Overview

The previous chapter focused on eliminating from consideration those strategies that were unattractive to a player regardless of the strategy profile collectively chosen by the other players, and also discarding those strategies that could only be justified if the other players behaved in an irrational manner. However as our examples at the beginning of this chapter will establish, there are many games in which these dominance principles in their various guises do not yield a unique solution, or even refine the set of strategy profiles in contention. Additional rules are required to yield tighter predictions about game outcomes. These rules are somewhat less plausible, and harder to defend as behavioral norms, which is why we have postponed their introduction until now.

This chapter analyzes the concept of equilibrium, the notion that each player's behavior can be viewed as the outcome of him optimizing an individual objective function that is partly defined by the solutions to optimization problems of the other players. In order to develop this concept we first define the notion of a best reply, which means a pure or mixed strategy that maximizes the player's payoff given a strategic profile of choices made by the other players in the game. Whether choosing a best reply is an attractive strategy or not depends on the conjecture the player is making about the strategies of the other players. For example thinking the other players will choose strictly dominated strategies, and forming a best reply based on that conjecture doesn't seem a very sensible course of action!

This leads to a discussion of a concept that is weaker than equilibrium, individual strategies that are rationalizable. Intuitively, a rationalizable strategy is a best response to some strategy profile of the other players, the elements of this profile are strategies that are also best responses, and so on. In two player games the set of rationalizable strategies exactly corresponds to those strategies that survive the iterative removal of strongly dominated strategies. However rationalizable strategies do further refine the the set of strategy profiles in games for more than two players. One of the reasons why rationalizable strategies are regarded as important is because they also fully exploit what players can achieve without some form of coordination. Further refinements of the solution strategy, found by applying an equilibrium concept for example, are less robust predictors of game outcomes.

Succinctly stated, a Nash equilibrium is a strategy profile for all the players, in which every player's strategy is a best reply to the strategy profile formed from the other players' actual choices. Having defined the Nash equilibrium concept, we explore its properties, again mainly by way of example. It is useful to distinguish between a Nash equilibrium where all the players choose pure strategies from one in which some or all the players choose mixed strategies. A pure Nash equilibrium can be interpreted as a self enforcing contract. If all the players in the game were agents told by a principal which pure strategy to choose, in a Nash equilibrium no player would have an incentive to unilaterally deviate from the principal's instructions. This theme is taken up in Part 6 of the book in Chapters 13 and 14, on mechanism design and contracts.

Not every game has an equilibrium in pure strategies, but some games have more than one pure strategy equilibrium. In games with multiple equilibria, one or more of them may violate the principle of weak dominance, and thus be disqualified as solution candidates. When these equilibria occur in perfect information games, they can sometimes be interpreted as an attempt to write a self enforcing contract based on a noncredible threat by one of players about how he will respond to another player's action later in the game. Nevertheless there are many games with multiple equilibrium and no convincing criteria for ranking their likelihood in games. Accordingly we investigate games with multiple equilibria played a repeated number of times in Chapter 11, to analyze how learning and coordination between the players might take place in those cases.

The last section of this chapter concentrates on mixed strategies. Strategies in which individual players randomize their choices, or belong to a group whose homogeneity is compromised only by the fact that its members make different choices, occur quite frequently in the real world. We provide examples of games lacking a pure strategy equilibrium. It has, however, been proved that every finite game has an equilibrium in mixed strategies. At the end of our discussion on this topic, we remark that in games with a unique pure strategy equilibrium the probability distribution describing the conditional choice probabilities is degenerate, whereas in a mixed strategy equilibrium, the probability distribution of choices is proper. Nonetheless through the use of examples, we show how the predictions of mixed strategy equilibrium can be tested in the same way that the predictions of a pure strategy equilibrium.

Parking Meter

The fact that many metropolitan areas employ parking meter patrols strongly suggests that many drivers would not pay parking meters if there were no deterrent effects. In the next example a shopper consider whether or not to feed the meter. At $1.00 an hour she requires 4 eight quarters to complete her 2 hour excursion, and if she does not pay, and the patrol checks her meter within that hour, a fine of $10.00 is levied. The patrol is rewarded on its success in discovering parking violations. An alternative use of the patrol's time would be to spend less time checking meters and more time in the air conditioned patrol car.

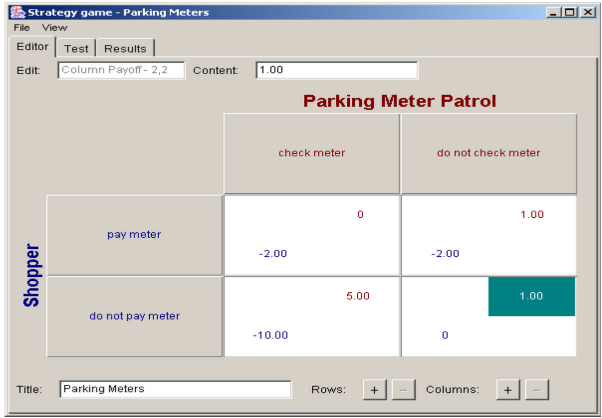


Figure 8.7
Parking Meters

Neither player has a dominant strategy in this game. Furthermore none of the cells support a self enforcing agreement. For example suppose the parking meter patrol is checking meters, the shopper should feed her meter $2.00 rather than pay a $10.00 fine. But if the shopper feeds the meter, the patrol is wasting his own item checking it, and values his best alternative time use engaging in some other activity. Now suppose the patrol does not check the meter: the shopper loses $2.00 by feeding it, so chooses not to. In that case the patrol should check it. The circle of possibilities is complete, Figure 8.8 showing the best replies!

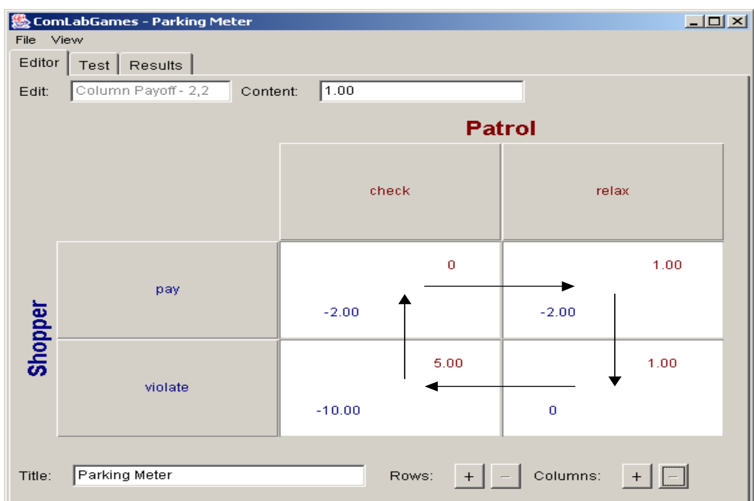


Figure 8.8
Best Replies in Parking Meter

This naturally begs the question of how the shopper and the patrol behave.

