling as the conjecture is convincing. the set of rationalizable strategies is based upon the notion as the conjecture itself.

Suppose that each player believes that every other player will form a conjecture and choose a best reply to it.

The first three rules don't help here. The idea of elimination by strict dominance does not help, because no strategy is strictly dominated. But suppose that player 1 expects player 2 to play L. Then 1's best response is to play U. If player 2 expects that player 1 plays U, then her best response is L. In this case we call the pair (U, L) a solution of the game.

A chain of conjectures

The chain of conjectures for this Nash equilibrium (U,L) is:

1 thinks that 2 plays L

=> 1 plays U;

2 thinks that 1 thinks that 2 plays L

=> 2 thinks that 1 plays U

=> 2 plays L;

1 thinks that 2 thinks that 1 thinks that 2 plays L

=> 1 thinks that 2 thinks that 1 plays U

=> 1 thinks that 2 plays L

=> 1 plays U;

and so on.

Then the conjecture each player makes about some other player should also reflect that the other player is forming a best reply to his conjecture as well.

Chains of best replies

the set of rationalizable strategies are those strategies that

There is a close connection between iterated dominance and rationalizable strategies in two player games. Every strategy that remains after iterated dominance in mixed strategies has been applied is a best reply to some mixed strategy. It follows that in an important class of games, that is those only involving two players, the use of rationalizable strategies does not refine the set of candidate solution strategy profiles at all over and above those strategy profiles that survive iterated dominance.

The chains of reasoning constructed over the different strategy profiles may of course not lead to much in the way of refinement. Example 8.2 shows two chains that cover the entire set of different strategic profiles.

It is not hard to see why the set of rationizable strategies is a subset of the those strategy profiles that survive iterated dominance. For suppose the contrary.

But is the set of rationalizable strategies are proper subset of the strategies that survive iterated dominance? In other words, are there strategies that cannot be eliminated using the arguments of iterated dominance, but are not rationalizable? For all two player games, the answer is no. Applying the principle of iterated dominance, reveals the set of rationalizable strategies. This provides a link between the principle of iterated dominance and rationalizable strategies. However in games involving three or more players, rationalizable strategies do further narrow the set of strategies one would predict for rational players.

The three player game we investigate to illustrate this point consists of two rivals in a high tech industry and the patent office.

The set of rationalizable strategies

We concluded that chapter by defining and analyzing rationalizable strategies, but noted that at least in two player games that did not further refine the set of strategic profiles beyond the refinement achieved by using the principle of iterated dominance.

Pure strategy Nash equilibrium

A pure strategy Nash equilibrium is a strategy profile that is also a rationalizable chain for each of the players. In other words it is a single rationalizable chain that contains only as many links as there are players. The arguments for selecting such a chain as being more plausible than other strategy profiles, whose elements are individually rational are based on notions of coordination that are not modelled as part of the game. Perhaps players communicate before playing the game and agree to a strategy profile

Corporate Plans

One might argue that rationalizable chains with many links are less likely to occur than shorter chains. Consider the following game. Conflict within the corporation about many We consider a problem the company faces as it mulls over possible directions of expansion. In this game reducing staff is a dominated strategy for the accounting department, but whether the department prefers to decentralize or to consolidate depends on where new sales are pitched.:

