Introduction

Iteratively removing strictly dominated strategies, selecting weakly dominant strategies, and then deriving the Nash equilibria for the remaining players in an appropriately defined reduced game, are principles that can be applied to predict the probability distribution for the outcomes of any game. This is because dominance and equilibrium concepts can be applied to the strategic form of any game. Putting this remark another way, although the previous two chapters only applied the techniques of dominance and equilibrium to simultaneous games, every game is strategically equivalent to a simultaneous move game.

There is, however, good reason for resisting the temptation to mechanically apply solution techniques developed for the strategic form to sequential games: The strategic form does not account for the timing of different moves. Timing has an important impact on how a player perceives the responses of the other players. We have already seen this in games of perfect information in Chapter 3, but this point holds more generally in sequential games with imperfect information. Within perfect information games the backwards induction rule shows how rational players anticipate future moves by their rivals and colleagues, but as we have seen, this technique cannot be applied directly in imperfect information games. Can backwards induction help to solve any sequential game with imperfect information? Is is possible to combine backwards induction with dominance and equilibrium principles to solve a bigger class of games than perfect information and simultaneous move games?

The purpose of this chapter is to restore the role of timing in determining strategic behavior, by combining all the principles we have developed in the previous chapters, to solve games of complete information. Thus the answer to both questions is "yes", and within the context of complete information games, this chapter explains how. Our explanation pivots around special decision nodes, which have two defining properties. First, they are information set singletons. This means that no other decision node is contained in the same information set, and implies that the player assigned to the node knows exactly what has happened in the game up until then. The second property is that every successor node belongs to information sets containing only other successor nodes. Hence everyone who makes a move after the special node knows the history of the game at least up until that point. Both properties are trivially satisfied by every decision node in perfect information games.

A decision node with these two properties can be interpreted as the initial node to a game that follows the structure of the original game from then onwards. The new game induced by the original game is called a subgame of the original game. The nodes and the branches of the subgame are the special node, and its successor nodes and branches as defined in original game. The players in the subgame are those players in the original game who have been assigned to make moves in the nodes defined by the subgame. Their information sets are defined as in the original

game. Finally the terminal nodes define the payoffs for the players in the subgame.

The significance of subgames is that they can be solved independently. When the solutions to all the subgames correspond to the solution of the whole game, we say the solution to the whole game is subgame perfect. This means that as the original game progresses, successively reaching initial nodes of smaller and smaller subgames, the players do not revise, but choose to implement, the contingent plans they made at the beginning of the game. Thus the principle of backwards induction applies subgame perfection to games of perfect information. Our approach is to make subgame perfection a requirement for every solution to a sequential game, in this way extending the principle of backwards induction to all sequential games.

The shortest subgames in a complete information game are either simultaneous games or single agent games with one decision node. Clearly the latter type of game can be solved, and the techniques developed in Part 3 of this book can be used to solve the formed type. Upon solving the shortest subgames, we substitute a terminal node for the initial node of the subgame with the solution payoffs to the subgame, analogous to the foldback procedure employed in the backwards induction algorithm. This procedure defines a reduced game, which can be solved recursively.

Having established a solution algorithm for solving games of complete information, the last sections of this chapter explore some topics in the theory of complete in formation games. First we show how information shouldbe viewed very differently than in adecsion theoretic context.

Finally we return to our claim at the beginning of this section, that ignoring the timing of moves in a game, and focusing exclusively on the strategic form solution, is unwise. By way of example we illustrate some pitfalls that one encounters by deviating from our suggested approach. In particular we remark on the close similarities between weak dominance and backwards induction, a similarity that might lead some to argue that the concept of subgame perfection is subsumed by a more aggressive application of dominance and equilibrium. Our examples show why this is not advisable, and that attempts to minimize or eliminate the important role of timing in games may yield misleading predictions.

Subgame perfection

We begin our discussion by focusing on two player games in which the first player moves only once. For example an employer might set a pay schedule, and then let the employee determine how the job is done. the president may set a command decision before moving on to other pressing matters of state and leave the implementation details to his staff. A pioneering firm with a large research and development division makes a decision about introducing a new product into the market, and then a firm that specializes in low production costs imitates the new service decides whether to enter the market.

Industrial Dispute

A dispute between an employer and employees (or thier trade union), usually about wages or conditions of working. Games of diplomacy have a familiar ring to them. Officials representing one country threaten, insult, another country or are at least of accused of doing this. The insulted country retaliates in a measured way that creates furhtr onfllict arather than leading to a resolution. This leads to furhter retaliation, until at some point the attention of the media is fully engaged, and many people proclaim that both parties are on teh edge of an abyss, straing as mutual destruction or something almost as bad. At that point the conflict , the sequence of confrontations seemingly disolve and a previous detnete is fully restored. Whether it is the Cuban crisis between the U.S.A. and the now defunct Soviet Union, the episodic violence between the Israelis and Palestinians in the Middle East, Indians and Pakistanis wrangling over Kashmir, or China and Taiwan arguing about the sovereignty of the latter, there is a familiar pattern of rhetoric and violence, that characterizes each of these conflicts.

The following example draws from a recent dispute between an Australian coal company and the union. The company is the world's largest private mining company. It has coal mines in the USA, Australia and Indonesia, with its main export mines which supply Asian markets being based in Australia. In the last two years the company has had a number of very serious industrial disputes with its employees in the Australian coal industry. This has resulted in considerable lost production. The most recent major dispute has been at the company's major mine which is a 5.5 million tonne per year operation with approximately 550 employees. The union has been seeking to achive a collective agreement at the mine for almost two years. However, the company has not been interested in a collective agreement. After almost two years of bargaining, the company started to offer individual employment contracts which included a pay rise of around 5%. It hoped that workers who were frustrated with the lack of a pay rise in two years would take up on the offer. The offer requires the individual to give up their right to be represented by the union in negotiations over pay and working conditions, and to work as directed by the company all the times. Company believed it would win 10% of the workforce to non-union individual contracts immediately, and then the majority of the workforce withing a few weeks. The union had a choice to ignore the company's action with respect to individual contracts or to initiate a strike to achieve a collective agreement quickly and to stop gradual defections from the union. Under Australian industrial law the union is allowed to strike indefinitely (and the company to lock out workers) during collective bargaining. The union sustained the striking workforce through the payment of strike pay which was financed by a levy of US$7 per week on all other members in the coal mining industry. The workers returned to work after the company agreed to a set of intensive bargaining talks. While precise levels of utility are hard to caculate for these kinds of

scenarios, it is relatively straightforward to infer what the rankings of orderings of the payoffs, and that is illustrated in Figure 10.1.

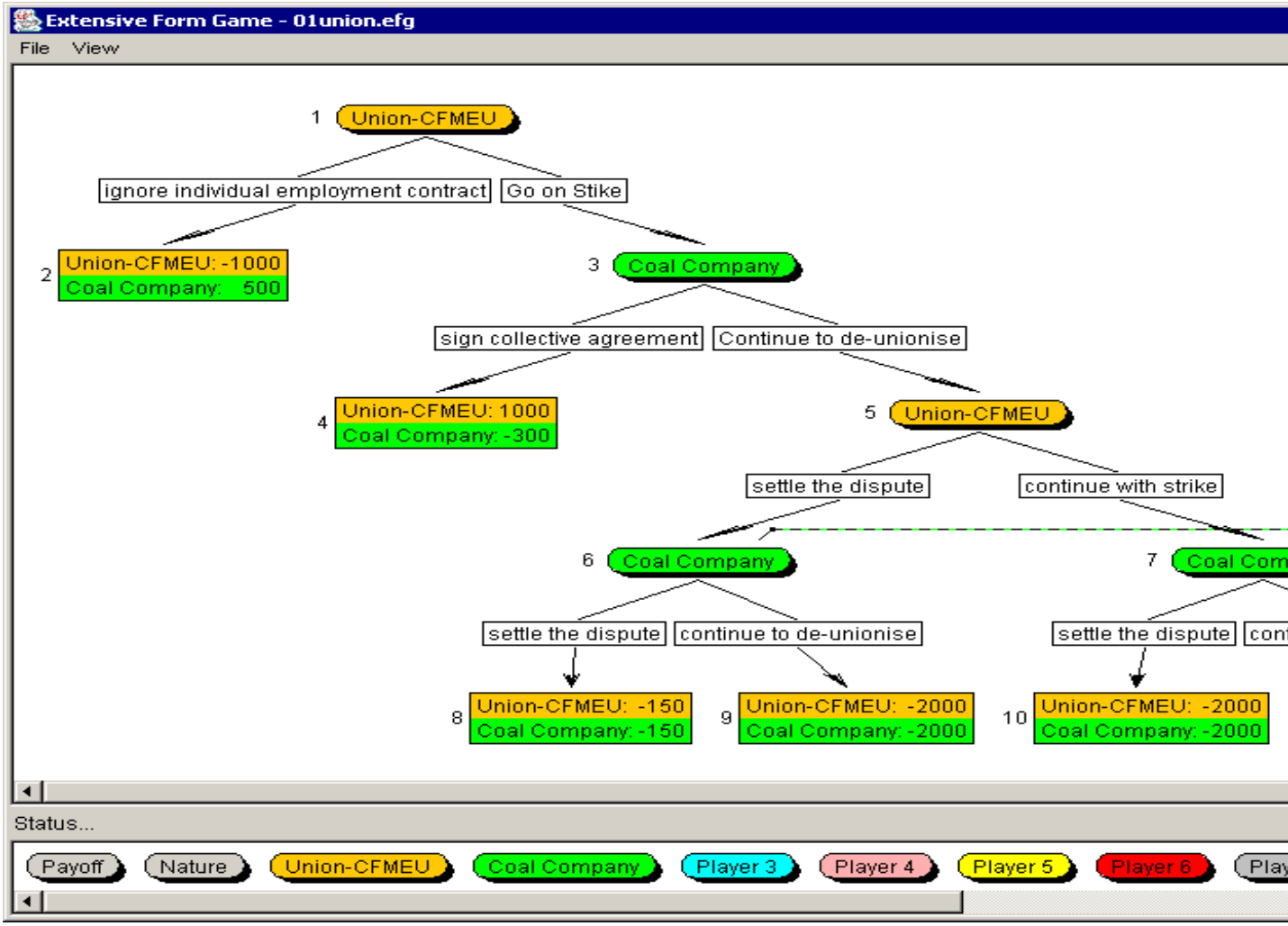


Figure 10.1
Industrial dispute

Since this is not a perfect information game, one's first instinct might be search for a strategic solution.The strategic form of the industrial dispute

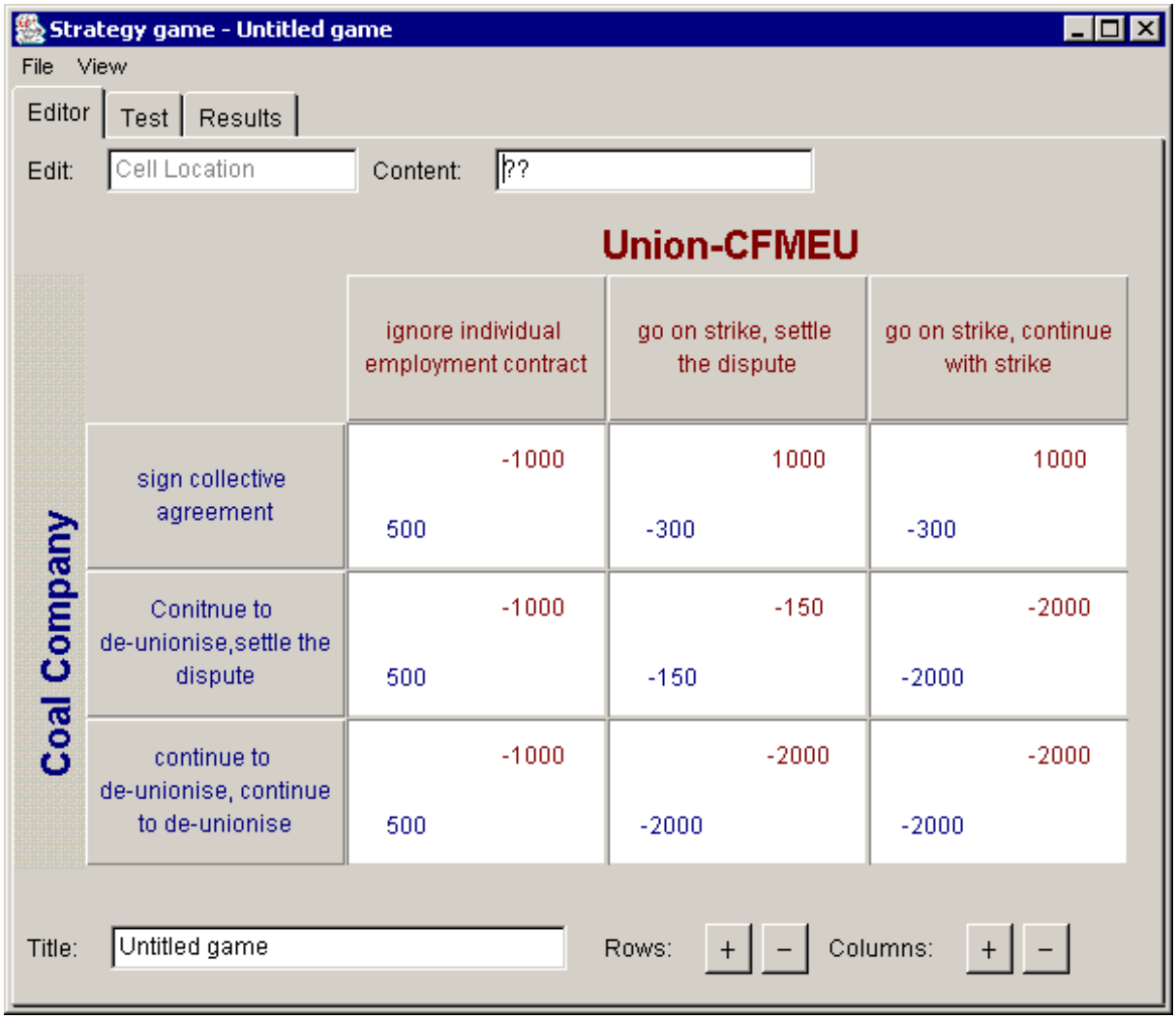


Figure 10.4
the strategic form

There are many Nash equilibrium for this game such as the pure strategy profiles:
(Sign collective agreement; go on strike, continue with strike)
(Continue to de-unionise, settle the dispute; go on strike, settle the dispute)
(Continew to de-unionise, continue to de-unionise; ignore individaul employment contract).