Exercise *Suppose the shopper will only take 30 minutes to complete her rounds. There is 8 minutes on the meter. She has a choice of feeding the meter for 15 minutes, 30 minutes or not at all. The parking patrol . Design the game, run it and tabulate the results.*

Defining a best reply

A definition of a best reply can be given for any finite game is strategic form. As before, let $u_n(s_n, s_{-n})$ denote the expected utility to the n^{th} player in a game when he plays strategy s_n and the other players strategy s_{-n}. For example, in the parking meter game, if the shopper is the n^{th} player, s_n means not paying the meter, and s_{-n} represents the patrol checking the meter, it follows that $u_n(s_n, s_{-n}) = -10.00$. Alternatively letting n stand for the male, s_n for taking a bus, and s_{-n} for the female taking the train, it follows that $u_n(s_n, s_{-n}) = 4$. We denote the set of strategies available to the n^{th} player by S_n. Then the strategy $s_n^* \in S_n$ is called a best reply to s_{-n} if

$$u_n(s_n^*, s_{-n}) = \max_{s_n \in S_n} [u_n(s_n, s_{-n})]$$

Following through with the examples above, if the n^{th} player is the shopper and s_{-n} represents the patrol checking the meter in the parking meter game, then s_n^* is paying the meter and $u_n(s_n^*, s_{-n}) = -2.00$. Using the same logic, if the n^{th} player is the male and s_{-n} means the female takes the train, then s_n^* is driving and $u_n(s_n^*, s_{-n}) = 8$. Note that the best reply is not necessarily uniquely defined. For example in the strategic form of the entry game in Figure 8.9, the best reply of the incumbent to the rival staying out is to either fight or accommodate.

Best replies for mixed strategies are defined in an analogous manner, and require us to define . Using this terminology, a mixed strategy for the n^{th} player is a d_n dimensional probability vector of the form $\pi_n \equiv (\pi_{n1}, \pi_{n2}, \ldots, \pi_{nd_n})$ such that $\pi_{nd} \geq 0$ for all $d \in \{1, 2, \ldots, d_n\}$ and

$$\sum_{d=1}^{d_n} \pi_{nd} = 1$$

where π_{nd} is interpreted as the probability that player n will pick strategy $s_n^{(d)}$.

$$u_n(s_n^*, s_{-n}) = \max_{s_n \in S_n} \left[\sum_{d=1}^{d_n} \sum_{d=1}^{d_n} \pi_{nd} u_n(s_n, s_{-n}) \right]$$

Exercise *Some general results*

1. *Prove that a weakly dominant strategy is a best reply*

2. *Prove that every strategy that survives the principle of iterated dominance is a best reply to some strategy profile by the other players in the reduced game, and in the original game.*

3. *Show that a best response is one or more pure strategy, and that every mixed strategy is a best response only puts weight on those pure strategy best responses.*

Corporate Plans

One might argue that rationalizable chains with many links are less likely to occur than shorter chains. Consider the following game. Conflict within the corporation about many We consider a problem the company faces as it mulls over possible directions of expansion. In this game reducing staff is a dominated strategy for the accounting department, but whether the department prefers to decentralize or to consolidate depends on where new sales are pitched.:

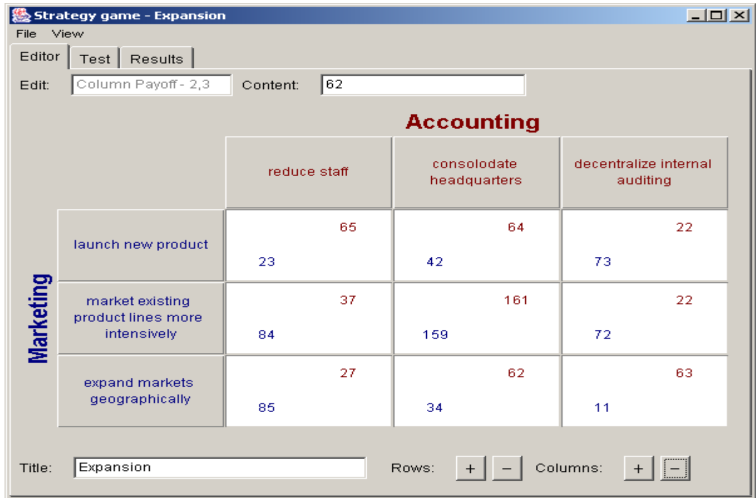


Figure 8.8
Corporate Plans

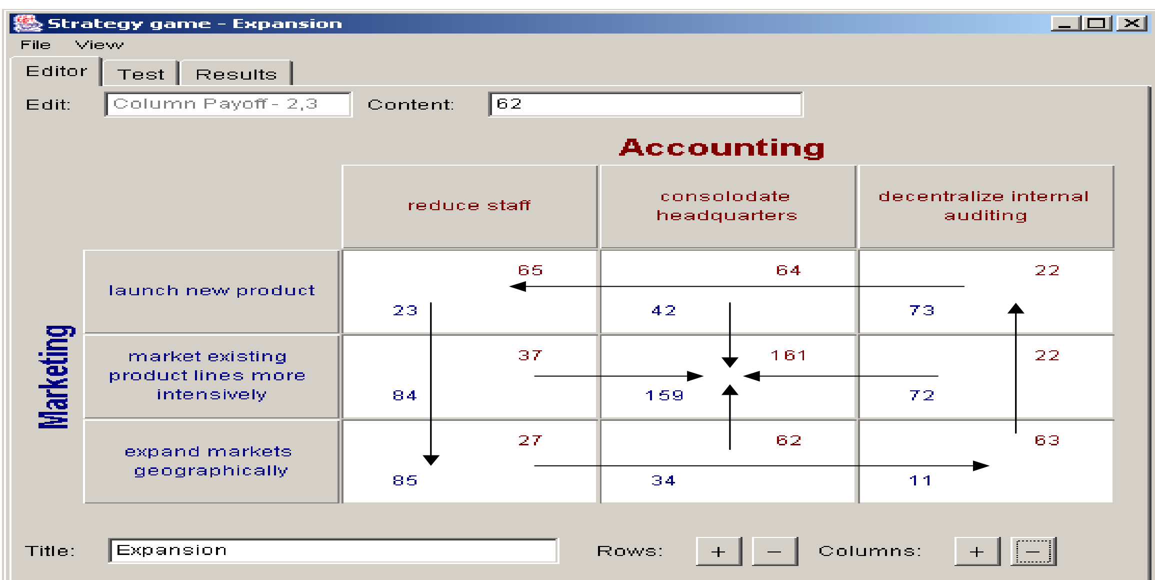


Figure 9.9
Best Replies in corporate plans

Patenting

Consider the following three player game.