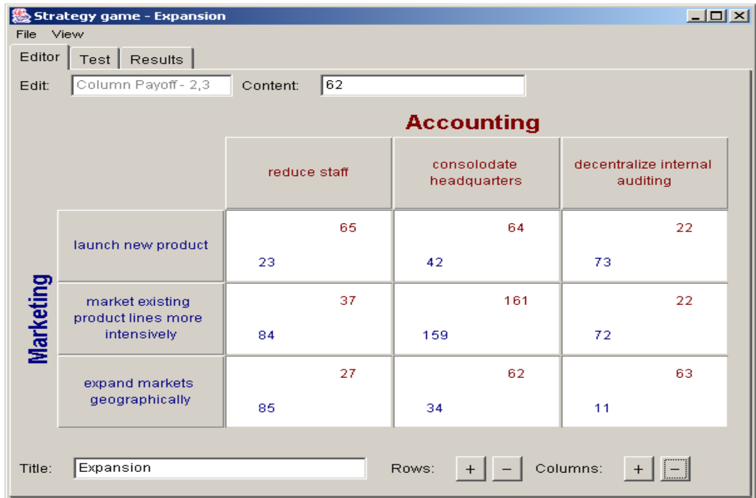


Figure 8.8
Corporate Plans

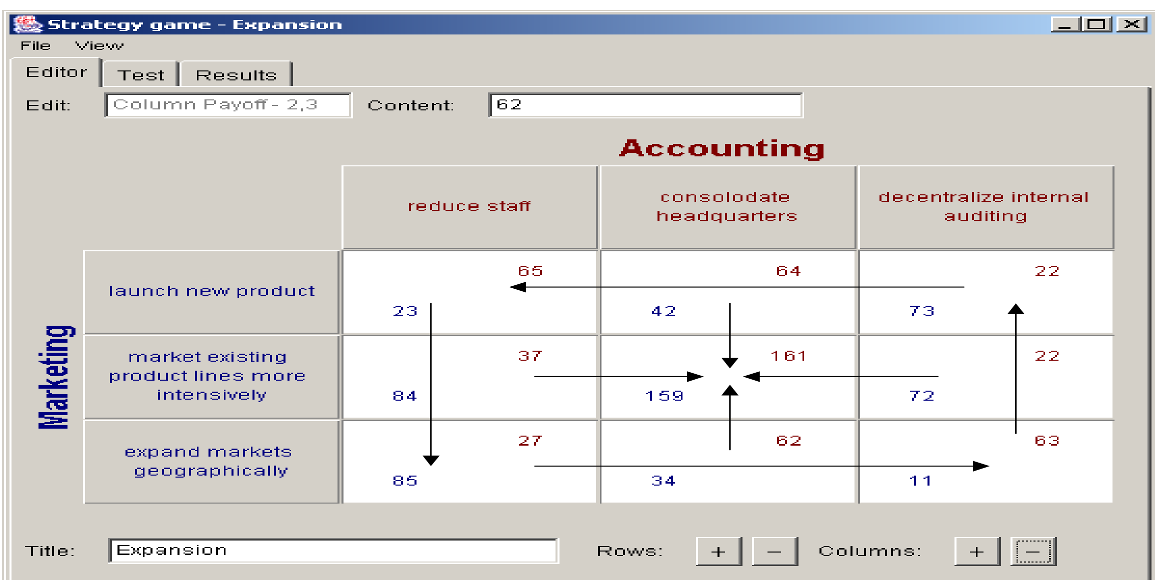


Figure 9.9
Best Replies in corporate plans

Patenting

Consider the following three player game.

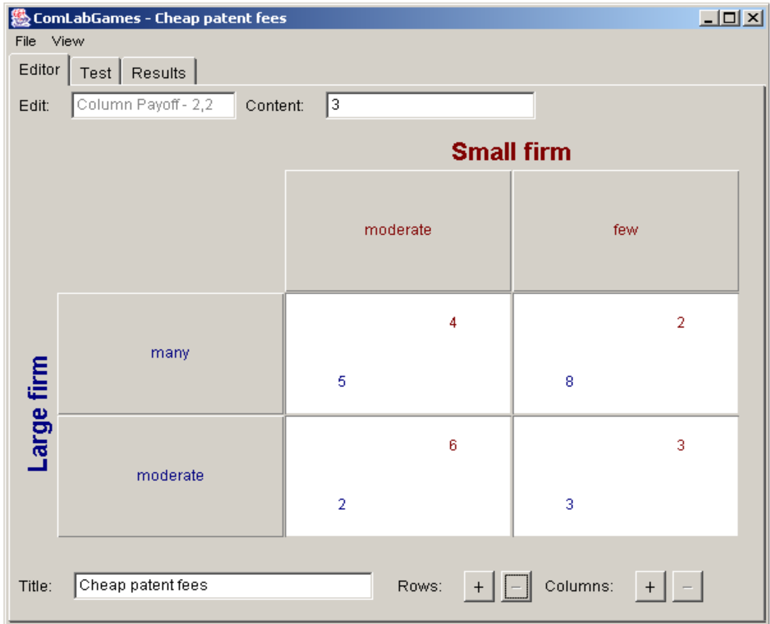


Figure 8.6
Cheap patenting fees

If the patent office charges expensive fees

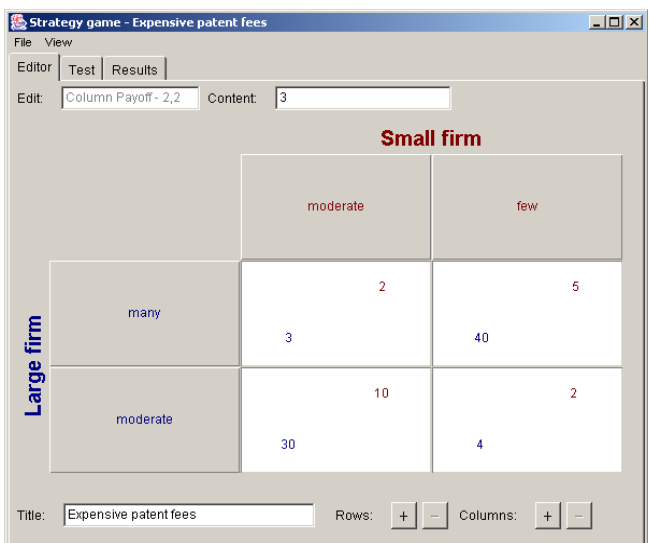


Figure 8.7
High patent fees

Formal definition

Using more basic terminology, consider a game for N players written in strategic form, where each player $n \in \{1, \ldots, N\}$ picks some strategy s_n from their choice set S_n. A strategy profile $s^* \equiv \{s_1^*, \ldots, s_N^*\}$ is a Nash equilibrium of the game if s_n^* is a best reply to $s_{-n}^* \equiv \{s_1^*, \ldots, s_{n-1}^*, s_{n+1}^*, \ldots, s_N^*\}$ for each $n \in \{1, \ldots, N\}$. Let $u_n(s)$ denote the expected utility of the game's outcome for player $n \in \{1, \ldots, N\}$ from the strategy profile

$s \equiv \{s_1, \ldots, s_N\}$. Thus a pure strategy Nash equilibrium s^* satisfies the inequalities

$$u_n(s^*) \geq u_n(s_1^*, \ldots, s_{n-1}^*, s_n, s_{n+1}^*, \ldots, s_N^*)$$

for all $n \in \{1, \ldots, N\}$ and $s_n \in S_n$.

Rule 5 If there is a unique Nash equilibrium, then play your own Nash equilibrium strategy.

This rule encompasses Rules 2 and 3 but not 1 and 4. Since 4 includes 1, 4 and 5 together encompass the first three. Use examples and show credibility issue again.