Notice continew to de-unionise, continue to de-unionise is weakly dominated by Sign collective agreement, and go on strike, settle the dispute weakly dominates go on strike, continue with strike. This leaves only (Continue to de-unionise, settle the dispute; go on strike, settle the dispute) as the unique strategic form solution.

In the industrial dispute example, the strategic form solution embodies elements of the backwards induction approach even though this example is not a perfect

information game. To explain this point we define a subgame as a game which starts from any singleton information set within the original game.

Sub-games in the industrial dispute example. The two proper sub-games in industrial dispute are:

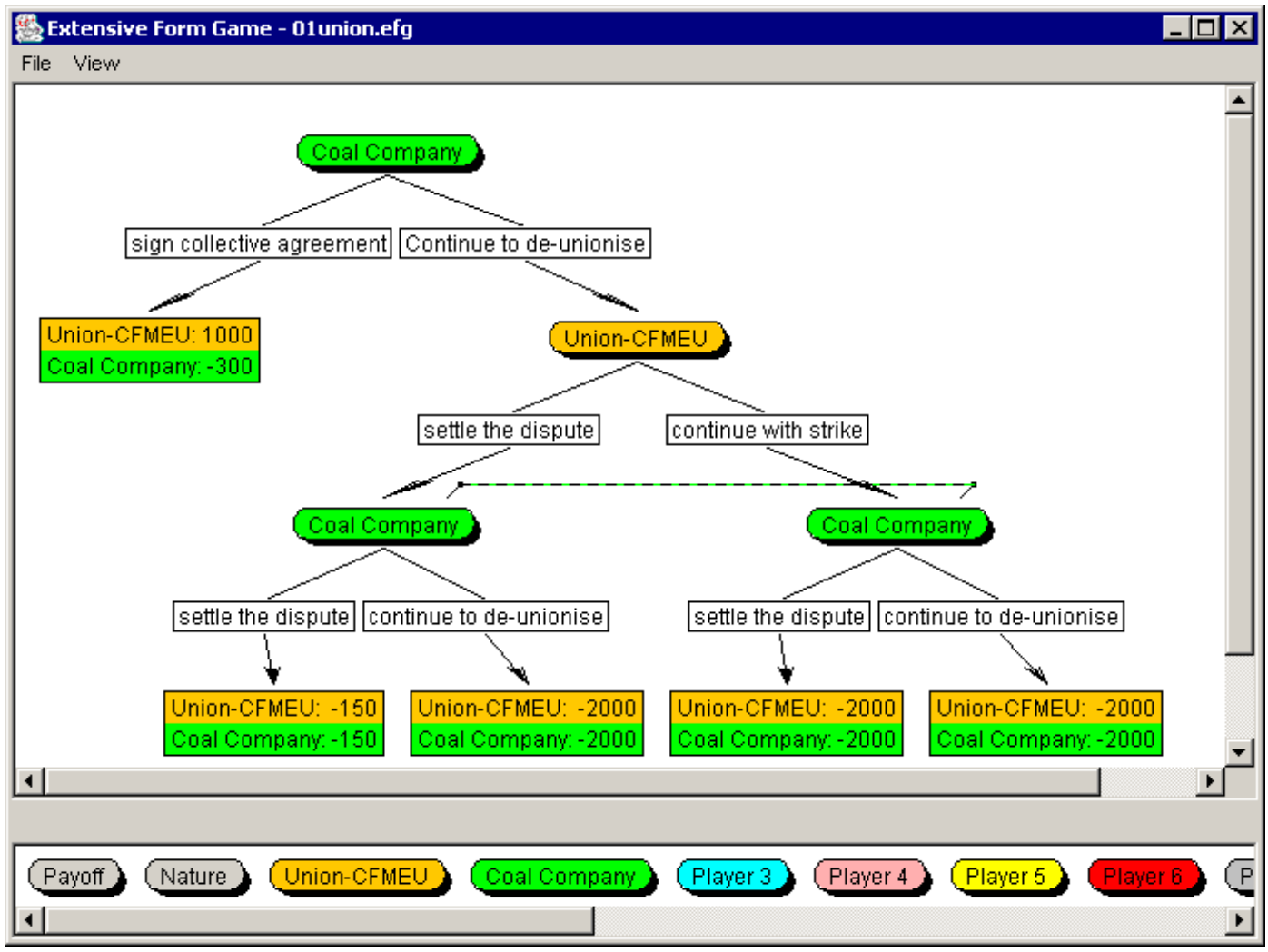


Figure 10.5
A subgame beginning with Coal Company deciding between agreeing ot union demands to continue to de-unionise

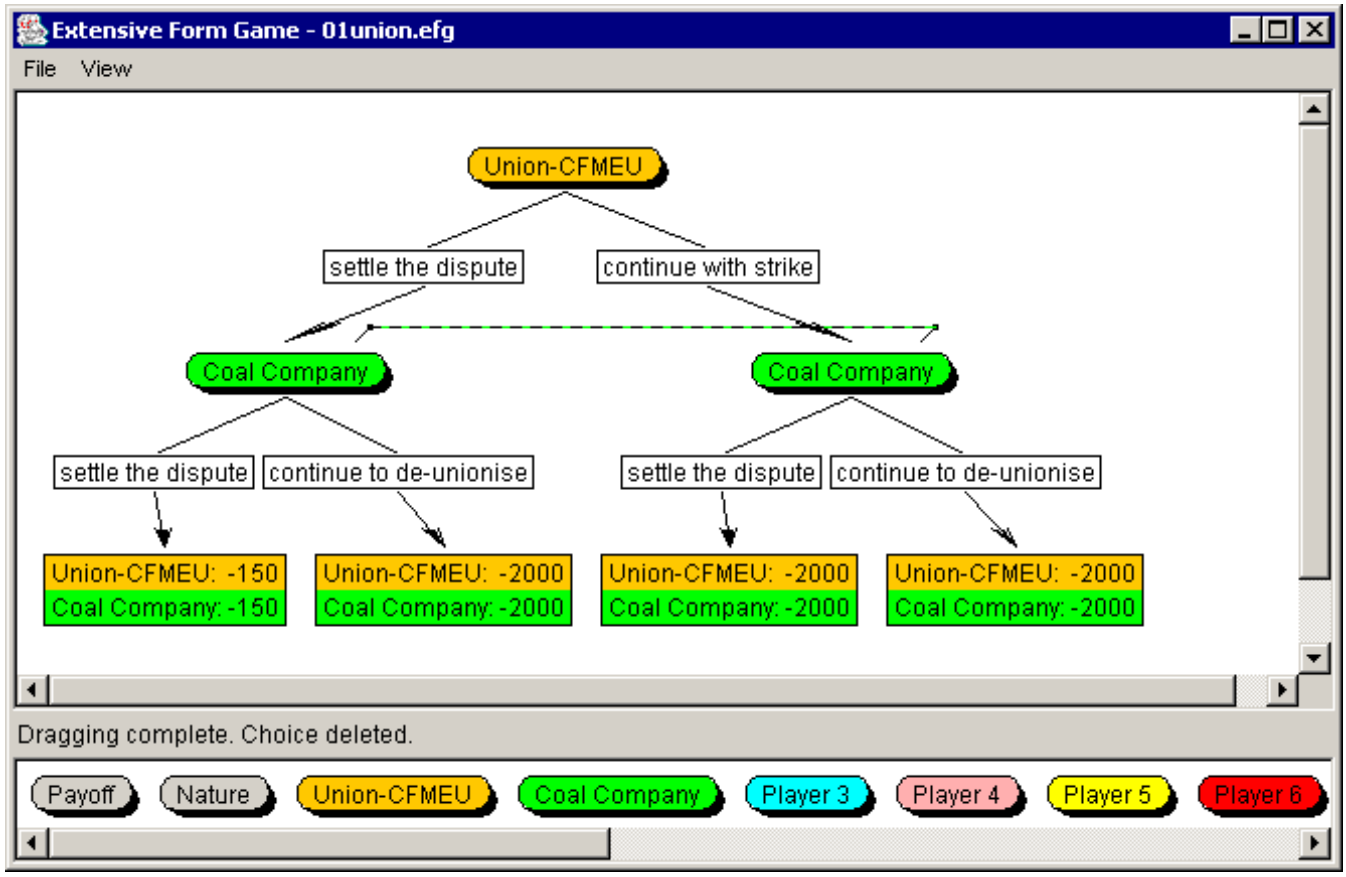


Figure 10.6
Another proper subgame

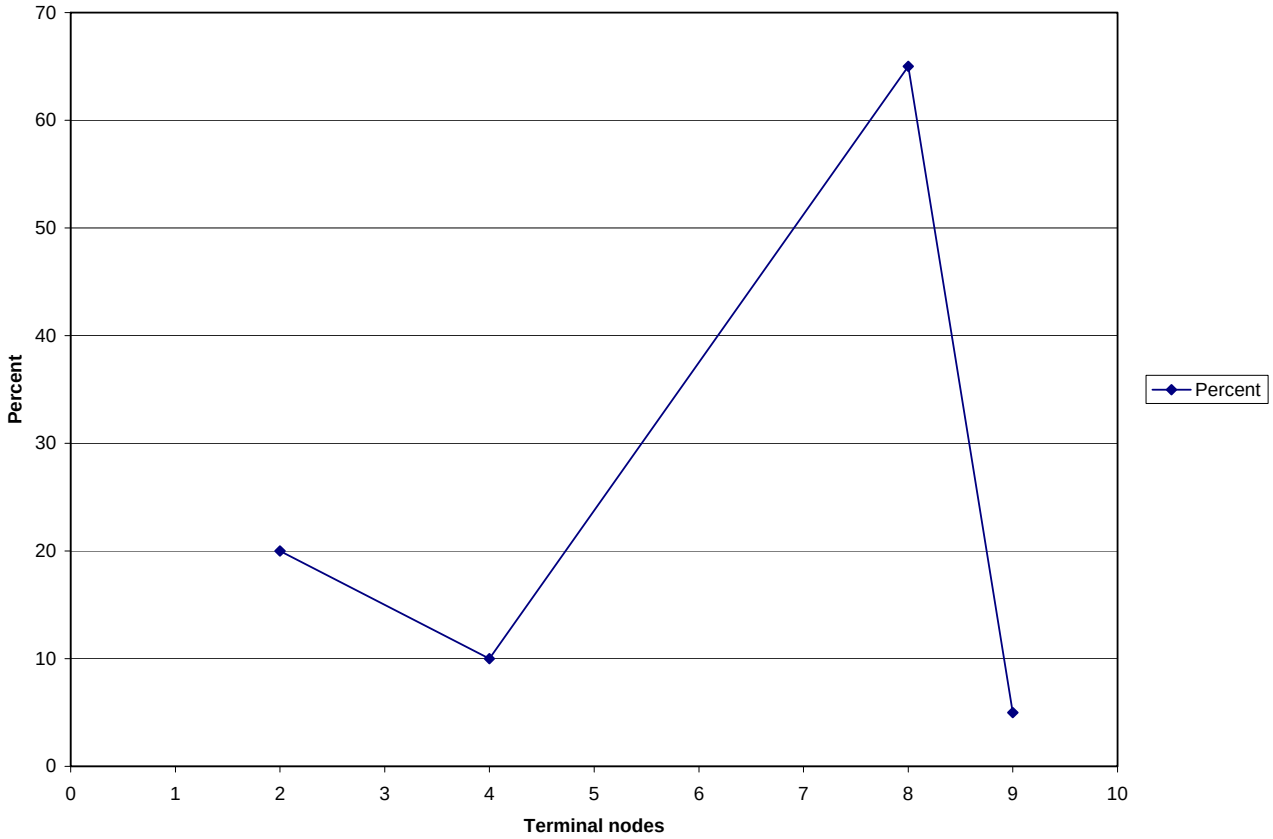
Sub-games – industrial dispute example: The final reduced form game

Figure 10.7
Reduced game

The unique strategic form solution to the Reduced form for the first sub-game is go on strike. Forming the reduced form to the original game, or just applying the backwards induction principle directly, the optimal move for the company is also to continue to de-unionise. This solution reflects a notion of credibility: the company knows that Union will go on strike in response to its own escalation, but that both will back down at the point of mutually assured destruction.

Experiment

Twenty subjects participated in the extensive form representation of the industrial dispute game that was played only once and subjects received detailed instructions how the game will be played. Subjects played the predicted outcome (node 8) 65 percent of the time in the only round they played. Four times they players who were assigned to be Union-CFMEU decided to ignore the individual contract 10 percent of the time.