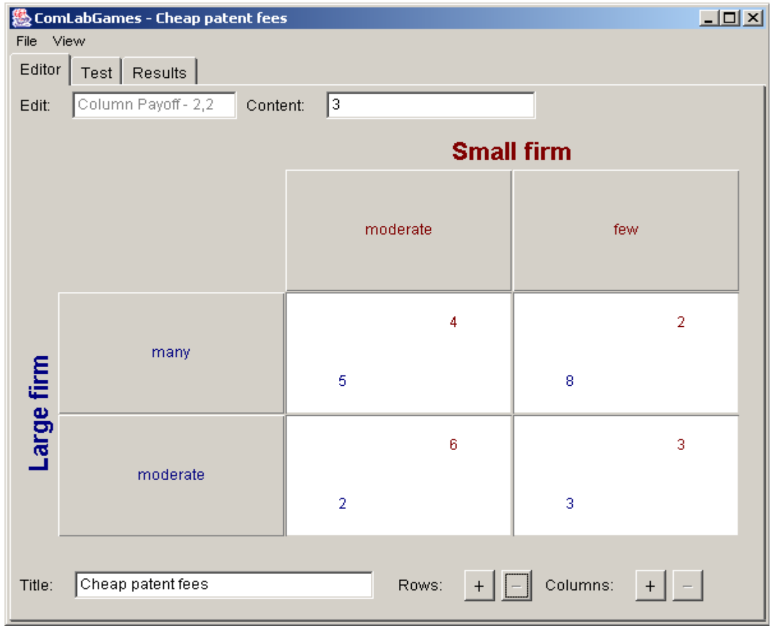


Figure 8.6
Cheap patenting fees

If the patent office charges expensive fees

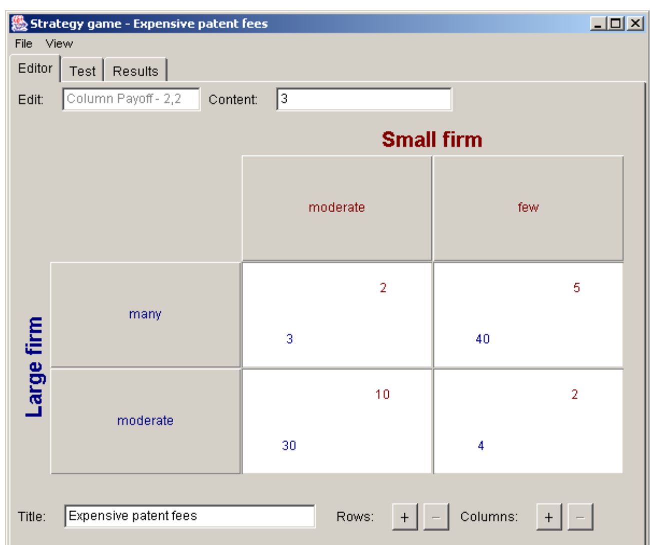


Figure 8.7
High patent fees

Formal definition

Using more basic terminology, consider a game for N players written in strategic form, where each player $n \in \{1, \ldots, N\}$ picks some strategy s_n from their choice set S_n. A strategy profile $s^* \equiv \{s_1^*, \ldots, s_N^*\}$ is a Nash equilibrium of the game if s_n^* is a best reply to $s_{-n}^* \equiv \{s_1^*, \ldots, s_{n-1}^*, s_{n+1}^*, \ldots, s_N^*\}$ for each $n \in \{1, \ldots, N\}$. Let $u_n(s)$ denote the expected utility of the game's outcome for player $n \in \{1, \ldots, N\}$ from the strategy profile

$s \equiv \{s_1, \ldots, s_N\}$. Thus a pure strategy Nash equilibrium s^* satisfies the inequalities

$$u_n(s^*) \geq u_n(s_1^*, \ldots, s_{n-1}^*, s_n, s_{n+1}^*, \ldots, s_N^*)$$

for all $n \in \{1, \ldots, N\}$ and $s_n \in S_n$.

Rule 5 If there is a unique Nash equilibrium, then play your own Nash equilibrium strategy.

This rule encompasses Rules 2 and 3 but not 1 and 4. Since 4 includes 1, 4 and 5 together encompass the first three. Use examples and show credibility issue again.

Exercise *Consider a two player game in which each player has two strategies to pick from. Suppose that none of the strategies are dominated. Show there is not a unique equilibrium in pure strategies.*

Mixed Strategy Equilibrium

There are two further limitations of the Nash equilibrium concept.. In contrast to less refined solution concepts, a Nash equilibrium in pure strategies might not exist. In two player games with a small number of strategies this is easy to check. There is however the second problem that like the refinements preceding it, there is no guarantee that a Nash equilibrium is unique. Unfortunately we cannot guarantee there is a unique Nash equilibrium in every simultaneous game.

Games lacking a Nash Equilibrium in Pure Strategies

Nor we cannot guarantee there is a Nash equilibrium in every simultaneous game, that is if players are restricted to picking a deterministic or pure strategy.

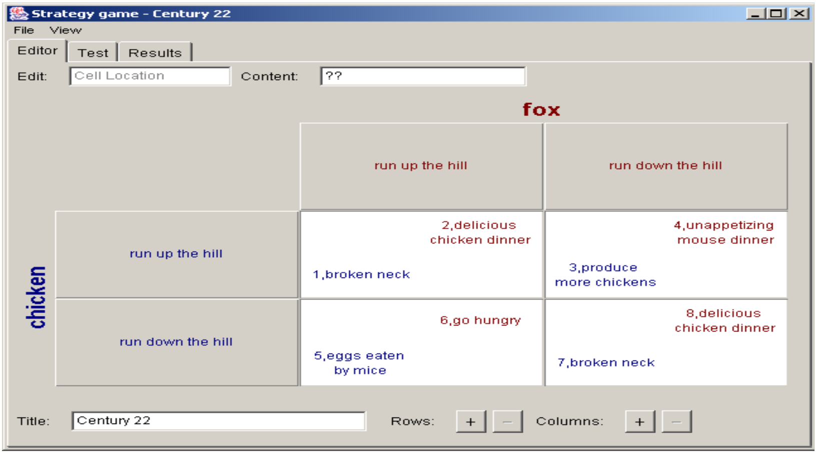


Figure 8.10

Predators and the quarry

Some games do not have any Nash equilibrium in pure strategies. For example neither player would make a predictable choice in the betting game of two up because the opponent would recognize this and win every round.

Exercise *Prove there is no pure strategy equilibrium in the parking meter game.*

More generally games modeling tax evasion, highway patrol, hunts and fights often do not support pure strategy Nash equilibrium profiles. In this case, instead of trying to predict the pure strategy profile chosen, it is more reasonable to predict the probabilities that each type of player assigns to the various strategies.

Pick on the feature that beats the competitor, ranking rotation

Off road capability, highway performance, comfort, price

This lecture begins with some examples to show that not every simultaneous game has an equilibrium in pure strategies. Then we show how to derive an equilibrium in mixed strategies. We state a theorem that states that every game supports at least on Nash equilibrium in pure and/or mixed strategies.

We conclude this topic with an example showing a game with 3 equilibrium, 2 with pure strategies and one with mixed strategies.

Matching pennies

Consider again the game of Matching Pennies we introduced in Chapter 3. For convenience the strategic from is displayed here as Figure 8.15, the arrows indicating the best responses to the other player's various choices. Following the chain of best responses, we see that if Player 1 plays Heads, Player 2 should play Heads, but if 2 plays Heads, 1 should play Tails. Therefore (Heads,Heads) is not a Nash equilibrium. Using a similar argument we can eliminate the other strategy profiles as being a Nash equilibrium, and simultaneously establish that the rationalizable set includes all the choices.