Exercise *Consider a two player game in which each player has two strategies to pick from. Suppose that none of the strategies are dominated. Show there is not a unique equilibrium in pure strategies.*

Multiple Equilibrium

An important result we state in the last chapter is that games that support an strategy that is arrived by iterated dominance is unique. Unfortunately there is an analogous result in the equilibirum. The existence of an equilibrium does not guarantee uniqueness.

Corporate Reorganization

Are pure strategy equilibrium more plausible than mixed strategy equilibrium? consider the corporate game

Suppose we change the payoffs drastically reducing the middle payoffs, but not affecting the best replies

A mixed strategy appears, although the pure strategy remains

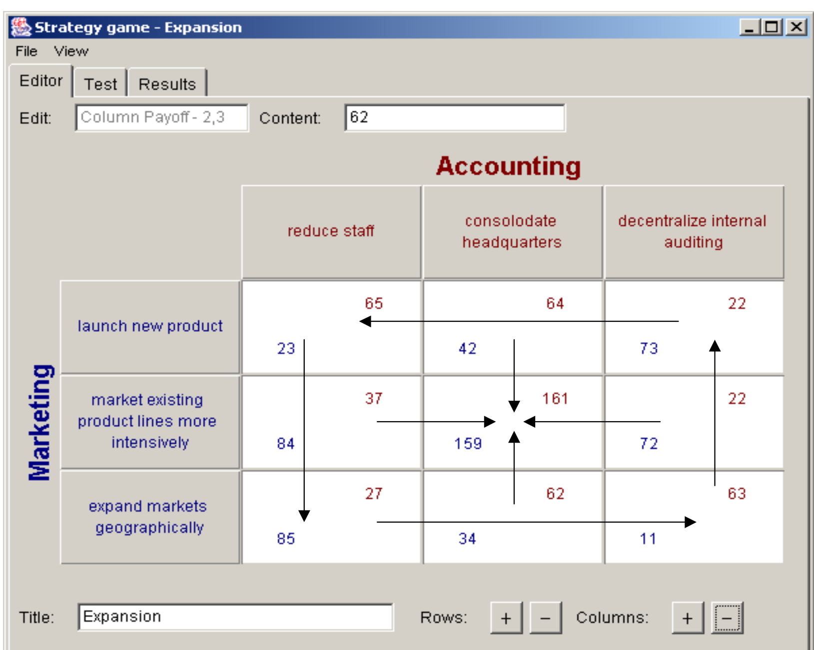


Figure 8.18

Revisions to corporate reorganization

Nor we cannot guarantee there is a Nash equilibrium in every simultaneous game, that is if players are restricted to picking a deterministic or pure strategy.

Exercise *Consider blind date. Find the mixed strategy equilibrium*

Entertainment

The following is a variation of the battle of the sexes. A couple enjoy the company of each other, but have different preferences over how they should spend it. We may be guilty of gender stereotyping by labeling the axis as theater and (ice) hockey.

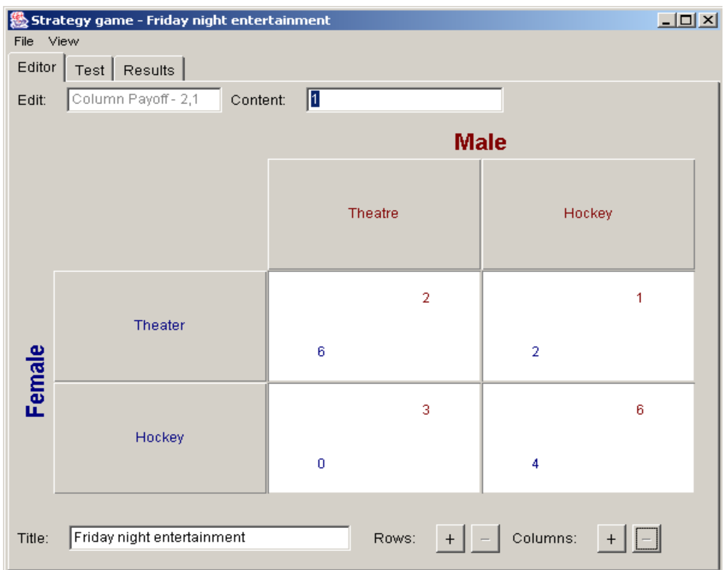


Figure 8.3
Friday Night Entertainment

Exercise *There are 2 pure strategy equilibrium in the Friday night entertainment game, as well as one mixed strategy equilibrium.*

1. *Find all three, and compare the payoffs to each partner in each equilibrium.*

2. *Is a pure strategy equilibrium more likely to occur. Play the game and compute the frequencies*

3. *If players have the opportunity to discuss the outcome beforehand is a Nash equilibrium more likely?*

A General Result

Theorem *Whenever there is more than one pure strategy equilibrium, there is at least one mixed strategy equilibrium as well.*

Nash equilibrium in perfect information games

So far we have only been discussing simultaneous move games

Entry Game

The two previous examples define best replies in simultaneous move games. In principle best replies can be defined for the strategic form of any game, but there is an important caveat to this statement. its usefulness and relevance is . which follows from the fact that the Since a best reply applies to the strategic form, it cannot capture the dynamic aspect that strategies are not binding commitments but merely detailed plans about choosing in all possible contingencies. The ramifications of this crucial difference are most evident in perfect foresight games

Accordingly, suppose an incumbent firm currently has a monopoly in a market, and at the initial node of the game, a rival chooses between end the monopoly and enter its market, or stay out. After the rival makes its decision, the incumbent firm chooses between producing less output to accommodate both firms and thus maintain the market price, much versus to produce the same output as before causing the price to fall sharply to prevent inventory accumulation. The extensive form of this entry game is depicted in Figure 8.7. Solving the game using backwards induction, the first rule of strategic play implies that the rival should enter and the incumbent should acquiesce.