Vertical Integration

We have already analyzed perfect information games, in which the second player to move knows exactly what the first player has done, and simultaneous move games,

in which the second player has no information about the first player's move. But there are intermediate cases that are of relevance as well. One possibility is that the second player has the opportunity to move at the same time as the fist but also retains an option to wait and see. For example a component supplier for a manufacturer that assembles

scale of a run versus the inventory costs easier to mark the part on a long run rather than produce supply on a

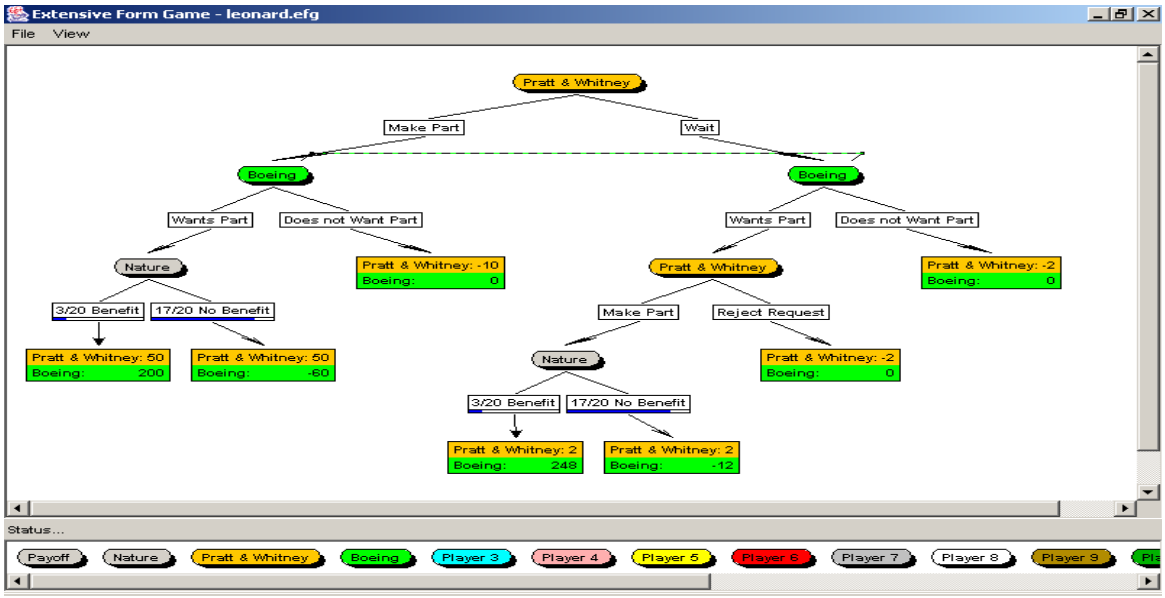


Figure 10.1
Flexibility

This game also lends itself to the foldback procedure used to solve the dispute game. The first step is to average over the payoffs in the terminal nodes that occur after the demand for the parts is revealed. If Boeing orders the part at Node 4, the expected net benefit to Pratt and Whitney and Boeing is respectively:

$$\frac{3}{20}50 + \frac{17}{20}50 = 50$$
$$\frac{3}{20}200 - \frac{17}{20}60 = 9 \qquad \#$$

while if Pratt and Whitney makes the part at Node 6, the net benefits to the two players are

$$\frac{3}{20}2 + \frac{17}{20}2 = 2$$
$$\frac{3}{20}248 - \frac{17}{20}12 = ? \qquad \#$$

Folding back the demand uncertainty we obtain a reduced game whose extensive form displayed in Figure 10.2.

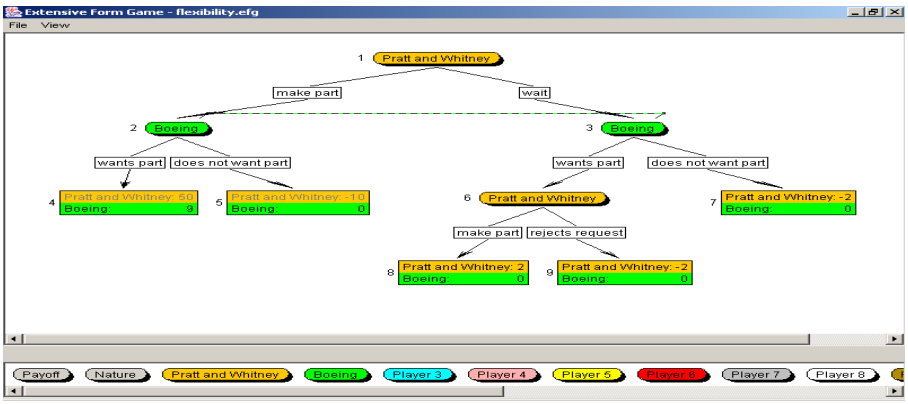


Figure 10.2
Reduced game for component supplier

At Node 6 we see that Pratt and Whitney would prefer to make the part rather than turn down Boeing's request. Folding back the payoffs of Node 8 into Node 6, we are left with a simultaneous move game to solve, which Figure 10.3 shows in strategic form.

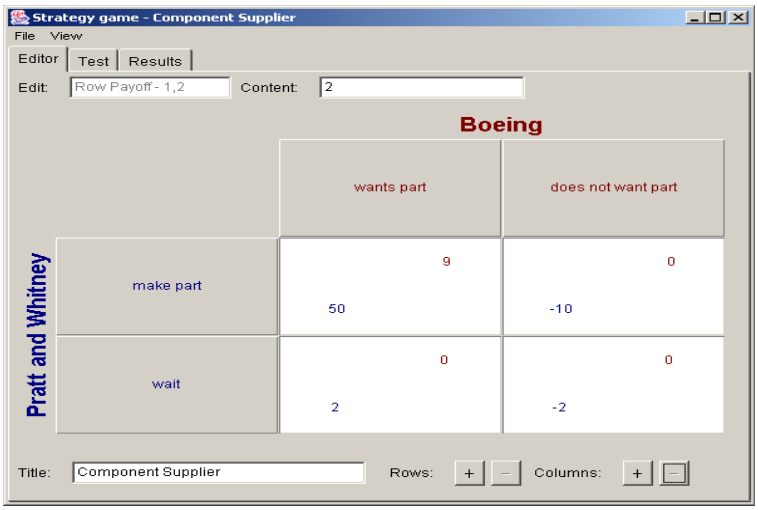


Figure 10.3
Strategic Form of Reduced Game

It is apparent from reviewing Boeing's payoffs that wanting the part is their dominant strategy. Applying the principle of removing strictly dominated strategies, Pratt and Whitney should eliminate Boeing's other strategy from consideration, and make the part, since $50 > 2$.

Strategic Partner

The previous example illustrated one aspect of how companies might benefit from coordinating their activities without fully integrating themselves through a merger or takeover. Several airlines coordinate their flight routes and ticketing to offer a more service to passengers using several of their domains on an extended trip taking them over . These arrangements between companies may take on a more formal character,

or alternatively might be based on informal understanding that is not fully articulated in any written agreement. In the following example we model a large company approaching a smaller company seeking to capitalize on its joint activities. The advantage of an informal partnership is its flexibility, that provides both companies with opportunities to increase profit. By way of comparison companies sometimes see the mutual value from coordinating their activities without directly forming a partnership. Sometimes a second mover has partial information about what a previous player has chosen. Figure 10.5 displays the extensive form of the strategic alliance game.

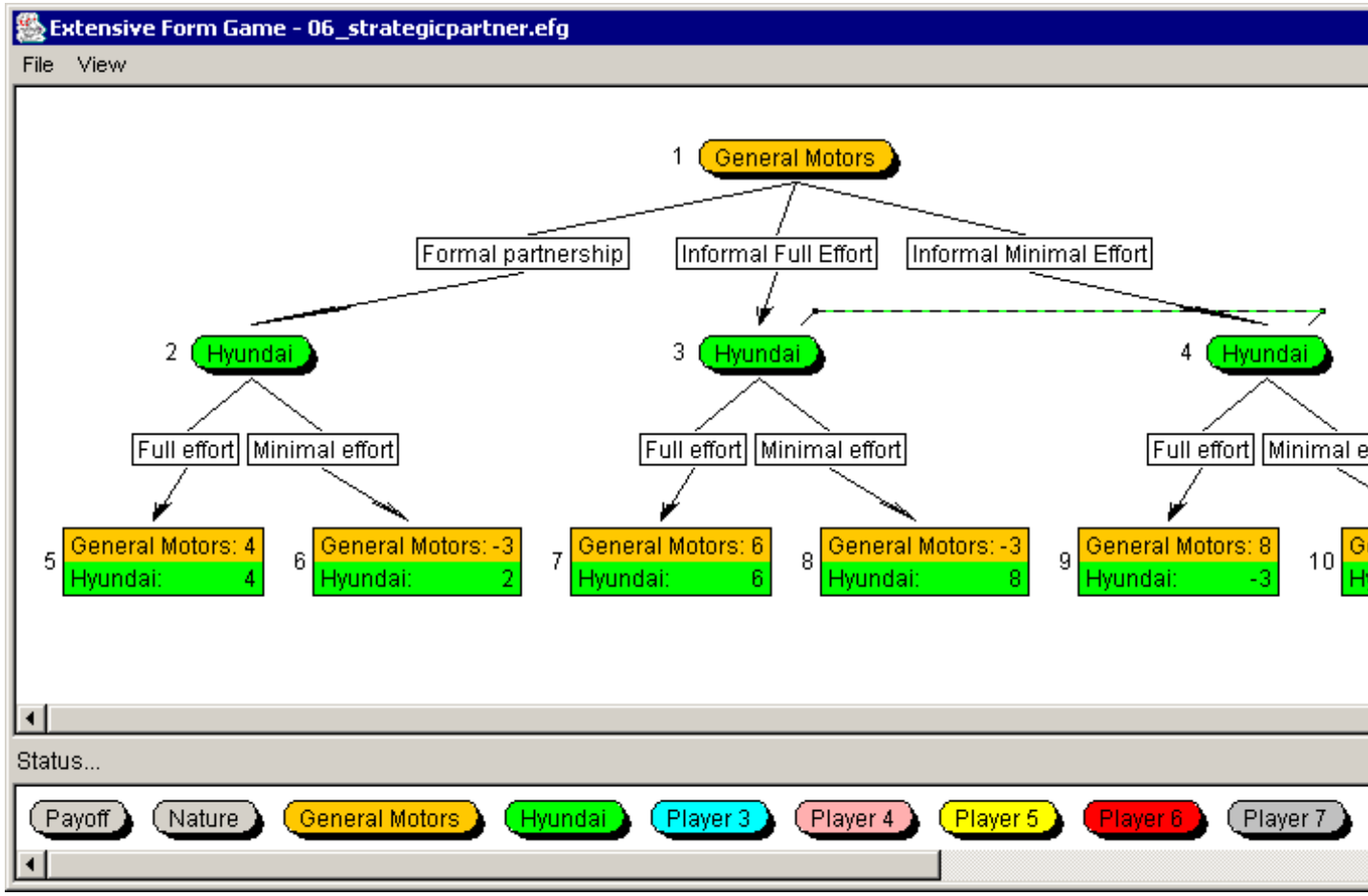


Figure 10.5
Strategic Partner

This game contains two proper subgames. The first subgame models the subgame that begins when the initiator approaches the responder about an alliance defined by a long term contract between the firms, while the other subgame determines how an informal alliance would work out. At node 2, Hunday chooses to full effort over minimal effort.

Figure 10.5
Effort choice by Hyundai in a formal agreement

Folding back the optimal decision by substituting Node 5 for Node 2, we obtain the reduced game shown in Figure 10.6.

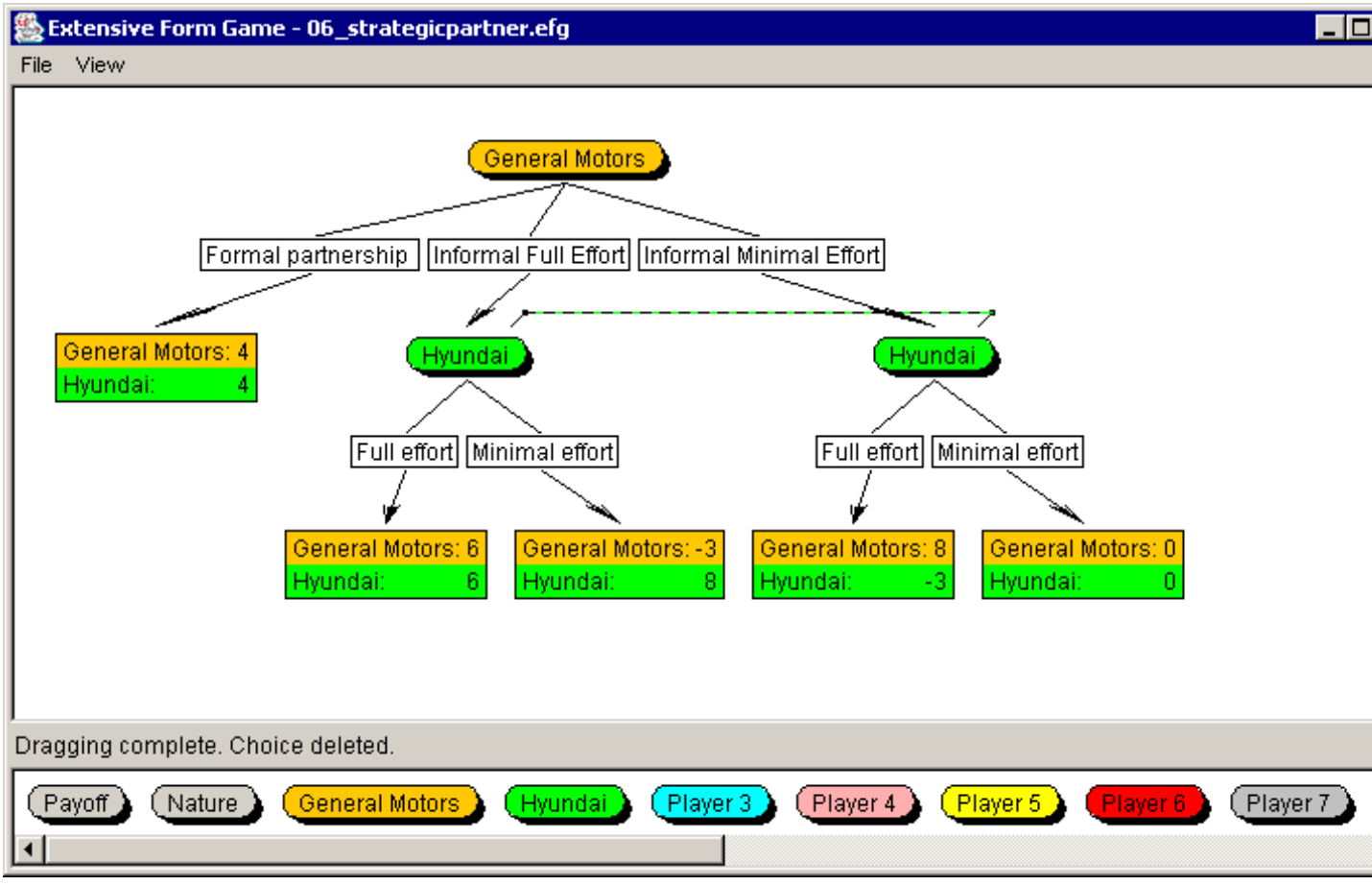


Figure 10.6
Reduced Game of Strategic Alliance

GM compares the value of having a formal agreement with Hyundai with a less structured approach, by considering the outcome of the simultaneous move subgame encompassing Nodes 1 and 3 through 8, with the outcome at Node 2.