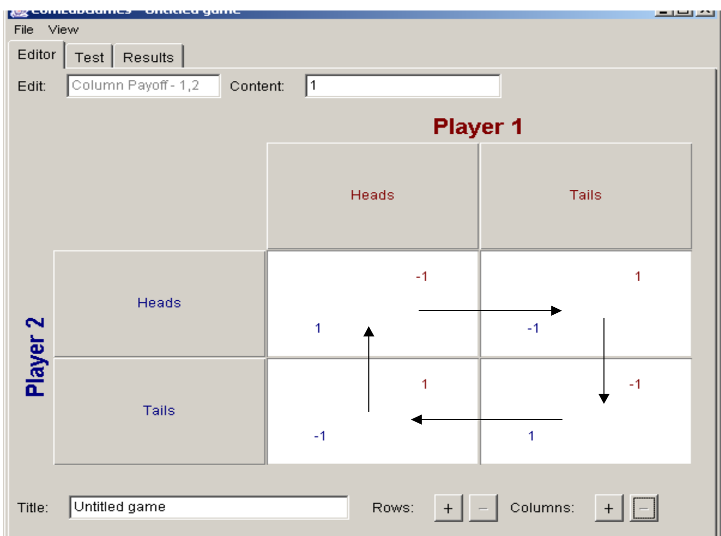


Figure 8.15
Matching pennies

What then is a solution of the game? To solve this game, we broaden our search by extending the strategy space to include mixed strategies. Supposing the first gambler plays Heads with probability π_1 and Tails with probability $(1 - \pi_1)$. We abbreviate by calling this strategy π_1. The second gambler payoff from playing Heads is then

$$1.\pi_1 + (-1)(1 - \pi_1) = 2\pi_1 - 1$$

while the payoff to the second gambler from playing Tails is

$$(-1)\pi_1 + 1.(1 - \pi_1) = 1 - 2\pi_1$$

Consequently, if the first gambler plays Heads with probability greater than 0.5, then the expected gain to the second gambler from playing Heads is positive, but negative from playing Tails. Therefore if $\pi_1 > 0.5$ the second gambler's best response to π_1 is Heads. Using a similar argument, if $\pi_1 < 0.5$ the second gambler's best response to π_1 is Tails. In both cases the expected payoff to the second gambler is strictly positive. Since this is a zero sum game, it now follows that if $\pi_1 > 0.5$ or $\pi_1 < 0.5$, then the first gambler expects to lose. To avoid expected losses incurred from the second gambler playing his best response, the first gambler must set $\pi_1 = 0.5$. In that case the second gambler cannot affect the expected payoffs regardless of his choices, and both gamblers expect to break even.

Since this game is symmetric this commentary applies to the second gambler. To avoid losses he must also mix Heads and Tails in equal proportions. This proves no equilibrium exists in which either player mixes in unequal proportions, or plays a pure strategy. But setting $\pi_1 = 0.5$ is one of many best response to $\pi_2 = 0.5$ and vice versa. We conclude that $(\pi_1, \pi_2) = (0.5, 0.5)$ is the unique Nash equilibrium to this game.

This equilibrium illustrates how a player introduces strategic uncertainty into the game by playing a mixed strategy, in order to make his moves unpredictable, and thus elude attempts by his rivals to pursue their own interest at the player's expense.

Exercise *Mixing versus Randomizing*

1. *To what degree are players are mixing strategies versus choosing randomized strategies.*

Taxation

Tax evasion and enforcement is another area where one might expect to observe both taxpayers and the collection agencies to choose mixed strategies. The following example seeks to explore some of the basic trade offs, and how they are reflected in the outcomes. Suppose a business recognizes that if it honestly reports is income over the tax period, it will be liable for $4 million. There are, however, two alternatives to truthful reporting. The business pays no tax if it fraudulently claims to have made no net income, and is not audited. If caught, it pays a penalty of 200 percent on its tax burden, and must pay $12 million in total. The second option is to make a plausible error that reduces but not eliminate the tax burden. This can be more easily detected by the collection agency but is only penalized by 100 percent of the tax burden. The accounting irregularity reduces taxes by $2 million. Thus the firm pays $6 million in total if audited or if its accounting arithmetic is checked. The objective of the collection agency is to maximize net revenue within its legislative charter. Checking for accounting regularities costs the collection agency $1 million, but conducting a full audit twice as expensive, $2 million. Figure 8.17displays the strategic form of the game, the arrows following our convention of indicating the best replies.

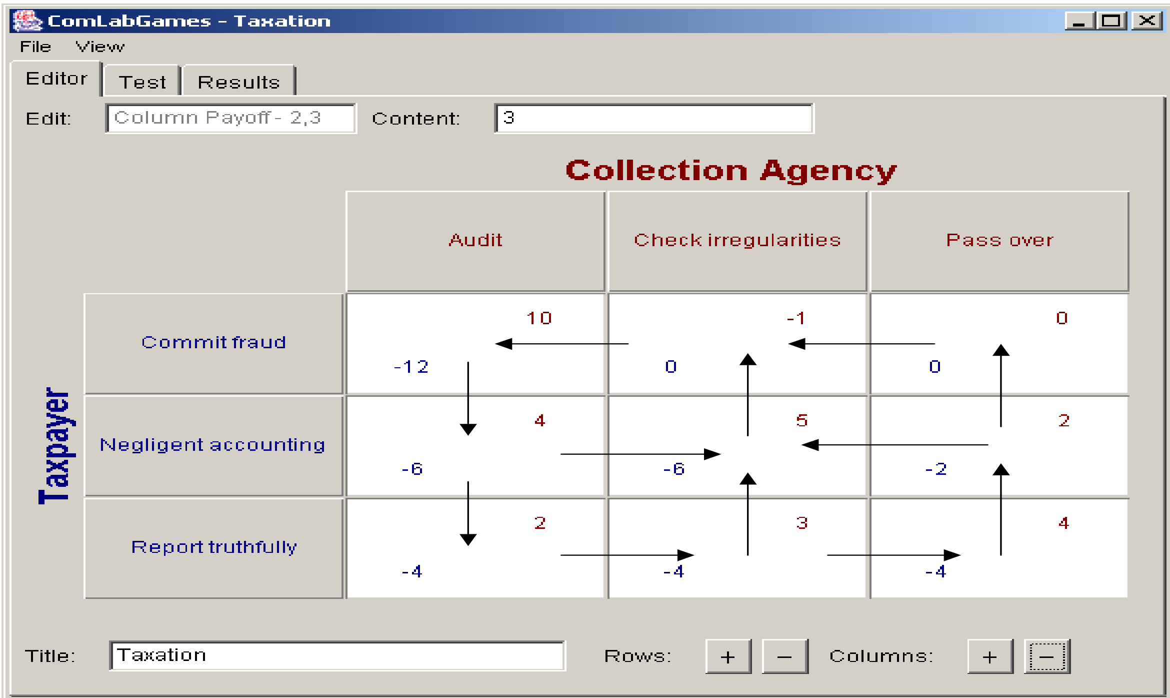


Figure 8.17
Taxation

There are no dominated strategies in this game. A visual inspection reveals that each element in the main diagonal of collection agency's payoff matrix is the maximum element in its column. Therefore every strategy available to the collection agency is a best reply to one of the taxpayer's strategies. Reporting truthfully is the best response to auditing, and committing fraud is the best reply to the other collection agency's strategies. To show that negligent accounting is not dominated, consider whether a mix of fraud versus truth telling might dominate negligent accounting, since by inspection negligent accounting is not dominated by either of the two pure strategies. Note that such a mix must place a probability of at least one half on fraud versus truth telling for the taxpayer to prefer the mix over negligent accounting, but the mix must be less than one quarter to yield a higher payoff if the collection agency audits. It is impossible to meet both conditions simultaneously, thus establishing that negligent accounting is not a dominated strategy.

Since this is a two person game, and there are no dominated strategies, it now follows from Theorem that all the strategies are rationalizable, and the arrows indicate the various thought sequences that each party might have about the other to justify any of their choice strategy choices. The configuration of the arrows shows there is no pure strategy Nash equilibrium either. This leaves the mixed strategy equilibria to investigate.