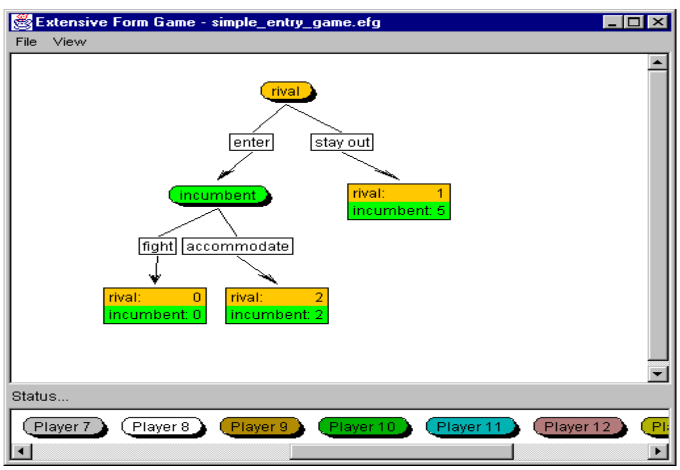


Figure 8.7
Entry Threat

There are two strategies for each firm. The rival chooses between enter and stay out, while the incumbent must fight or acquintense if the rival enters the industry. The strategic form of this game is shown in Figure 8.8. Noting that enter is a weakly dominant strategy, the backwards induction solution is redhead using the third rule of strategic play

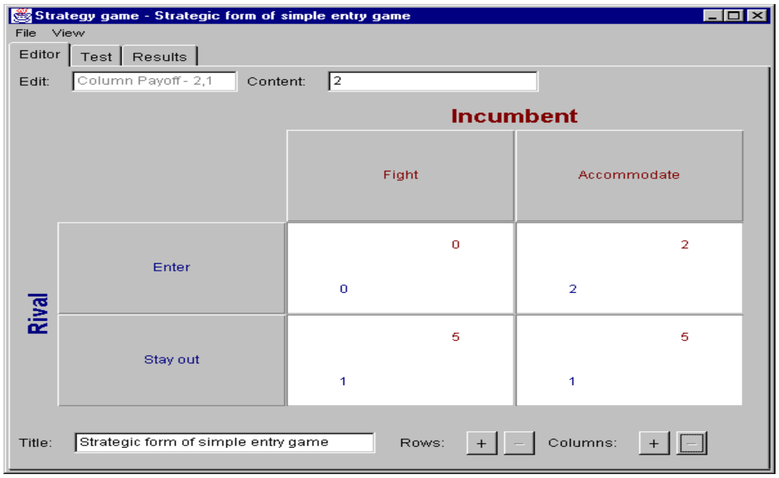


Figure 8.8

Strategic Form of Entry Game

How does this prescription correspond to our to our analysis of best replies this far? The best reply mapping for the rival is:

1. Enter if the incumbent accommodates.
2. Stay out if the incumbent fights.

The best reply correspondence for the incumbent is:

1. Accommodate if the rival enters.
2. Fight or accommodate if the rival stays out.

The best replies are illustrated in Figure 8.9, the horizontal dotted line indicating the incumbent's indifference between his own choices when the rival stays out.

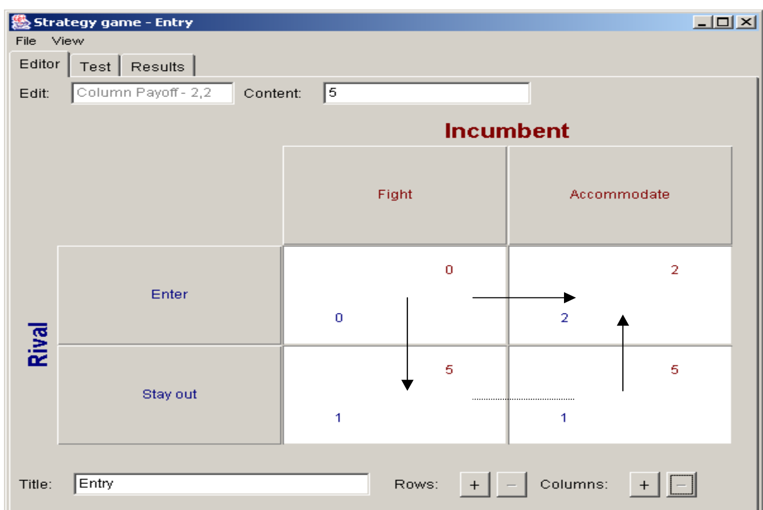


Figure 8.9

Best Replies in Entry

The strategic form shows there are two incentive compatible strategy profiles, namely (enter, accommodate) which is the solution to this game, and (stay out, fight) which is not a solution. This allocation corresponds to the incumbent committing himself to fighting should the entrant come in. Yet the only way such commitment can be made is if the the game is a simultaneous move game, in which the incumbent chooses his strategy without knowing what the rival has chosen. In this case the extensive from would look like Figure 8.10.