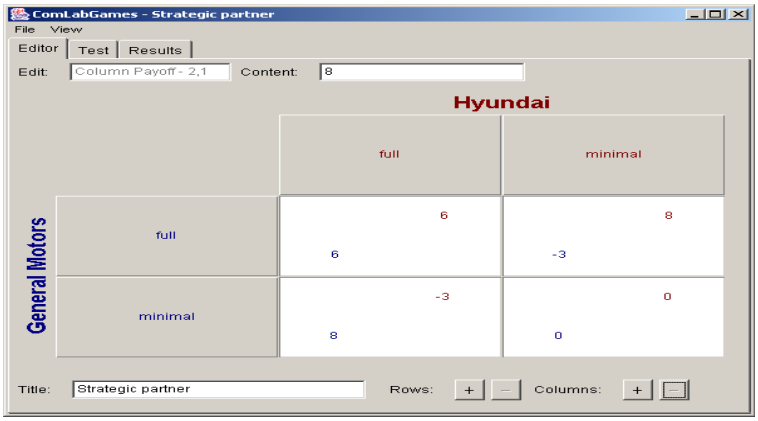


Figure 10.7
Strategic Form of Alliance

Note that the dominant strategy of both firms is to expend minimal effort on an informal alliance, thus leaving the synergistic benefits unexploited. Comparing the payoff of zero with four, which comes from forming a partnership, the companies set up the formal partnership.

Complete Information games lacking proper subgames

The reduced game in Strategic Partner illustrated in Figure is one example of a complete information game, that is neither a simultaneous move nor a perfect information game. There are many other examples of complete information games that have this property.

Not all games with the same skeleton of the first or the second one have the interpretation we have provided. For example But suppose the second player has some but not all

The first type of game is a perfect information game and the second Games that are neither

Investment broker

In many relationships between a professional and her client, the client cannot tell precisely what the professional is doing, but does have other information that relate to the professional's activities that might not be accessible to the professional. For example when a client entrusts her wealth to her broker to invest, even though he might assure her, and be able to establish that her funds are well diversified across broad categories, how much information he has about the future performance of firms within those categories is typically not something she would know. In this game the client of an investment firm does not know how much attention her broker pays to anticipating changes in future returns to tech stocks versus industrials. At the initial

node the broker chooses which type of stock he will follow most closely. Then nature reveals to his client whether the fluctuations in the economy are indicative of radical shift in technology or merely a prospect that ultimately was not realized. Since she follows business news herself the client is more savvy than most investors, and thus has the opportunity to liquidate her position before the ramifications of the shock are apparent to everybody. At this point nature could have another turn with probabilities that techs versus industrials do well, and then the payoffs could be realized. Folding back the last stage we obtain Figure 10.15.

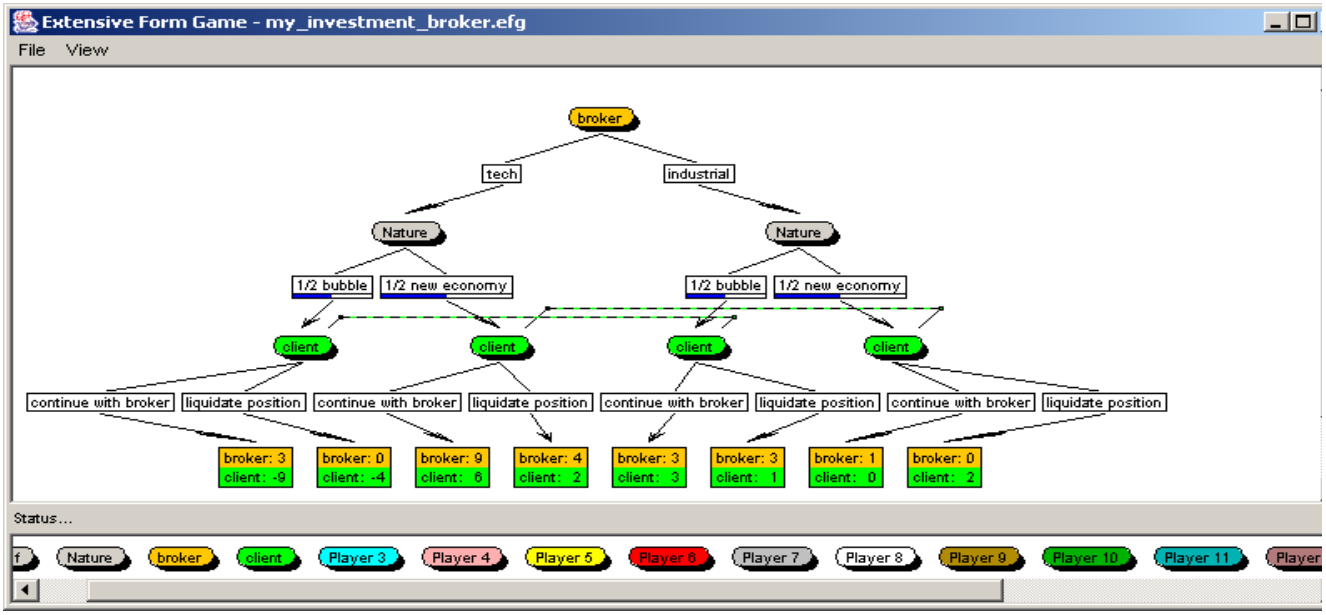


Figure 10.15
Investment broker

Suppose the investor is an expected wealth maximizer, then the strategic form looks like:

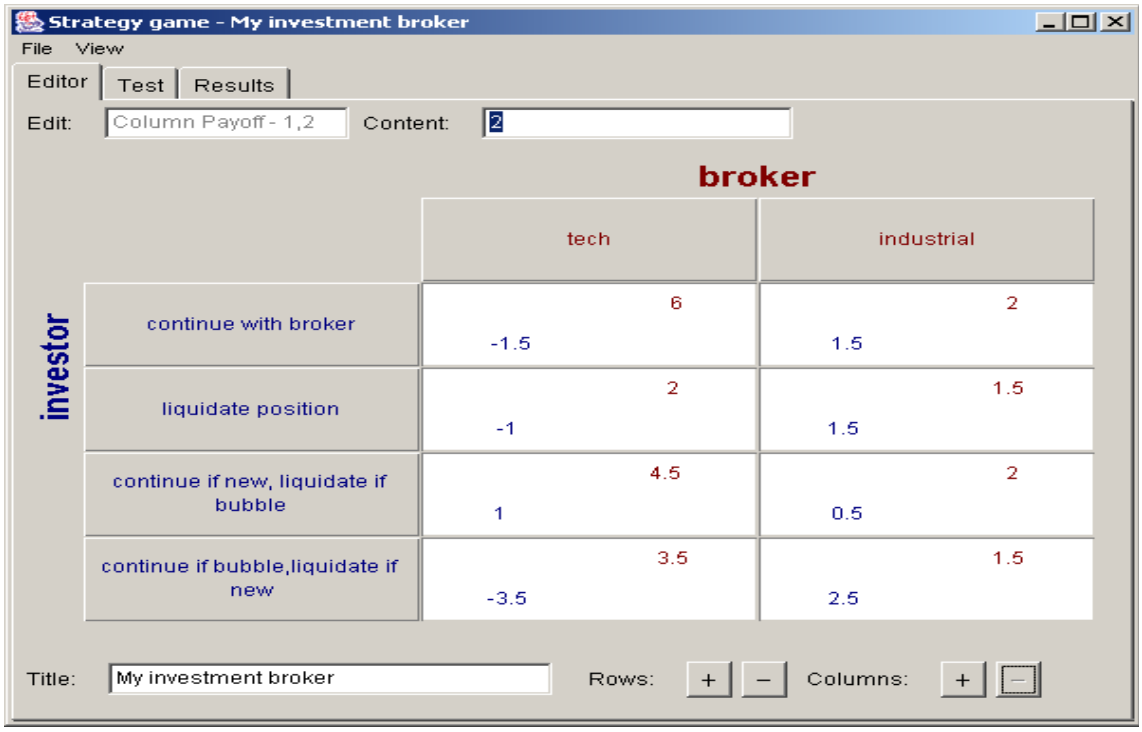


Figure 10.16
Strategic Form of Investment Broker

The investor's top strategy, of continuing with her broker, is strictly dominated by mixing her second strategy, of liquidating her position, with an arbitrarily small proportion of her bottom strategy, continuing with her broker only if the economy is in a bubble. Appealing to the principle of iterated dominance, the investor's top strategy can be discarded, and the strategic form of the reduced game is represented as Figure 10.17.

Figure 10.17
Strategic Form of Reduced Game

In the reduced game, concentrating in tech stocks is a dominant strategy for the broker. Applying iterated dominance once again, the investor should continue with her broker only if the economy is new.

Insurgency

Another reason why a game might not contain any subgames, and yet not be a simultaneous move game, is that after two players have moved simultaneously, a third player might observe the actions of one but not both of the first two players. This arises in the game of insurgency. In this game a Central American government initially chooses between three options, to seek peace with an rebel group demanding more autonomy from the central government, to engage them without attracting attention from other developed countries upon which they rely on foreign aid, or to give the conflict international recognition by asking for aid from a larger developed country, the United States in this example, to quash the independence movement. If the wealthy power refuses to come to the aid of the country, then it revisits the decision about how to proceed. But having asked the wealthy power for help it raises the public consciousness about the public, and in view of the country's dependence of on the wealthy power, this reduces its bargaining power with the rebels and also their methods of containment. All these decisions are made in confidence, and communications network within the rebel group is poor. consequently their decision to resist or seek a peaceful outcome is not contingent on the government's or the wealthy power. Figure 10.15 displays the extensive form of the game.

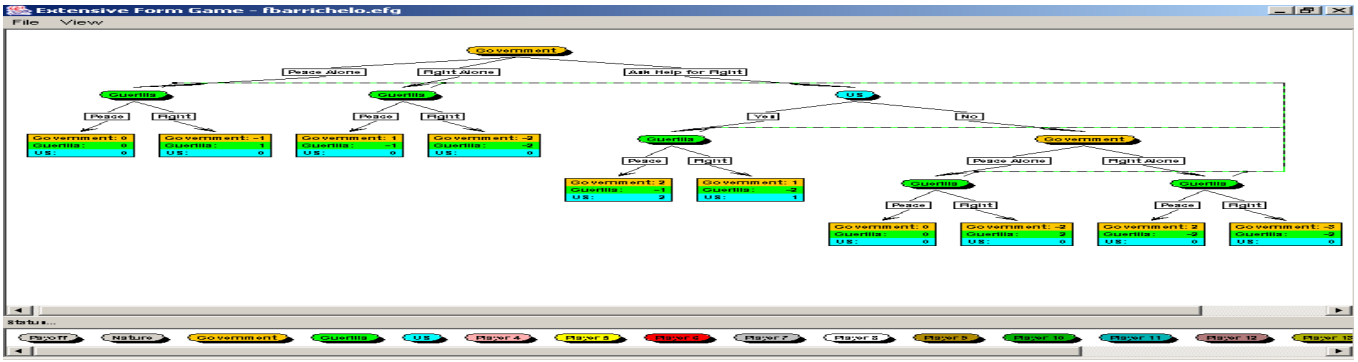


Figure 10.15
Insurgency

From the perspective of the guerrillas, this is a simultaneous move game in which both they and the other do not.

Conditional on

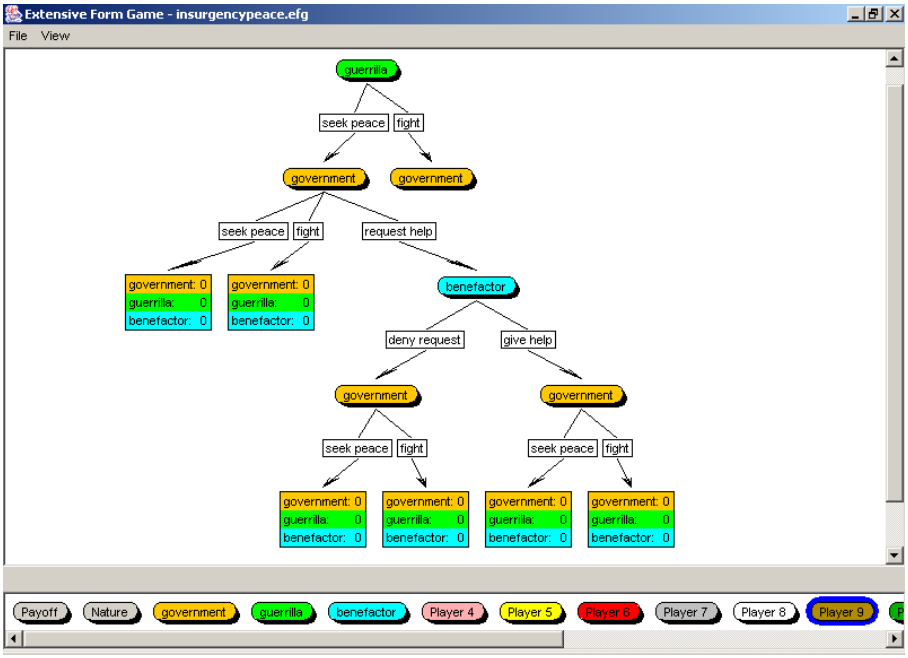
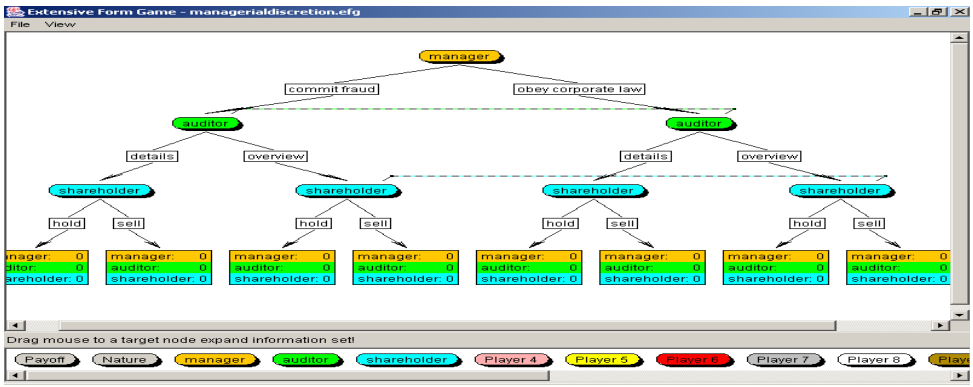
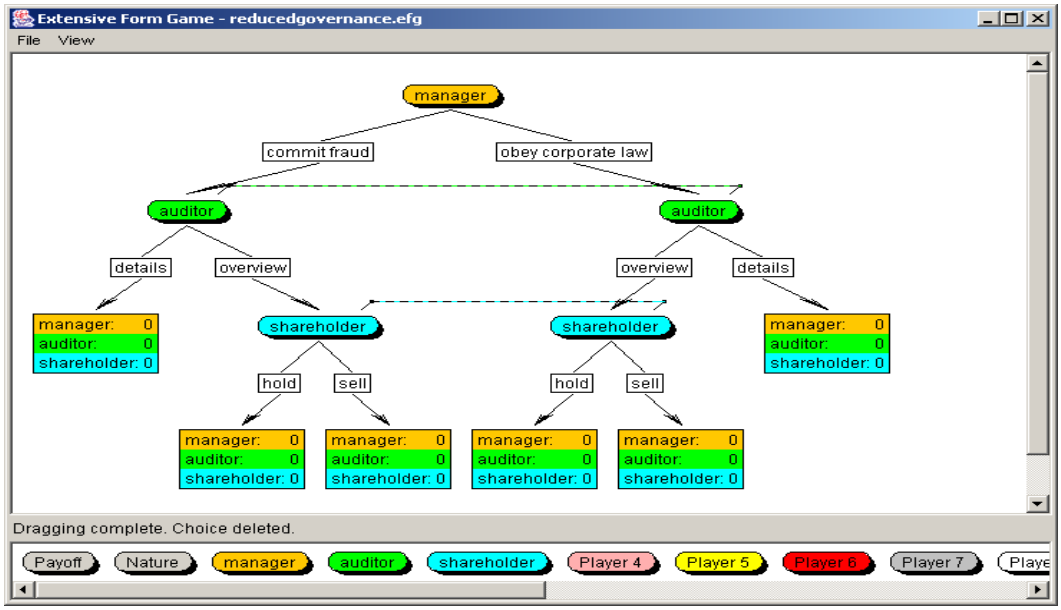


Figure 10.16
The guerillas seek peace

Corporate Governance





An algorithm for solving finite horizon complete information games

Define all the sub-games by the singleton information sets.