Much can be gleaned about the solution structure before mechanically using linear algebra to solve for the equilibrium mixing probabilities. To fix notation, let π_{11} denote the probability of the taxpayer committing fraud, π_{12} denote the probability of engaging in a negligent accounting practice, and π_{13} denote the probability of full disclosure. Similarly, denote by π_{21} the probability of an audit, π_{22} the probability of checking for an accounting irregularity, and π_{23} as the probability that the collection agency passes over the return. We also use a superscript to denote strategies chosen in equilibrium:

1. Note that if the agency does not audit at all and sets $\pi_{21} = 0$, then the taxpayer's dominant strategy is to commit fraud, meaning $\pi_{11} = 1$. From Figure 8.17, this is not an equilibrium strategy, because the unique best response to fraud is to audit. Therefore $\pi_{21}^{e} > 0$.

2. Committing fraud is also part of every equilibrium strategy. Otherwise auditing would be dominated by checking for accounting irregularities, not used in equilibrium, and thus contradicting our first point. Therefore $\pi_{11}^{e} > 0$.

3. If full disclosure is never practiced, and $\pi_{13} = 0$, then passing is dominated by auditing, and hence $\pi_{23} = 0$. By iterative dominance, this implies full disclosure dominates negligent accounting, thus contradicting the premise. Therefore full disclosure is practiced with strictly positive probability, or $\pi_{13}^{e} > 0$.

4. Suppose negligent accounting is not part of the support. Then checking irregularities is dominated by a mixed strategy of auditing and passing in which more than more than 50 percent is apportioned to passing. This would leave both players mixing between two strategies. In this case the collection agency must pass with probability $2/3$ to make the taxpayer indifferent between full disclosure and committing fraud. In this case negligent accounting dominates both strategies, contradiction the premise. Therefore $\pi_{12}^{e} > 0$.

5. The previous three remarks imply the taxpayer positively weights all three strategies. Consequently the taxpayer is indifferent between his three available strategies. Reviewing the payoffs in Figure 8.17, this can only be accomplished by placing strictly positive probabilities on all three strategies of the agency.

6. Since the solution to the two sets of three equations is unique, there is a unique mixed strategy equilibrium in which all outcomes have strictly positive probability.

Equating the expected utilities of the collecting agency

$$10\pi_{11} + 4\pi_{12} + 2(1 - \pi_{11} - \pi_{12}) = -\pi_{11} + 5\pi_{12} + 3(1 - \pi_{11} - \pi_{12})$$

and

$$10\pi_{11} + 4\pi_{12} + 2(1 - \pi_{11} - \pi_{12}) = 2\pi_{12} + 4(1 - \pi_{11} - \pi_{12})$$

Solving these equations in two unknowns we obtain the mixed strategy:

$$\pi_1^e \equiv \begin{pmatrix} \pi_{11}^e \\ \pi_{12}^e \\ \pi_{13}^e \end{pmatrix} = \begin{pmatrix} 1/12 \\ 1/4 \\ 2/3 \end{pmatrix}$$

Similarly the taxpayer only mixes between those pure strategies that yield the same expected utility. the taxpayer only mixes between all three strategies if:

$$-12\pi_{21} = -6\pi_{21} - 6\pi_{22} - 2(1 - \pi_{21} - \pi_{22})$$

and

$$-12\pi_{21} = -4$$

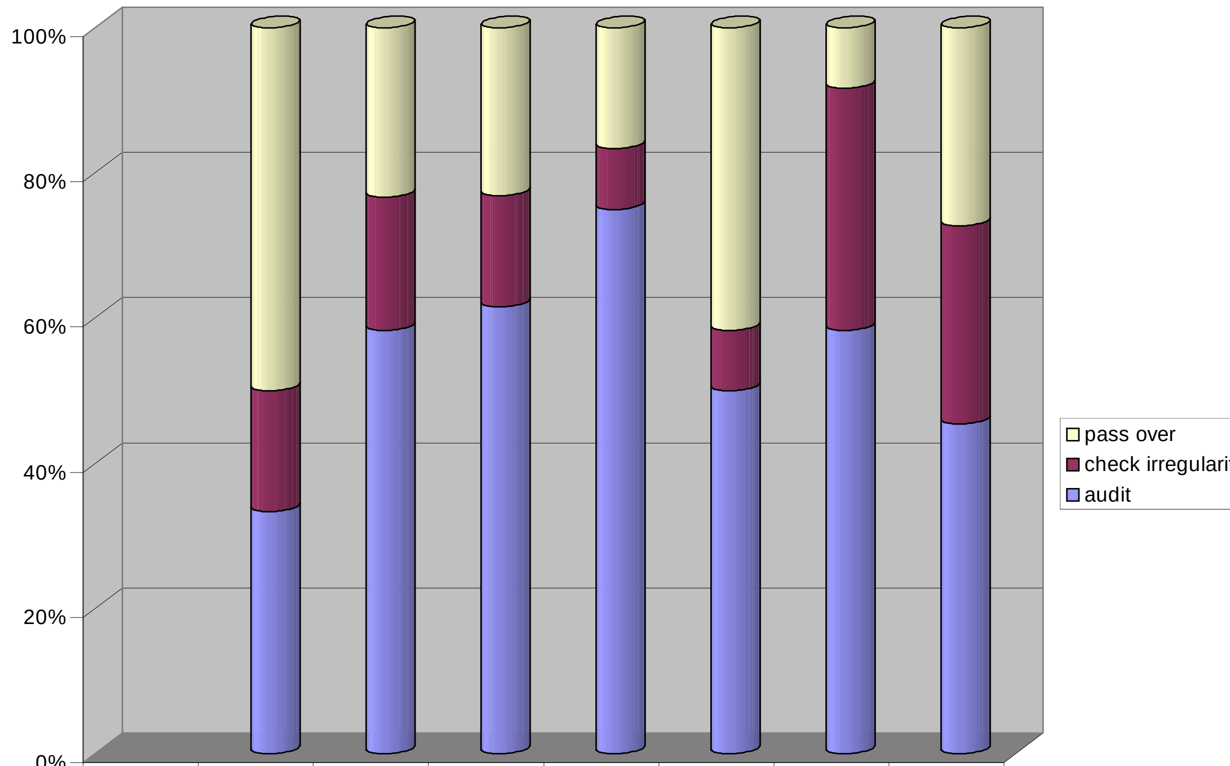
Defining the only strategy that leaves the taxpayer indifferent between all three choices is:

$$\pi_2^e \equiv \begin{pmatrix} \pi_{21}^e \\ \pi_{22}^e \\ \pi_{23}^e \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1/6 \\ 1/2 \end{pmatrix}$$

By construction (π_1^e, π_2^e) is a mixed strategy equilibrium. Although at $2/3$, full tax compliance is not overwhelming practice, the conviction rate for tax evasion is $11/72$, with taxes collected on $4 million owing are in total of which are due, plus penalties