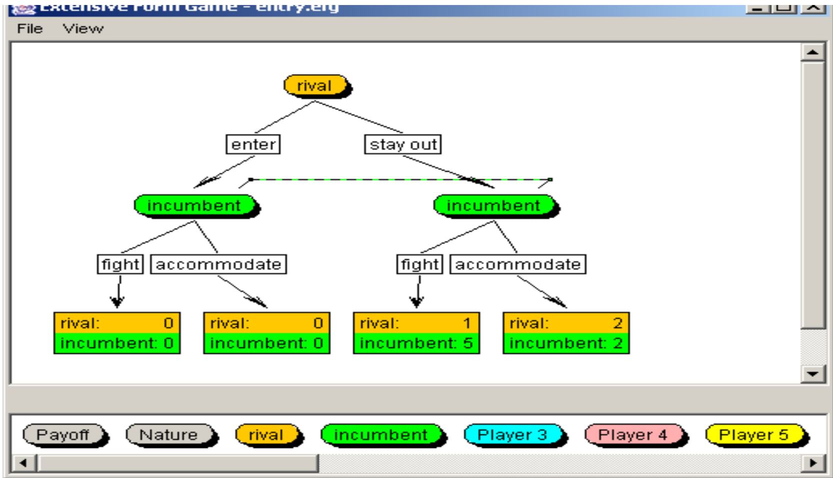


Figure 8.10
Simultaneous Entry Game

The fact that the concept of a best reply applies to the strategic form of the game and not its extensive form limits its usefulness. Unless one knows that the game is a simultaneous game, rival should question whether the strategy of fight is truly a best reply.

Summary

These examples demonstrate that after iteratively removing dominated strategies, some games do not yield a unique strategy profile. In order to provide sharper predictions about the outcomes of games, more controversial criteria must be developed and applied. We showed that in two of the examples discussed above a subset of the strategy profiles were self enforcing agreements, whereas the strategy profiles were not. In one of the examples the self enforcing agreement was unique, in the second example two strategy profiles had this property, while in the third example none of the strategy profiles were self enforcing agreements. It immediately follows that depending on the rules of the game, or more broadly the organizational structure and the nature of rivalry between different organization, self enforcing agreements may not exist or in the case of nonuniquenes do not fully resolve how the game will be played. In this important respect the notion of a self enforcing agreement is deficient.

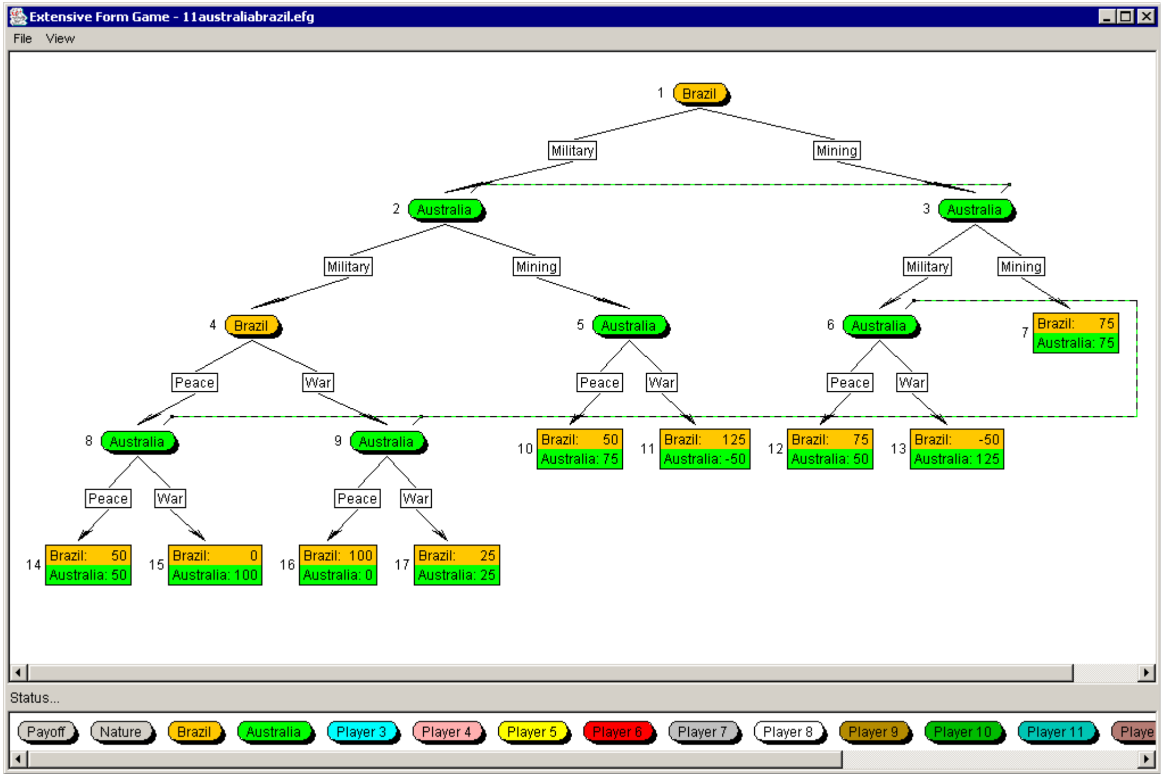
There is a further criticism. In our discussion of backwards induction and dominance, we noted that these principles could be applied unilaterally, and did not require any centralized decision making. A precondition for an agreement, on the other hand, is communication between players. It is worth asking whether one can go further than the notion of dominance in seeking more precise predictions about the outcomes of games without appealing to such a centralist notion as a self enforcing agreement.