Find the strategic form solution for all the sub-games that contain terminal nodes.

Successively substitute the solutions into the payoffs to form reduced games.

The resulting solution, called the subgame perfect Nash equilibrium, corresponds exactly to the strategic form solution in complete information games.

Rule 1 Revised Look forward, solve the shortest sub-games, and fold back.

The value of information in complete information games

The chapter on perfect information discussed the value of information within a decision theoretic context.

Industry Newsletter

Consider two rival firms are competing with each other to discover and introduce a new product. Often companies do not know precisely how much competition they will face before launching a new product, and how much effort should be put in marketing.

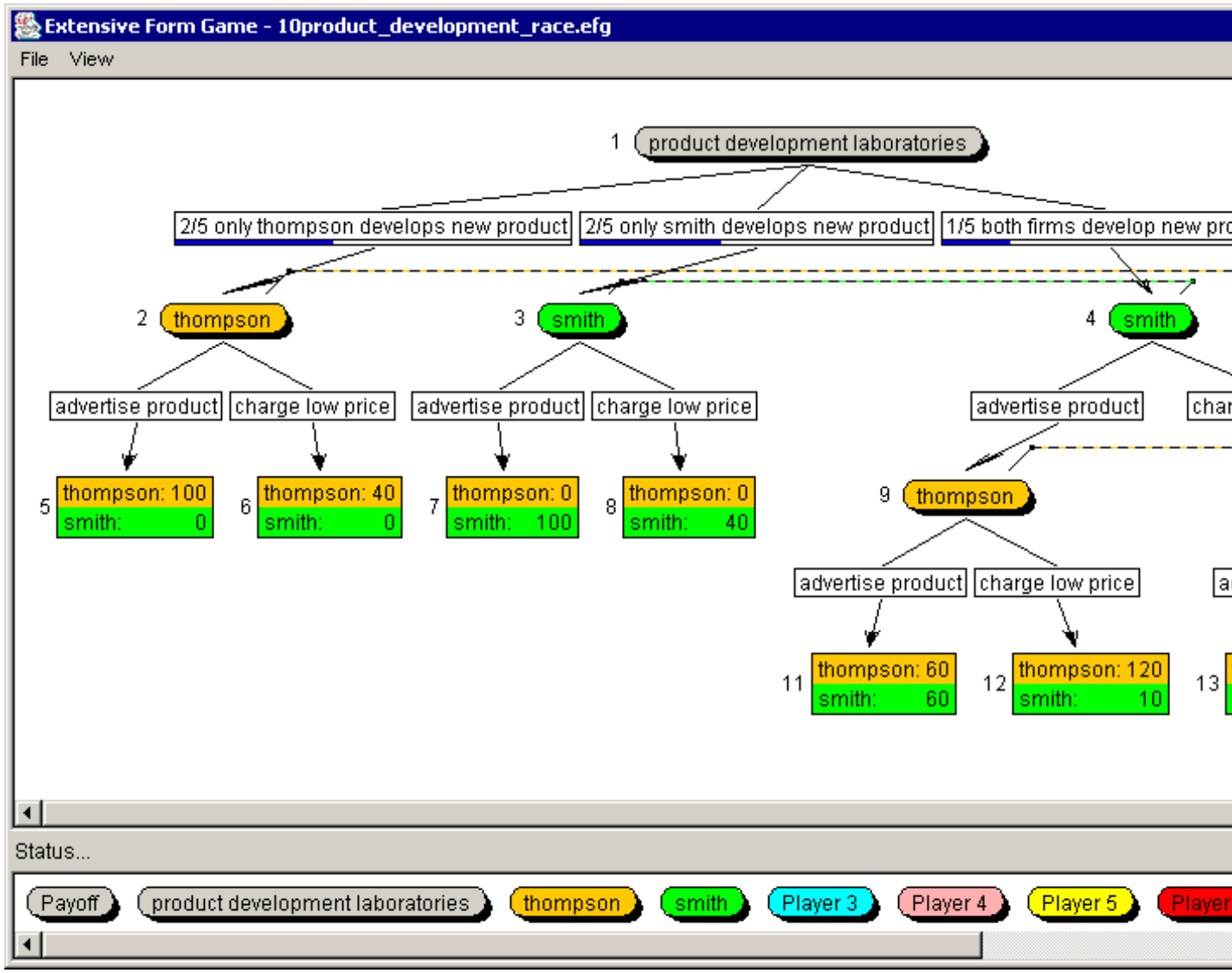


Figure 11.24
Product Development Race

Both Thomson and Smith have two strategies to advertise product or to charge low price. The strategic form representation is shown in Figure 11.25. Both firms have a dominant strategy to advertise the product, which determines the unique solution to this game.

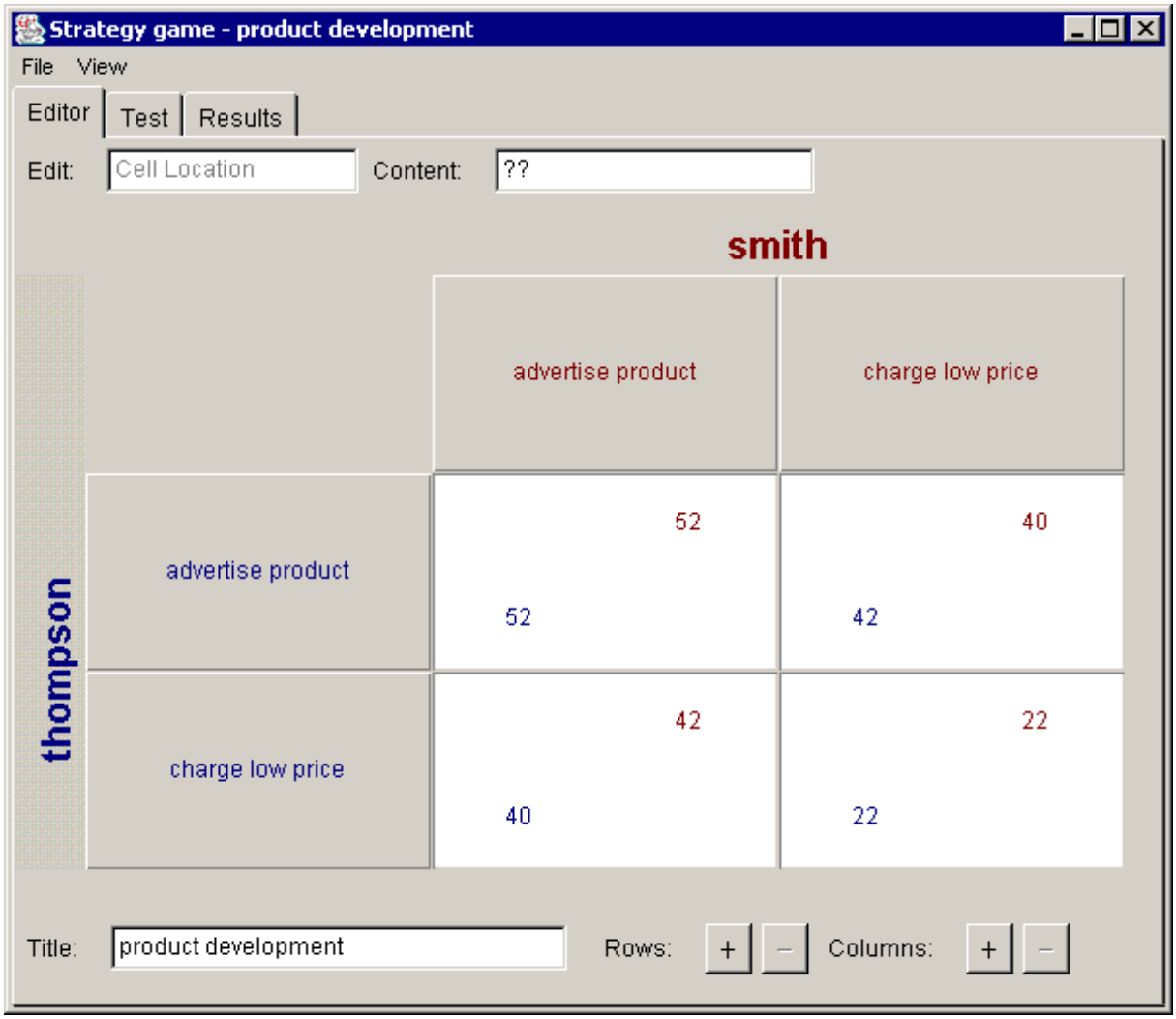


Figure 11.25
Strategic Form of Product Development Race

Now suppose a newsletter is produced to keep firms abreast of the latest developments. The extensive form becomes:

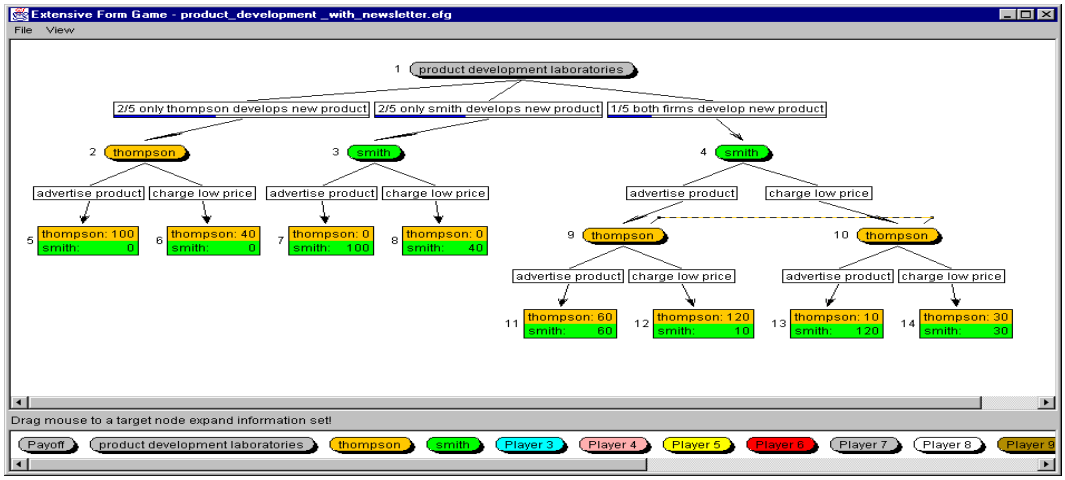


Figure 11.26
Industry Newsletter

Sub-games

There are three proper sub-games beginning at nodes 2, 3, and 4. If Thompson is the only firm to develop the product, it should advertise rather than choose a low price, and similarly for Smith.

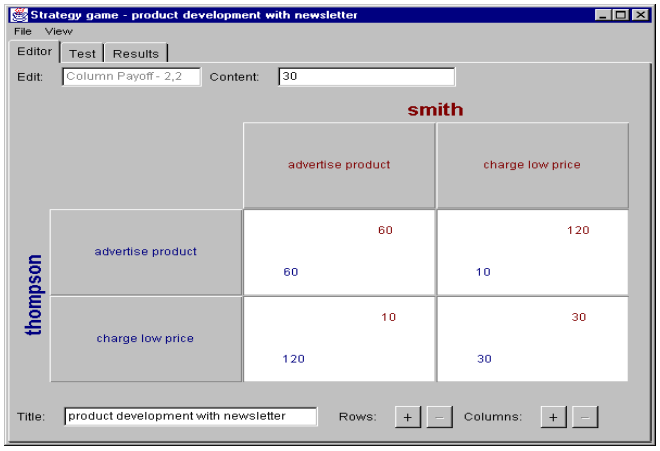


Figure 11.27
A Subgame

The sub-game starting node 4, when both firms develop the product, illustrates the prisoners' dilemma. The unique solution is for both firms to charge the low price.