Experimental results

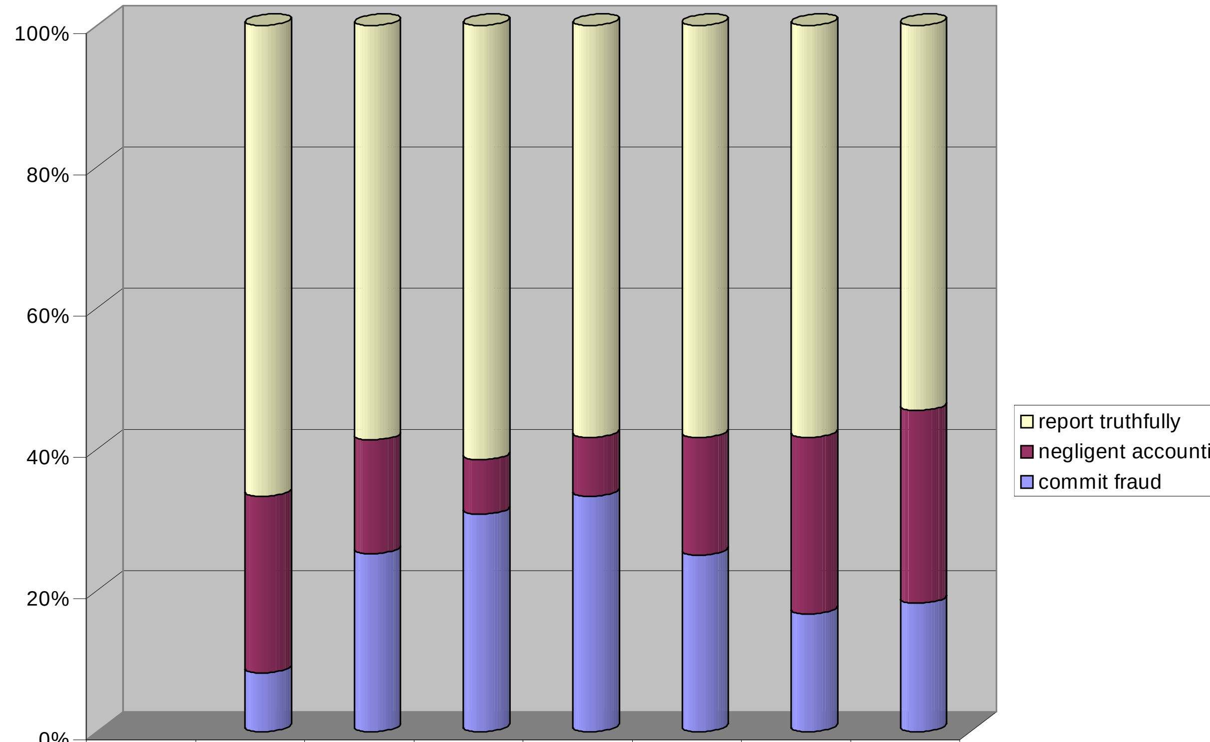
Figure 8.17a shows the theoretical probabilities that collection agency would choose and the empirical probabilities that were calculated by playing the experiment. Subjects selected audit on average 25 percent more than it is predicted by the theory. The χ^2 statistics reject the hypothesis at $\alpha = 0.05$ that subjects mixing in the proportion predicted by the theory. $\chi^2 = \frac{(35-60*0.3333)^2}{60*0.3333} + \frac{(11-60*0.1667)^2}{60*0.1667} + \frac{(14-60*0.5)^2}{60*0.5} = 19.89$ and the tabulated critical value of χ^2 is 5.991 with 2 degrees of freedom.



	theoretical prediction	all round together	round 1	round 2	round 3	round 4	round 5
pass over	50	23.33	23.08	16.67	41.67	8.33	27.27
check irregularities	16.67	18.33	15.38	8.33	8.33	33.33	27.27
audit	33.33	58.33	61.54	75	50	58.33	45.45

Figure 18.17a: Theoretical and empirical probabilities for collection agency

Similarly the calculations for the tax payer were performed.



		theoretical prediction	all round together	round 1	round 2	round 3	round 4	round 5
report truthfully		66.7	58.33	61.54	58.33	58.33	58.33	54.55
negligent accounting		25	16	7.69	8.33	16.67	25	27.27
commit fraud		8.3	25	30.77	33.33	25	16.67	18.18

Exercise *Taxation*

1. *Suppose the auditing costs of the collection agency double and the penalties for fraud are doubled in response. What is the new strategic form of the game.*

2. *Map in the best replies, and find the rationalizable strategies.*

3. *Solve for the new equilibrium.*

4. *How much is collected in the new equilibrium. What proportion are fines versus revenue? How much do total expenditure on the agency change?*

5. *Conduct experiments on both*

6. *Test the hypothesis that the frequencies are the same*

7. *Test the hypotheses that the preceded distribution from the theory*

8. *Do the proportions move in the way predicted by the theory?*

An Existence Result

For N player games

One can show that if a game does not support pure strategy equilibrium, then there is at least one mixed strategy equilibrium. In fact a stronger results holds: there always exists one or more Nash equilibrium (in pure and/or mixed strategies) where no weakly dominated strategy is played. For convenience we call such equilibrium solutions to the game, thus concluding our analysis of the strategic form's theoretical properties.

Multiple Equilibrium

An important result we state in the last chapter is that games that support an strategy that is arrived by iterated dominance is unique. Unfortunately there is an analogous result in the equilibirum. The existence of an equilibrium does not guarantee uniqueness.

Corporate Reorganization

Are pure strategy equilibrium more plausible than mixed strategy equilibrium?

consider the corporate game

Suppose we change the payoffs drastically reducing the middle payoffs, but not affecting the best replies

A mixed strategy appears, although the pure strategy remains

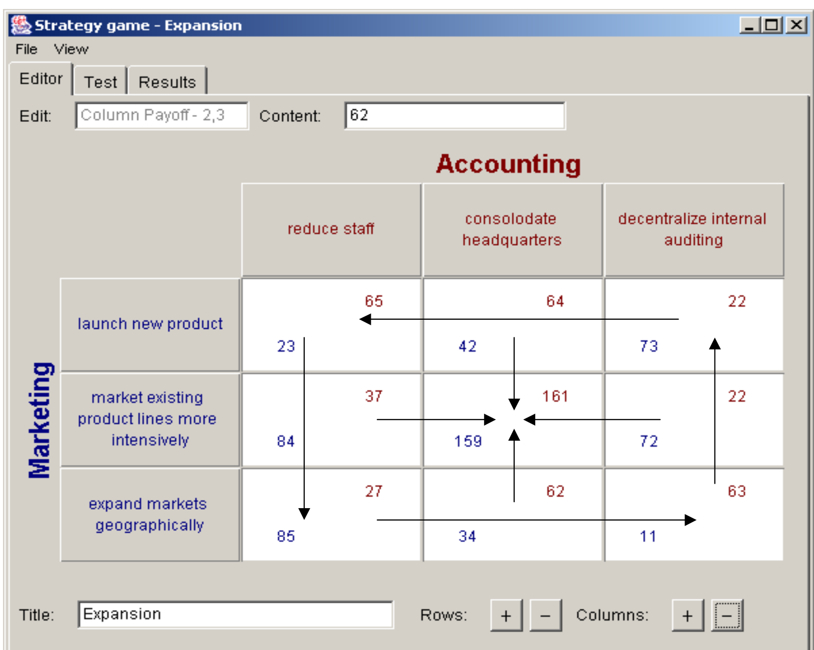


Figure 8.18

Revisions to corporate reorganization

Nor we cannot guarantee there is a Nash equilibrium in every simultaneous game, that is if players are restricted to picking a deterministic or pure strategy.