This chapter develops both these issues, existence and uniqueness of , and the decentralization, in more depth. At the heart of this development is the notion of a best response. This chapter elaborates on criteria for such games, and explains how they are related to dominance. defines and exposits Nash equilibrium. Therefore a stronger notion of rationality is required to differentiate between the remaining ones and thus generate sharper predictions. This remark motivates the definition of a best reply. This concept embodies the idea that the strategy each player picks might be optimal given that player's beliefs about the strategies all the other players are picking. Best responses are the building blocks for successive refinements to iterative dominance, namely rationalizable strategy profiles, Nash equilibrium, and discarding Nash equilibrium that contain any weakly dominated strategies. The chapter works through each of these concepts in turn, illustrating them with examples from business strategy at each stage of refinement.

Sample Test

1. The Game between Brasil and Australia

In this game Brazil has an option between Military and Mining and later between war and peace , and similarly Australia.



Comments and Solution

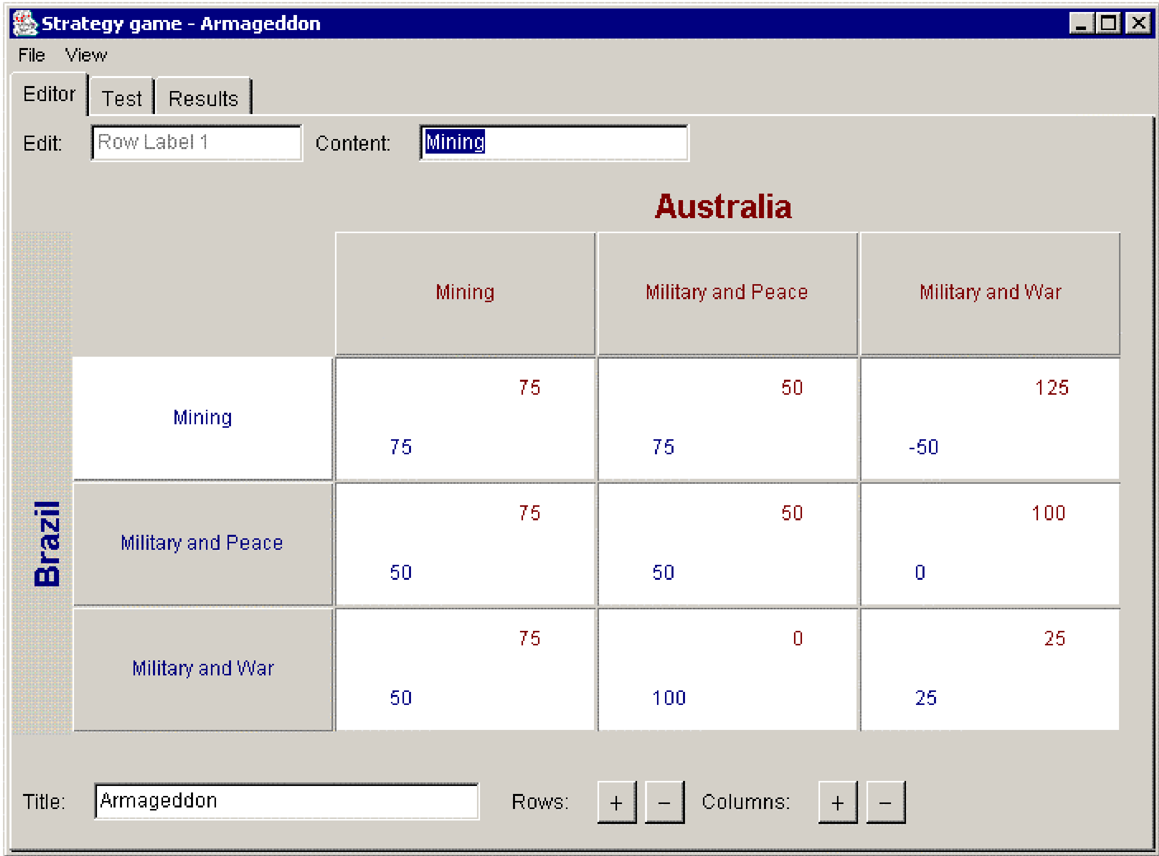
Strategies for Brazil are:

- Mining
- Military and Peace
- Military and War

Similarly for Australia:

- Mining
- Military and Peace
- Military and War

The strategic form representation of the game is:



First, we can discard Military and Peace for both players since they are strongly dominated by Military and War (in case of Australia) and weakly dominated by Military and War (in case of Brazil). The game is now reduced to Mining and Military and War. There is no pure strategy equlibria in this reduced game. If Brazil selects Mining, Australia will reply with Military and War. Brazil will then respond wih Minitary and War and Australia will switch to Mining and Brazil will respond with Mining. We are back were we started. The next step is to test if there are any mixed strategy equlibria. Let p be probability that Brazil will select mining and let q be the probability that Australia selects Mining.

Brazil \ Australia	probability	q	1- q	
		Mining	Military and War	EU_{Brasil}:
p	Mining	75, 75	125, -50	75*q – 50*(1-q)
1- p	Military and War	75, 50	25, 25	50*q + 25*(1-q)
$EU_{Australia}$:		75*p + 75*(1-p)	125*p + 25*(1-p)	

Australia will only mix between Mining and Military and War if EU(Mining) = EU(Military and War). Since Brazil is using a mixed strategy Australia faces an expected values:

75*p + 75*(1-p) = 125*p + 25*(1-p) -> p = ½

where p and (1-p) are the probabilities with which Brazil will mix between Mining and Militarry and War. By knowing p=1/2, we are now in the position to compute Australia's expected value. Expected payoff for Australia, with p= ½ is 75