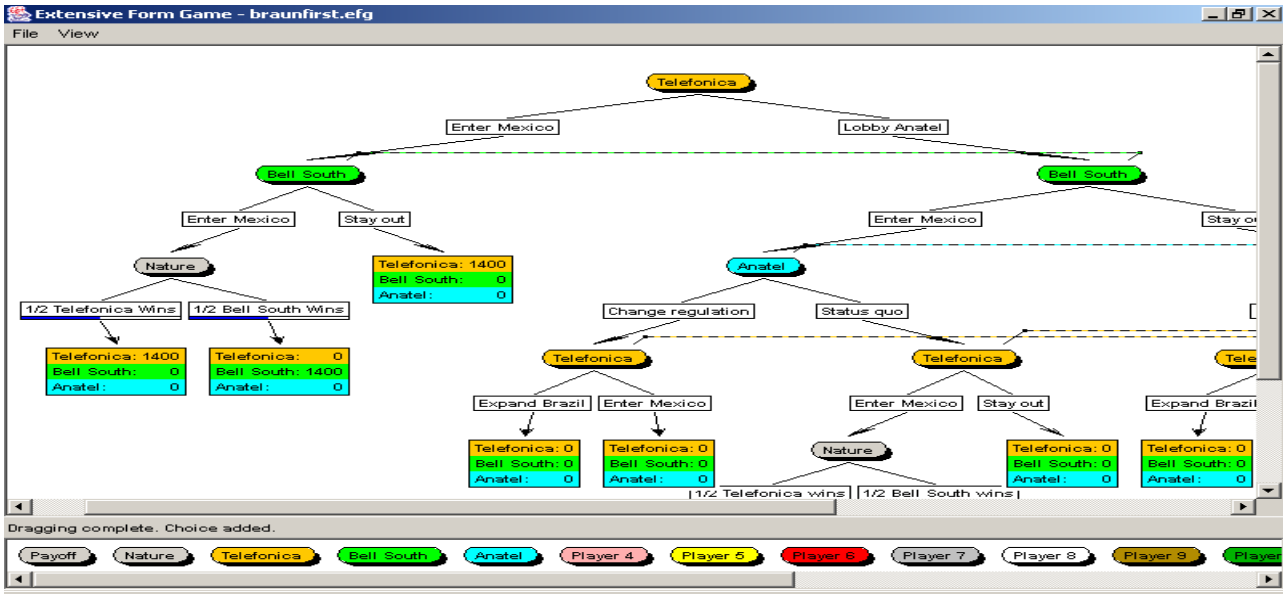
The strategic cost of better information is the difference between the payoffs that the players can expect under the full information (30) and the payoffs that they can

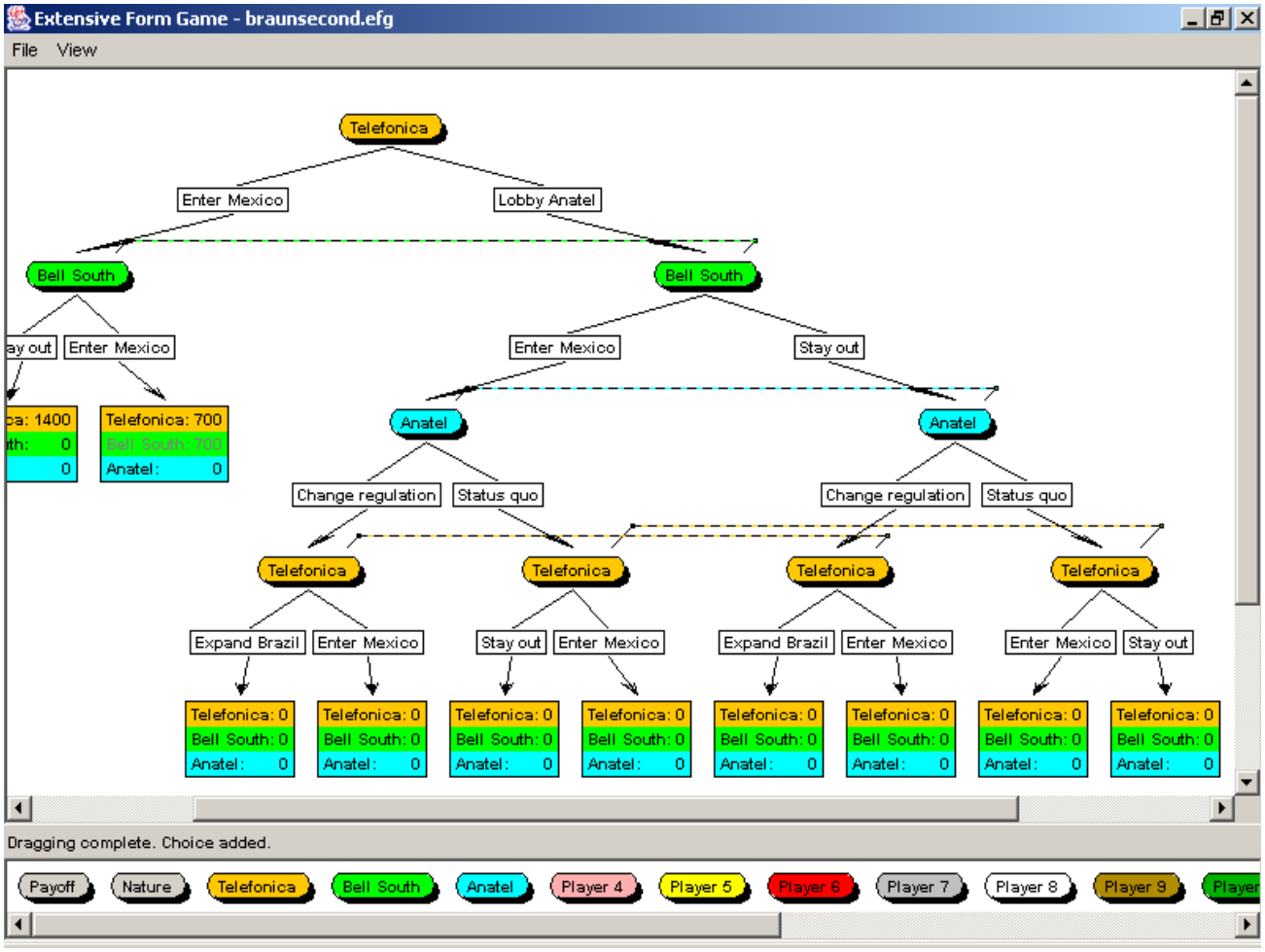
expect when the information is not revealed (52). The cost of information is in our example 22.

This example shows that more information about an industry could sometimes hurt it. Additional information helps firms to identify situations where are their positions are opposed to each other, and induce competition that might lead to the detriment of all firms.

Privatization

A central American government is divesting itself of its telephone operations. there are two potential bidders for the publicly owned company companies





What would Telefonica pay to have information about Bell south?

Is it in the interest of Bell South to declare intentions to Ana and /or Bell South? should it be spending resources to hide its intentions?

Nash equilibrium in sequential games

So far we have only been discussing simultaneous move games

Entry Game

The two previous examples define best replies in simultaneous move games. In principle best replies can be defined for the strategic form of any game, but there is an important caveat to this statement. its usefulness and relevance is . which follows from the fact that the Since a best reply applies to the strategic form, it cannot capture the dynamic aspect that strategies are not binding commitments but merely detailed plans about choosing in all possible contingencies. The ramifications of this crucial difference are most evident in perfect foresight games

Accordingly, suppose an incumbent firm currently has a monopoly in a market, and

at the initial node of the game, a rival chooses between end the monopoly and enter its market, or stay out. After the rival makes its decision, the incumbent firm chooses between producing less output to accommodate both firms and thus maintain the market price, much versus to produce the same output as before causing the price to fall sharply to prevent inventory accumulation. The extensive form of this entry game is depicted in Figure 8.7. Solving the game using backwards induction, the first rule of strategic play implies that the rival should enter and the incumbent should acquiesce.

Figure 8.7
Entry Threat

There are two strategies for each firm. The rival chooses between enter and stay out, while the incumbent must fight or acquiescence if the rival enters the industry. The strategic form of this game is shown in Figure 8.8. Noting that enter is a weakly dominant strategy, the backwards induction solution is reached using the third rule of strategic play

Figure 8.8
Strategic Form of Entry Game

How does this prescription correspond to our to our analysis of best replies this

far? The best reply mapping for the rival is:

1. Enter if the incumbent accommodates.
2. Stay out if the incumbent fights.

The best reply correspondence for the incumbent is:

1. Accommodate if the rival enters.
2. Fight or accommodate if the rival stays out.

The best replies are illustrated in Figure 8.9, the horizontal dotted line indicating the incumbent's indifference between his own choices when the rival stays out.

Figure 8.9
Best Replies in Entry

The strategic form shows there are two incentive compatible strategy profiles, namely (enter, accommodate) which is the solution to this game, and (stay out, fight) which is not a solution. This allocation corresponds to the incumbent committing himself to fighting should the entrant come in. Yet the only way such commitment can be made is if the game is a simultaneous move game, in which the incumbent chooses his strategy without knowing what the rival has chosen. In this case the extensive from would look like Figure 8.10.

Figure 8.10
Simultaneous Entry Game

The fact that the concept of a best reply applies to the strategic form of the game and not its extensive form limits its usefulness. Unless one knows that the game is a simultaneous game, a rival should question whether the strategy of fight is truly a best reply.

Weakly dominated strategies

Although we have been wary about suggesting that weakly dominated strategies should be iteratively eliminated, and in that way have carefully distinguished between strict and weak dominance, the notion that one should use weak dominance has some appeal.

Manufacturing aircraft

In this example we consider the negotiations between a developer and a community who would both benefit from the new proposal the developer has put forward. The net economic benefit of the project to both parties is $6 million, but the benefits to the parties depend on the way financing the project is structured. Under one option, the net benefit to the developer is $4 million and to the community is $2 million. Under the other option the benefits to each party are equal to $3 million. The negotiations proceeding the final outcome prevent either party from committing to a plan that is contingent on what the other party does. Consequently the bargaining that determines the outcome of the game can be modeled as a simultaneous move game. There is, however some asymmetry in the bargaining positions of the two parties. At a cost of $1 million the community can hire a lawyer to draft legislation that prohibits the developer from choosing the option that give him the $4 million share of the project. This legislation is of limited value because it would certainly be defeated by the community council, thus retouring the original bargaining game. Figure 10.10 illustrates the game.

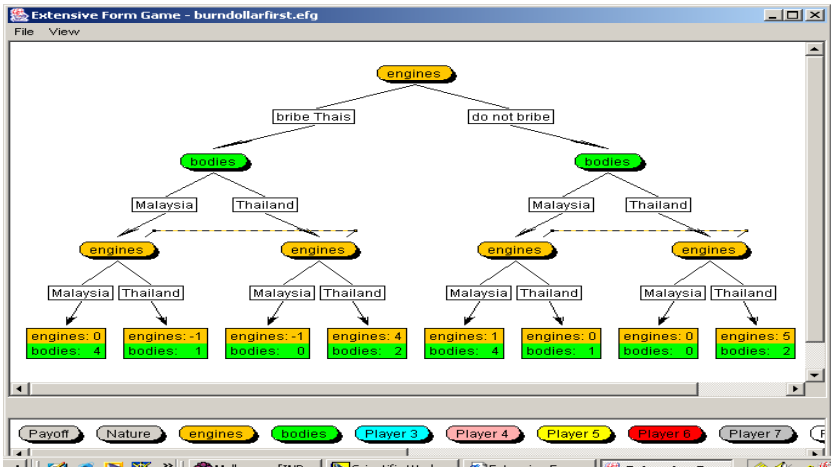


Figure 10.10
Manufacturing

There are four pure strategies open to each player. The engine manufacturer could bribe the Thai government and then locate in either country, or alternatively decide against bribing Thailand and locate one of the two countries. The airframe manufacturer could make a location decision that is independent of whether Thailand is bribed or not, it could locate in Malaysia if and only if the Thais are not bribed, or it might choose to locate in Malaysia if and only if the Thais are bribed. Figure 10.11 lists the strategies as rows and columns of a matrix, and then completes the strategic form by filling in the payoffs form different pure strategy profiles.

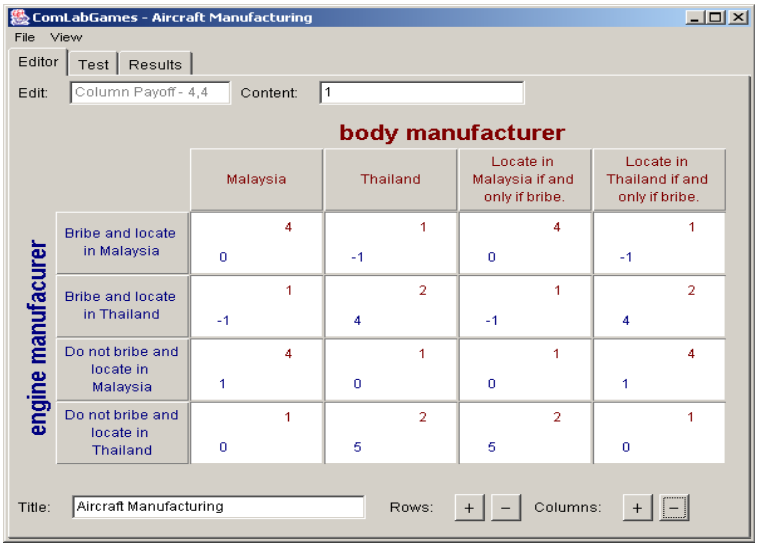


Figure 10.11
Strategic form of Direct foreign Investment

The strategic form is almost dominance solvable. First note that the strategy of bribing Thailand and then locating in Malaysia is dominated by mixing the two strategies of not bribing in any proportion. The strategic form for the reduced game is

displayed in Figure 10.12.

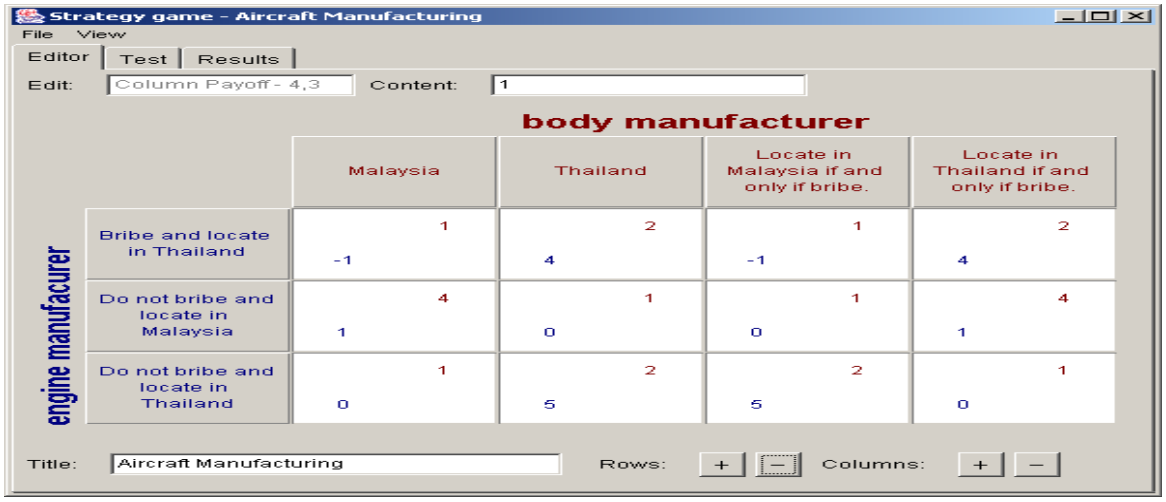


Figure 10.12
Reduced Game of Aircraft Manufacturing

The next step in the chain is its weakest link. We remark that there are no strictly dominated strategies in the reduced game.