Exercise *Consider blind date. Find the mixed strategy equilibrium*

Entertainment

The following is a variation of the battle of the sexes. A couple enjoy the company of each other, but have different preferences over how they should spend it. We may be guilty of gender stereotyping by labeling the axis as theater and (ice) hockey.

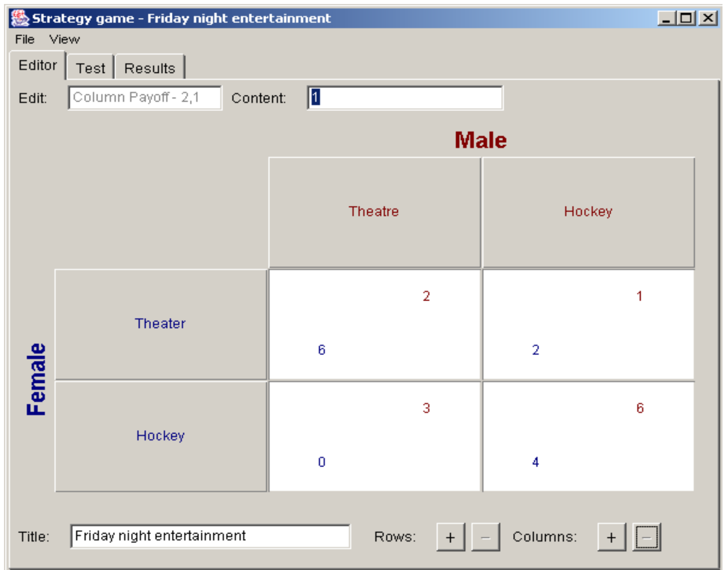


Figure 8.3
Friday Night Entertainment

Exercise *There are 2 pure strategy equilibrium in the Friday night entertainment game, as well as one mixed strategy equilibrium.*

1. *Find all three, and compare the payoffs to each partner in each equilibrium.*

2. *Is a pure strategy equilibrium more likely to occur. Play the game and compute the frequencies*

3. *If players have the opportunity to discuss the outcome beforehand is a Nash equilibrium more likely?*

Broadband

When firms overinvest in a new market, industry watchers are often critical of the naive expectation held by each firm in not properly anticipating the actions of its rivals. The recent collapse of the broadband cable laying market is just a recent illustration of this phenomenon. However the following analysis castes doubt on the hypothesis that such problems would not arise if the managers involved in these debacles were replaced by more intelligent and strategically savvy players. In the broadband game' we assume that if only firm enters, then the value of the firm will rise by $10 billion, but if two firms enter both will both suffer losses of $x billion, where x is a positive number less than 10.

There are two pure strategy equilibria in this game as indicated by the arrows in Figure 8.16 which show the best replies. In each pure strategy equilibrium one firm enters and the other stays out. An unsettling feature of both equilibria is why either firm would be comfortable being left out, and which equilibrium is likely to prevail. Notice that if one firm plays a pure strategy, then the best reply is a pure strategy, and it is unique. Therefore neither firm plays a pure strategy in equilibrium or both do.

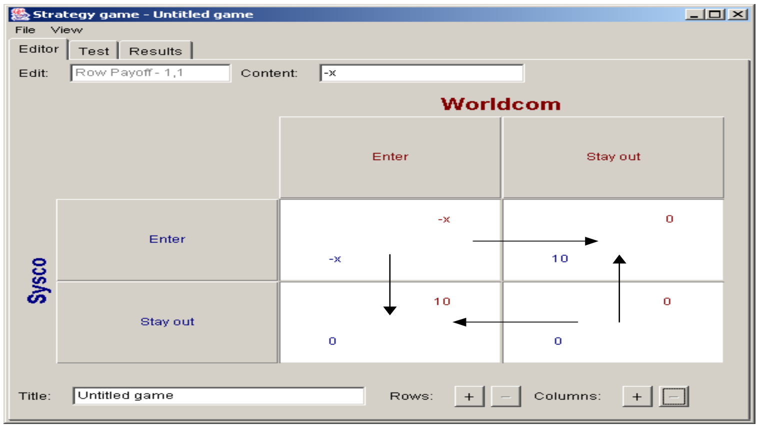


Figure 8.16
Broadband

This leads us to consider possibility that both firms play a mixed strategy in equilibrium. If Worldcom enters the new market with probability π_1, then the benefit from Sysco entering is

$$-\pi_1 x + (1-\pi_1)10 = 10 - (x+10)\pi_1$$

If Syco is playing a mixed strategy as well then it must be indifferent between entering and staying out. equating benefits calculated above with zero, we find that

$$\pi_1 = \frac{10}{(x+10)}$$

Since Worldcom solves an identical problem to Sysco, it follows that the probability of Worldcom entering is also π_1.

The main features of this mixed strategy equilibrium is that neither firm expects to increase its value from playing this game regardless of the value of x,, profit opportunities are sometimes not exploited at all (with probability $x^2/(x+10)^2$). With probability $100/(x+10)^2$ both firms suffer losses, and only with probability $2x/(x+10)^2$ would industry observers see the efficient number of firms entering. Thus the ratio of observing overexpansion to efficient expansion is $50/x$ in this game. If the losses are relatively small from overexpansion compared to the gains from establishing a monopoly, then overexpansion occurs quite frequently in this equilibrium.

A General Result

Theorem *Whenever there is more than one pure strategy equilibrium, there is at least one mixed strategy equilibrium as well.*

Nash equilibrium in perfect information games

So far we have only been discussing simultaneous move games

Entry Game

The two previous examples define best replies in simultaneous move games. In principle best replies can be defined for the strategic form of any game, but there is an important caveat to this statement. its usefulness and relevance is . which follows from the fact that the Since a best reply applies to the strategic form, it cannot capture the dynamic aspect that strategies are not binding commitments but merely detailed plans about choosing in all possible contingencies. The ramifications of this crucial difference are most evident in perfect foresight games

Accordingly, suppose an incumbent firm currently has a monopoly in a market, and at the initial node of the game, a rival chooses between end the monopoly and enter its market, or stay out. After the rival makes its decision, the incumbent firm chooses between producing less output to accommodate both firms and thus maintain the market price, much versus to produce the same output as before causing the price to fall sharply to prevent inventory accumulation. The extensive form of this entry game is depicted in Figure 8.7. Solving the game using backwards induction, the first rule of strategic play implies that the rival should enter and the incumbent should acquiesce.