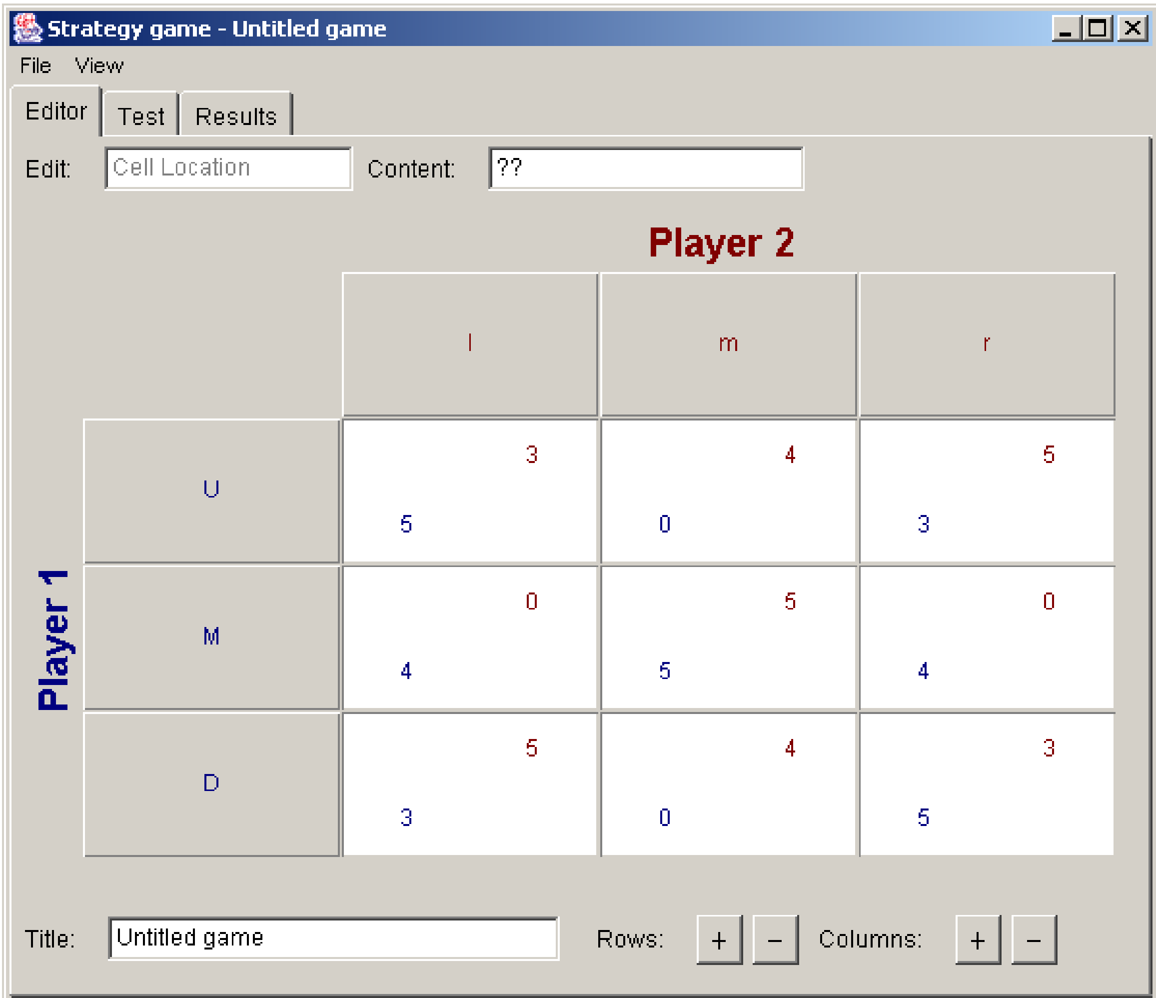
Brazil will only mix between Mining and Military and War if EU(Mining) = EU(Military and War). It requires to Australia to mix so that:

75*q - 50*(1-q) = 50*q + 25*(1-q) è p = ¾

The expected payoff for Brazil, with p= ¾ is 43.75.

2. Nash equlibrium

This game was designed just to test the knowledge of best reply (i.e. Nash equilibrium) . In this game you could not use the strict dominance or weak dominance rule to delete some of the strategy.



Solution

1. Player 1 should select M and player 2 should select m. (M, m) is Nash equilibrium with payoffs (5,5). Why? If I Player 1 selects U, player 2 will reply with r, if Player 2 replies with r then Player 1 will move to M, if Player 1 selects M then Player 2 will select m; if player 2 selects m then Player 1 will stay with M.

Bibliography

Anderson, Simon P., Jacob Goeree, and Charles A. Holt (1998) “Control Costs and Equilibria in Games with Bounded Rationality,” University of Virginia, Discussion Paper, presented at the Summer 1998 ESA Meeting. Abstract: Van Damme’s notion of control costs is that it is more costly to implement decisions more precisely. This paper derives the relationship between control costs and noisy approaches to equilibrium in games. In two-by-two games, quantal response equilibria are equivalent to Nash equilibria with control costs. Extensions to N-person matrix games are also discussed.

Anderson, Simon P., Jacob K. Goeree, and Charles A. Holt (1998) “Logit Equilibrium Models of Anomalous Behavior: What to Do when the Nash Equilibrium Says One Thing and the Data Say Something Else,” in Handbook of Experimental Economics Results, edited by C. R. Plott and V. L. Smith, New York: Elsevier Press, forthcoming.

Anderson, Simon P., Jacob K. Goeree, and Charles A. Holt (1999) “Properties of Logit Equilibria in Games with Rank-Based Payoffs,” University of Virginia, Discussion Paper.