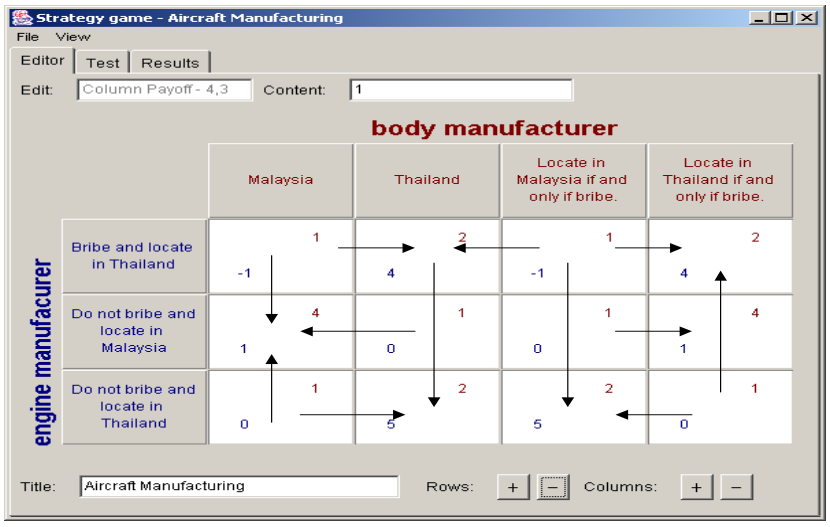


Figure 10.13
Best Replies in the Strategic Form

that the airframe manufacturer locating its facilities in Malaysia unconditionally is weakly dominated by locating in Malaysia only if the engine manufacturer does not bribe the Thais, suppose that we eliminated that strategy from consideration. In this case we are left with a further reduction, displayed in Figure 10.13.

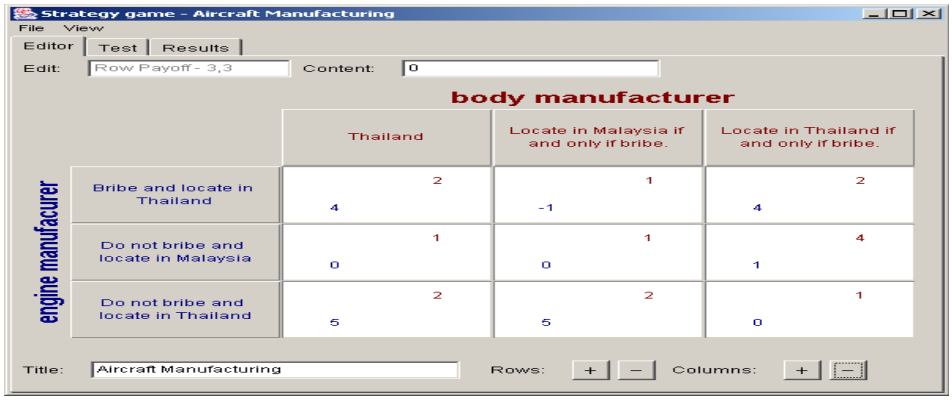


Figure 10.13
Removing a weakly dominated strategy

The remainder of the game can now solved using the principles we have advocated in the earlier chapters, namely iteratively eliminating strictly dominated strategies until a weakly dominant strategy is revealed for one of the players. Specifically, not bribing and locating in Malaysia is dominated by any mix of the other two strategies placing between one fourth and five sixths on bribing and locating in Thailand. Upon eliminating the middle row strategy in Figure 10.3, choosing Thailand is a weakly dominant strategy for the airframe manufacturer. Therefore the unique solution to the strategic form game illustrated in Figure 10.3 has the engine manufacturer not bribing Thailand but choosing to locate there, and the airframe manufacturer doing the same thing.

A Word of Caution

Our last example shows that while backwards induction and dominance principles are very powerful in yielding predictions, the predictions are sometimes quite sensitive to subtle changes in the structure of the game. We begin by considering a game in which a pioneer and an imitator in the air service support industry, on line travel agents or new conference/resort owners for example, find themselves choosing simultaneous move game, choosing whether to target their product towards business or vacation customers. Because of commitments already made, the debt on servicing capital is unlikely to make more than one of the firms optimal. If both firms seek the business market, they will incur heavy expenditures and yet generate only modest demand, having to split the market between them. Profits can be make by one of them serving business clients, only if the other relinquishes that market and concentrates on the vacation industry, which yields smaller losses than going head to head for demand from business. Finally if both opt for the vacation demand, both make more losses than if only one does. The extensive form for this simultaneous move game are displayed in Figure.

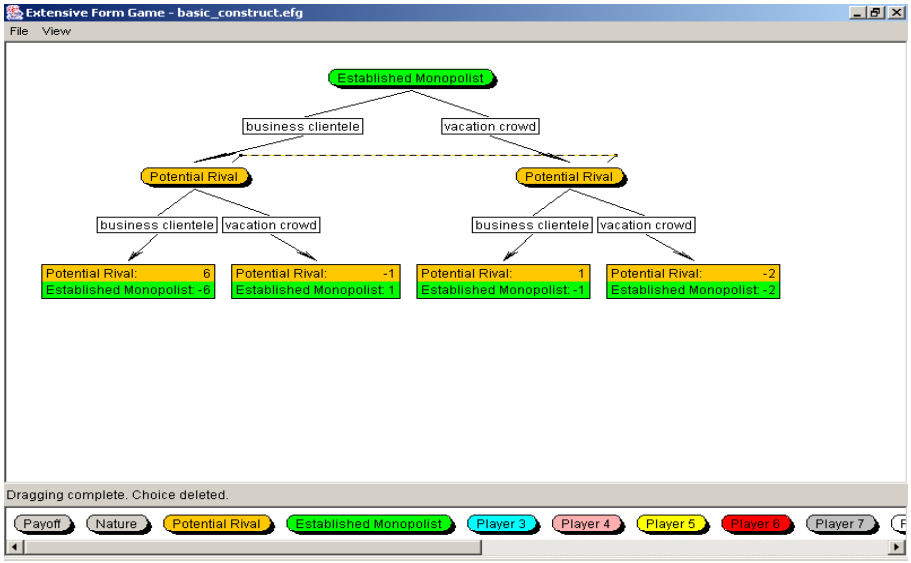


Figure
Basic Entry Model

The game supports two pure strategy Nash equilibrium solution, where one firm targets the vacation crowd and the other firm targets business clients, There is also a symmetric mixed strategy equilibrium, with each firm targeting business clients with probability $3/8$. The fact that three out of the four outcomes to this game have negative payoffs might lead one to question whether earlier decisions by both firms have lead to the initial node of this game, and if so whether the preceding play might have an effect on the outcome of this subgame.

In this example we suppose that the imitator previously had an opportunity to enter the market after seeing a pioneer enjoying the fruits of this monopoly position. The extensive form of an extended game is depicted below.

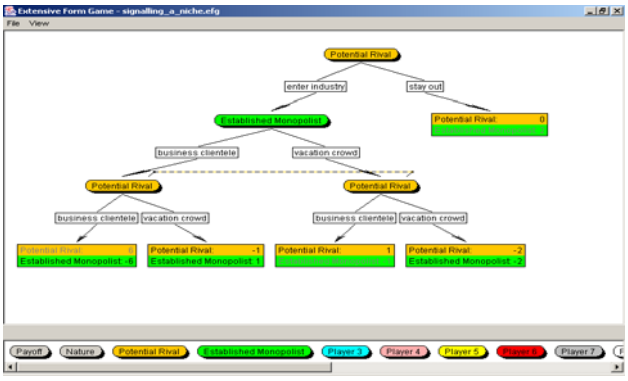


Figure
Entry and Product Niche

This game is dominance solvable. To establish that point we display strategic form

of the extended game in Figure . The imitator has three strategies, to stay out, to enter and then target business clients, and to enter and then target the vacation crowd. The pioneer only has two strategies: conditional upon entry by the imitator, the pioneer chooses between targeting the business versus vacation markets.

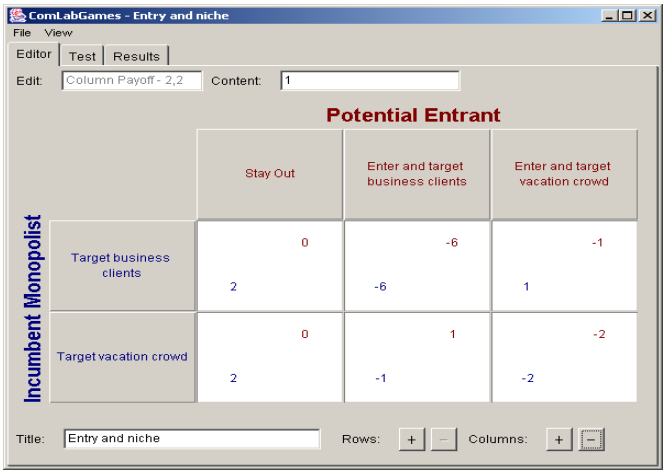


Figure
Strategic Form of Entry and Product Niche Game

First note that the third strategy of the potential entrant, targeting the vacation crowd, is dominated by the first strategy of staying out. Eliminating from that strategy from consideration we are left with the following bimatrix:

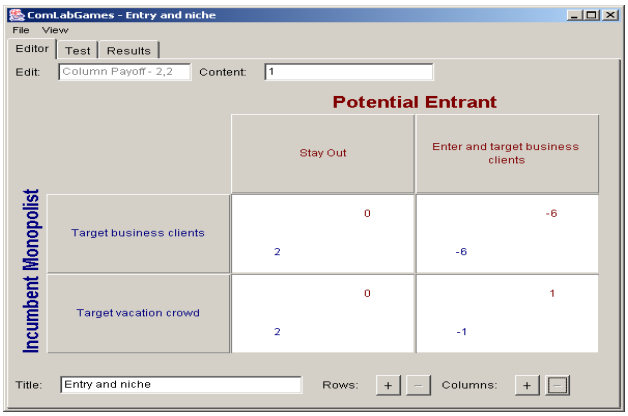


Figure
Reduced Strategic Form

In the reduced strategic form the pioneer has a weakly dominant strategy to target the vacation crowd (if the potential entrant decides to invade the market). Recognizing this the potential entrant invades the market and targets the business clientele.

There is an unsettling feature of this game. Suppose we step the game back one more, and consider the pioneer's initial decision to enter. We get:

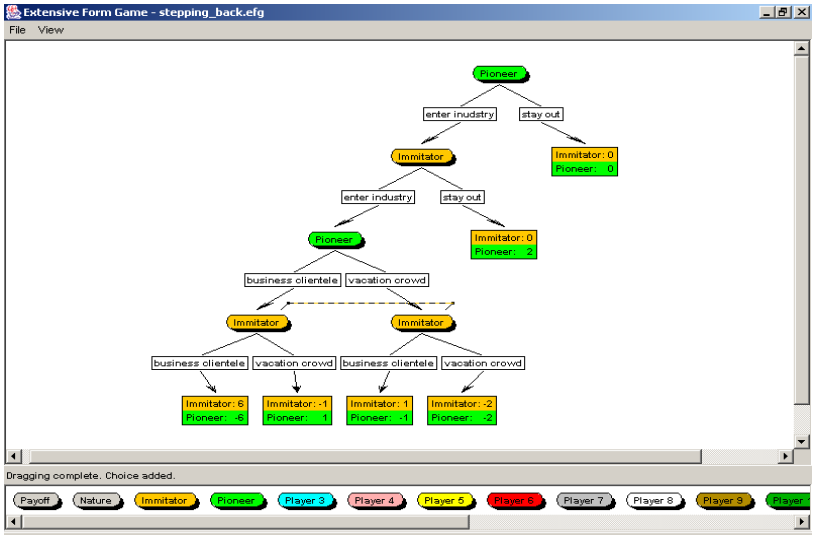


Figure
A Further Extension

Using the procedure of folding back, there are only two possibilities. Either the pioneer will take the lead knowing that the incumbent will stay out, or the pioneer will stay out, so that no one enters the industry, since the pioneer recognizes that if he enters the imitator will stay out. Hence whenever we observe the imitator about to move in an industry that is already occupied we should conclude that it will not enter at all.

The sharply contrasting predictions surely makes us pause before jumping to conclusions. Does the pioneer anticipate that the imitator would realize that the pioneer would choose the profitable segment of the market? Supposing the imitator enters, why should the pioneer accept the argument made in the first extension, when the imitator rejected the same argument initially. We lack sufficient confidence to provide a definitive prediction about what will happen in this game. Perhaps the art of strategy is to convince a rival to see the solution to the conflicts in the world in the way that is most favorable to the player.