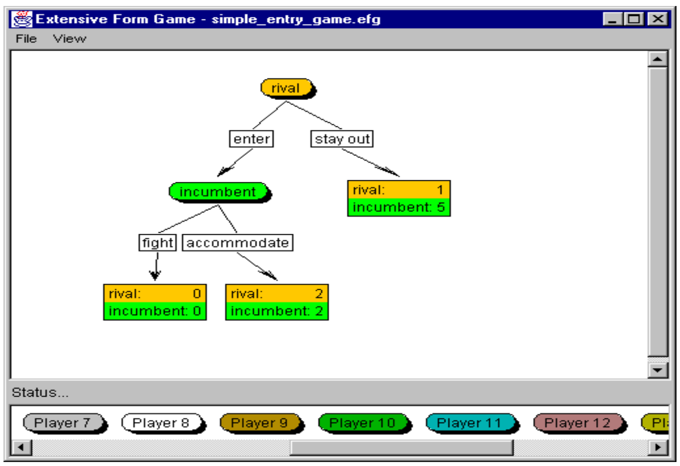


Figure 8.7
Entry Threat

There are two strategies for each firm. The rival chooses between enter and stay out, while the incumbent must fight or acquintense if the rival enters the industry. The strategic form of this game is shown in Figure 8.8. Noting that enter is a weakly dominant strategy, the backwards induction solution is redhead using the third rule of strategic play

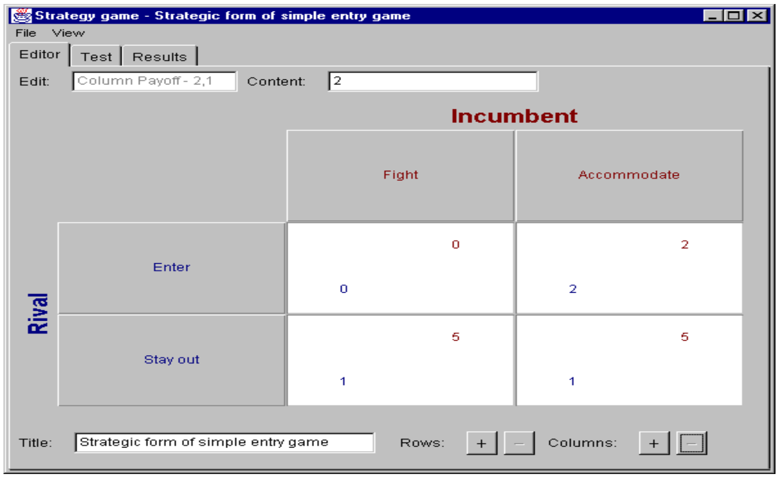


Figure 8.8
Strategic Form of Entry Game

How does this prescription correspond to our to our analysis of best replies this far? The best reply mapping for the rival is:

1. Enter if the incumbent accommodates.
2. Stay out if the incumbent fights.

The best reply correspondence for the incumbent is:

1. Accommodate if the rival enters.
2. Fight or accommodate if the rival stays out.

The best replies are illustrated in Figure 8.9, the horizontal dotted line indicating the incumbent's indifference between his own choices when the rival stays out.

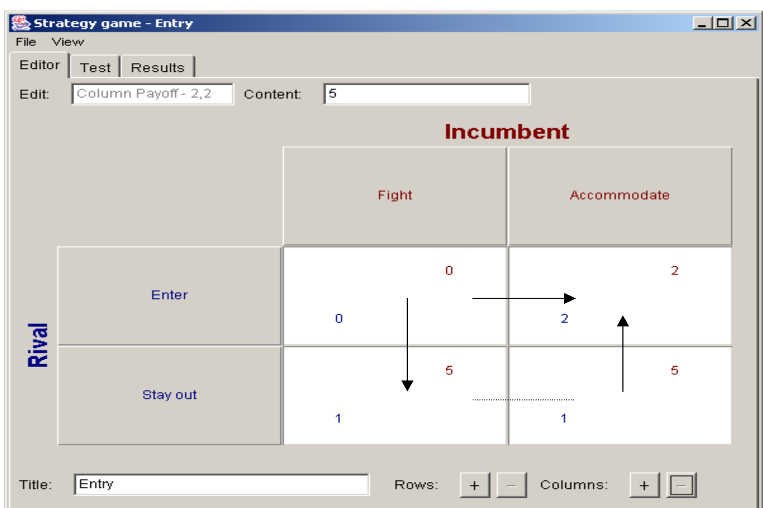


Figure 8.9
Best Replies in Entry

The strategic form shows there are two incentive compatible strategy profiles, namely (enter, accommodate) which is the solution to this game, and (stay out, fight) which is not a solution. This allocation corresponds to the incumbent committing himself to fighting should the entrant come in. Yet the only way such commitment can be made is if the the game is a simultaneous move game, in which the incumbent chooses his strategy without knowing what the rival has chosen. In this case the extensive from would look like Figure 8.10.

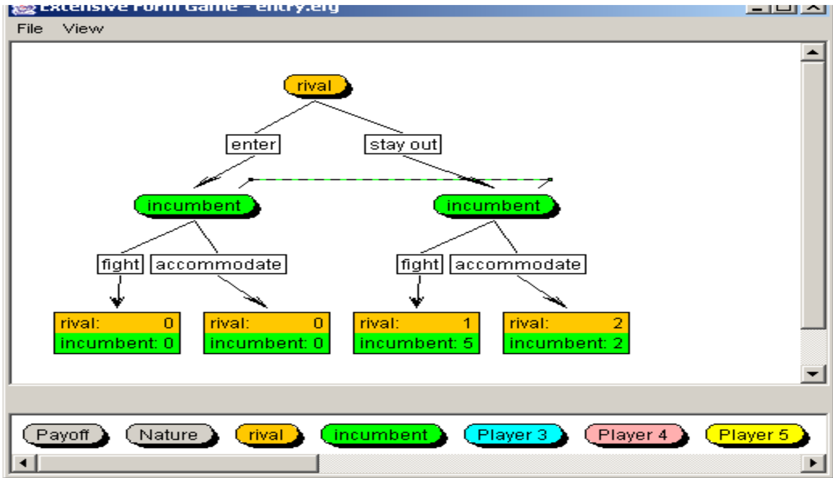


Figure 8.10
Simultaneous Entry Game

The fact that the concept of a best reply applies to the strategic form of the game and not its extensive form limits its usefulness. Unless one knows that the game is a simultaneous game, rival should question whether the strategy of fight is truly a best reply.

Summary

These examples demonstrate that after iteratively removing dominated strategies, some games do not yield a unique strategy profile. In order to provide sharper predictions about the outcomes of games, more controversial criteria must be developed and applied. We showed that in two of the examples discussed above a subset of the strategy profiles were self enforcing agreements, whereas the strategy profiles were not. In one of the examples the self enforcing agreement was unique, in the second example two strategy profiles had this property, while in the third example none of the strategy profiles were self enforcing agreements. It immediately follows that depending on the rules of the game, or more broadly the organizational structure and the nature of rivalry between different organization, self enforcing agreements may not exist or in the case of nonuniquenes do not fully resolve how the game will be played. In this important respect the notion of a self enforcing agreement is deficient.

There is a further criticism. In our discussion of backwards induction and dominance, we noted that these principles could be applied unilaterally, and did not require any centralized decision making. A precondition for an agreement, on the other hand, is communication between players. It is worth asking whether one can go further than the notion of dominance in seeking more precise predictions about the outcomes of games without appealing to such a centralist notion as a self enforcing agreement.

This chapter develops both these issues, existence and uniqueness of , and the decentralization, in more depth. At the heart of this development is the notion of a best response. This chapter elaborates on criteria for such games, and explains how they are related to dominance. defines and exposits Nash equilibrium. Therefore a stronger notion of rationality is required to differentiate between the remaining ones and thus generate sharper predictions. This remark motivates the definition of a best reply. This concept embodies the idea that the strategy each player picks might be optimal given that player's beliefs about the strategies all the other players are picking. Best responses are the building blocks for successive refinements to iterative dominance, namely rationalizable strategy profiles, Nash equilibrium, and discarding Nash equilibrium that contain any weakly dominated strategies. The chapter works through each of these concepts in turn, illustrating them with examples from business strategy at each stage of refinement.