Brown, J. N., and Robert W. Rosenthal (1990) “Testing the Minimax Hypothesis: A Re-examination of O’Neill’s Game Experiment,” Econometrica, 58:5 (September), 1065-1081.

Blume, Andreas, and Andreas Ortmann (1999) “The Effects of Costless Pre-play Communication: Experimental Evidence from a Game with Pareto-ranked Equilibria,” University of Iowa, Discussion Paper, presented at the Summer 1999 ESA Meetings. Abstract: The experiment introduces costless preplay communication into a multi-player coordination game with multiple Pareto-ranked equilibria. This communication results in immediate convergence to the Pareto-efficient outcome.

Chen, Hsiao-Chi, James W. Friedman, and Jacques-Francois Thisse (1996) “Boundedly Rational Nash Equilibrium: A Probabilistic Choice Approach,” Games and Economic Behavior, 1832-54. Abstract: This paper uses the Luce probabilistic choice rule to analyze behavior in matrix games.

Cooper, Russell, Douglas V. DeJong, Robert Forsythe, and Thomas W. Ross (1989) “Communication in the Battle of the Sexes Game: Some Experimental Results,” Rand Journal of Economics, 20:4 (Winter), 568-587.

Cooper, Russell, Douglas V. DeJong, Robert Forsythe, and Thomas W. Ross (1990) “Selection Criteria in Coordination Games: Some Experimental Results,” American Economic Review, 80:1 (March), 218-233.

Cooper, Russell, Douglas V. DeJong, Robert Forsythe, and Thomas W. Ross (1992) “Communication in Coordination Games,” Quarterly Journal of Economics, 107:2 (May), 739-771.

Cachon, Gerard P., and Colin F. Camerer (1996) “Loss-Avoidance and Forward Induction in Experimental Coordination Games,” Quarterly Journal of Economics, 111:1 (February), 165-194.

Goeree, Jacob K., and Charles A. Holt (1999) “Ten Little Treasures of Game Theory, and Ten Intuitive Contradictions,” University of Virginia, Discussion Paper. Abstract: The "treasures" are ten static and dynamic games where behavior matches the Nash equilibrium or relevant refinement, and the contradictions are variations of the same game that produces anomalous behavior patterns. In some games, Nash seems to work only by coincidence, e.g. if deviation losses are symmetric or very high. In other games the data are repelled from the Nash prediction and pile up on the opposite side of the set of feasible decisions.

Harless, David W., and Colin F. Camerer (1995) “An Error Rate Analysis of Experimental Data Testing Nash Refinements,” European Economic Review, 39:3-4 (April), 649-660.

Holt, Charles A., and Fernando Solis-Soberon (1992) “The Calculation of Equilibrium Mixed Strategies in Posted-Offer Auctions,” in Research in Experimental Economics, Vol. 5, edited by R. M. Isaac, Greenwich, Conn.: JAI Press, 189-229. Abstract The "how-to" for calculating equilibrium mixed strategies in the complicated step-function and capacity constrained environments often used in laboratory experiments. The paper discusses ways to deal with multiple steps, cost and capacity asymmetries, risk aversion.

Huck, Steffen, Hans-Theo Normann, and Jörg Oechssler (2000) "Does Information About Competitors' Actions Increase or Decrease Competition in Experimental Oligopoly Markets?," International Journal of Industrial Organization, 18:1 (January), 39-57. Abstract: Evidence from Bertrand and Cournot type experiments indicates that more information about others' actions does not promote tacit collusion. There is some tendency for subjects with good information to imitate actions that earned high payoffs, which tends to make the outcomes more competitive.

McKelvey, Richard D., Thomas R. Palfrey, and Roberto Weber (1997) "The Effects of Payoff Magnitude and Heterogeneity on Behavior in 2x2 Games with Unique Mixed Strategy Equilibria," California Institute of Technology, working paper.

Morgan, John, and Martin Sefton (1998) "An Experimental Investigation of Unprofitable Games," Princeton University, Discussion Paper, presented at the Fall 1998 ESA Meetings. Abstract: An unprofitable game is one in which maximin strategies do not constitute a Nash equilibrium, yet they guarantee the same payoff as players earn in a Nash equilibrium. In two-person matrix games with three strategies, the data patterns do not correspond to either Nash or maximin play.

My, K. Boun, Marc Willinger, and A. Ziegelmeyer (1998) "Local Interaction and Equilibrium Selection: An Experimental Investigation," BETA, Discussion Paper, presented at the Summer 1998 ESA Meeting. Abstract: This experiment evaluates the effects of local versus global interaction on behavior in two-person 2x2 coordination games. Other treatment variables include the size of the local neighborhoods.

Ochs, Jack (1995) "Coordination Problems," in The Handbook of Experimental Economics, edited by J. H. Kagel and A. E. Roth, Princeton, N.J.: Princeton University Press, 195-249. Abstract This paper surveys the literature on coordination game experiments.

Shubik, Martin (1996) "Why Equilibrium? A Note on the Noncooperative Equilibria of Some Matrix Games," Journal of Economic Behavior and Organization, 29:3 (May), 537-539.

Stahl, David O. (1988) "On the Instability of Mixed Strategy Nash Equilibria," Journal of Economic Behavior and Organization, 9:1 (January), 59-69.

Van Huyck, John B., Raymond C. Battalio, and Richard O. Beil (1990) "Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure," American Economic Review, 80:1 (March), 234-248. Abstract: This paper shows that behavior may converge to the Pareto-inferior Nash equilibrium in multi-person coordination games with multiple decisions. Coordination failure is more prevalent with large numbers of participants.