Exercise *Design a game with a two pure strategy equilibria*

Summary

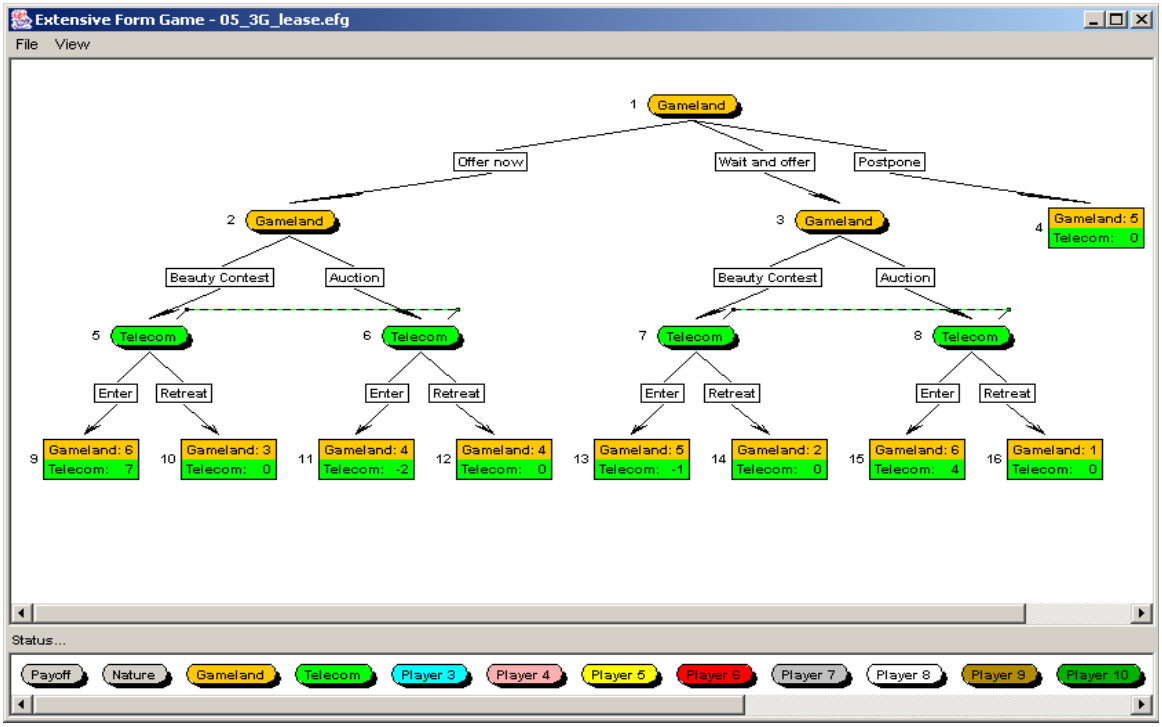
In complete information games the information available at successive moves does not shrink. This implies that a player who is about to move has at least as much information as everyone who has already moved. Complete information games include simultaneous move and perfect information games. The solution to any complete information game is found by decomposing it into a sequence of solutions to smaller reduced games, using the tools developed for perfect information games and simultaneous move games. In we extended the process extending the concept of backwards induction to subgame perfection.

Test examples

1. Licencing the third mobile phone services

Background:

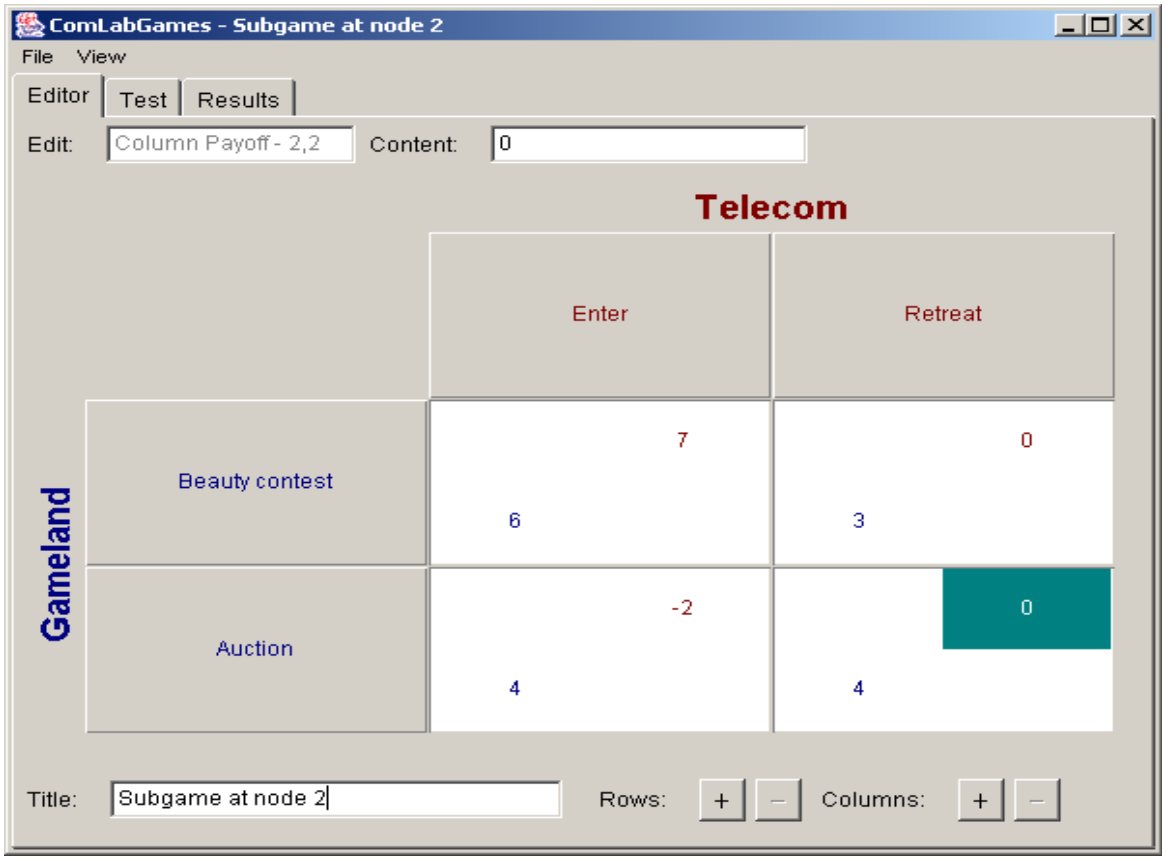
This game represents the decision that European governments faced with licensing the third generation mobile phone services. Government (Gameland) can offer license now or later or postpone it indefinitely. If they decide to go with it then they have to decide on the type of the auction: ("Beauty Contest" means that other factors affect the winning apart from the price and the rules are not clearly specified; "Auction" means that the rules are clearly stated in advance and it is known to everybody that the person with the highest offer wins the auction. This is the example of the complete information game.



Solution

The first singleton decision nodes going from the terminal nodes up are: node 2 and node 3. We will first solve the simultaneous move game for subgame at node 2:

1. **The strategic form game for subgame at node 2 is:**



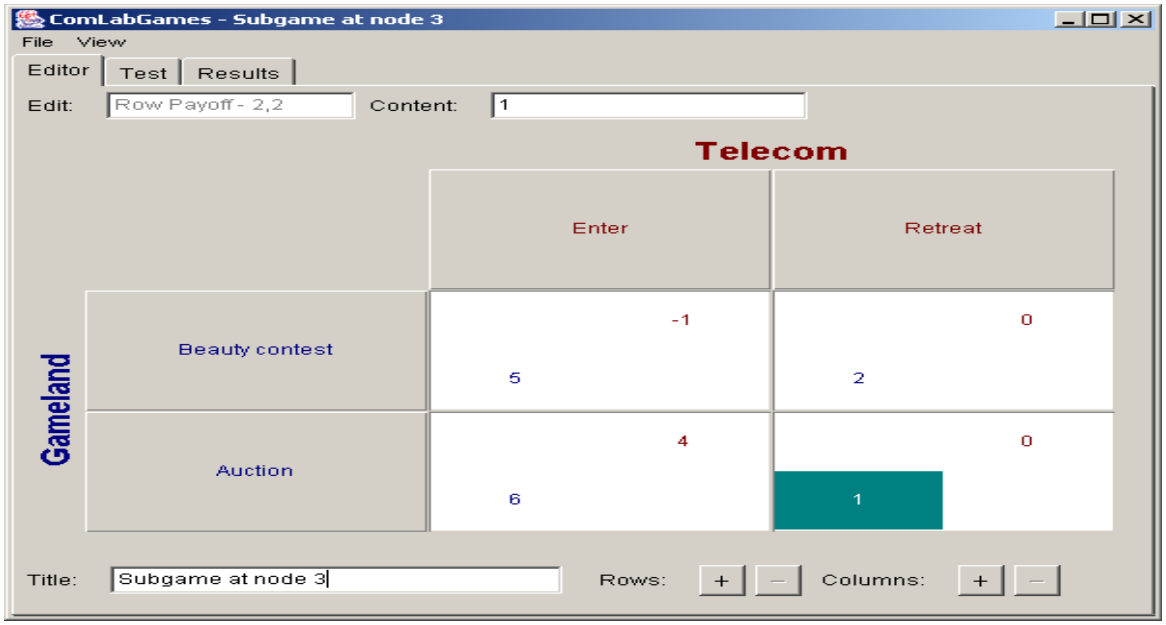
There are two Nash equilibria in this game:

· Beauty contest for Gamland and Enter for Telecom that leads to payoffs (6, 7)

· Auction for Gameland and Retreat for Telecom with payoffs (4, 0).

However the first equilibrium (6, 7) dominates the second one.

2. The strategic form game for subgame at node 3 is:



There are two Nash equilibria in this game as well:

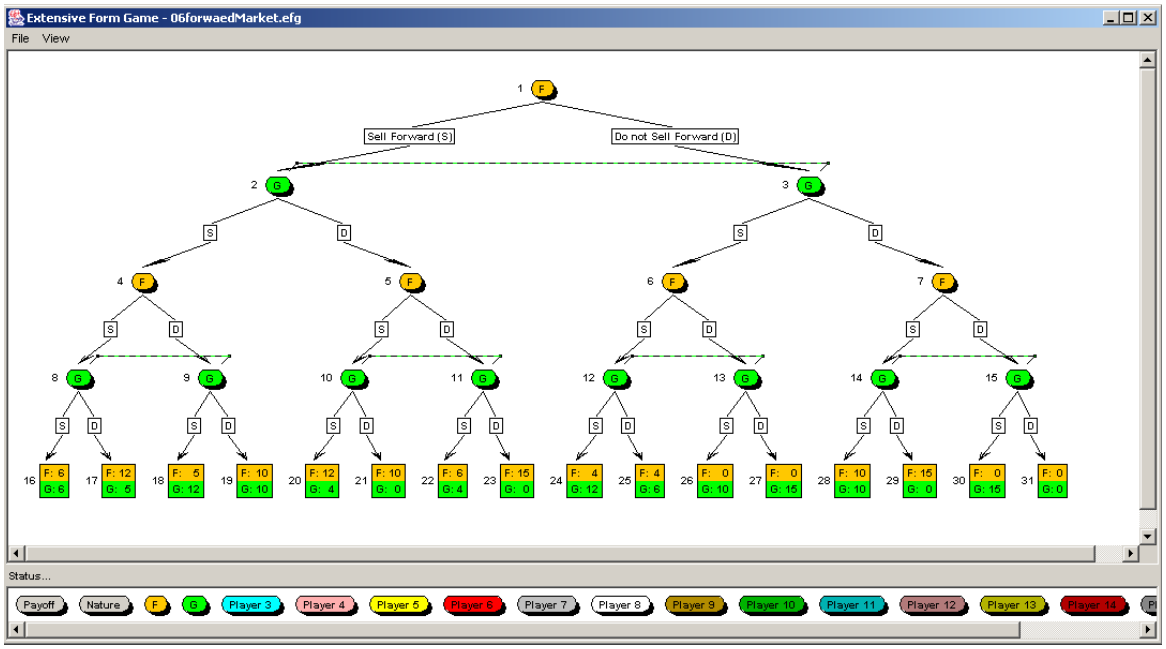
- Auction for Gameland and Enter for Telecom that leads to payoffs (6, 4)
- Beauty contest for Gameland and Retreat for Telecom with payoffs (2, 0).

However the first equilibrium (6, 4) dominates the second one.

Gameland compares payoffs (6,4) at node 3 with payoffs (6, 7) at node 2. Both of these equilibria are feasible. However offering Auction will reduce the payoff to Telecom. There are also mixed strategy equilibria in this game. We played only one shot games and for this reason we do not need to calculate how we were supposed to mix between the two.

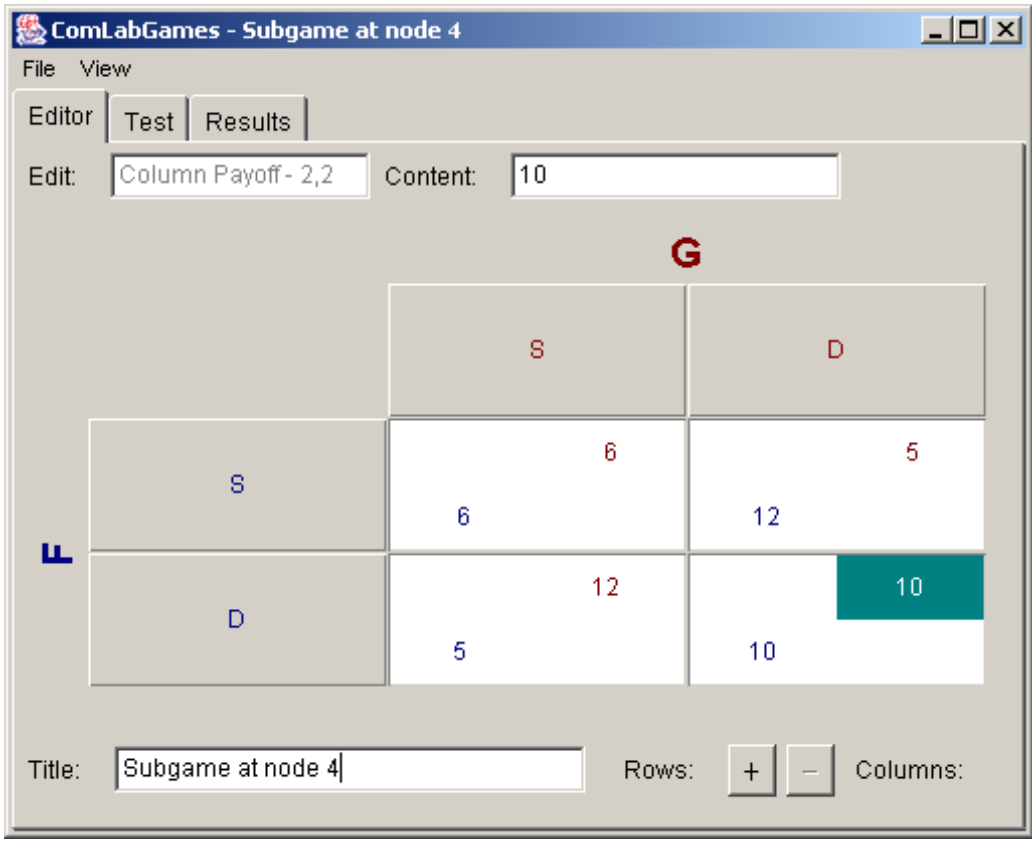
2. Bidding game

This is an example of the complete information game.



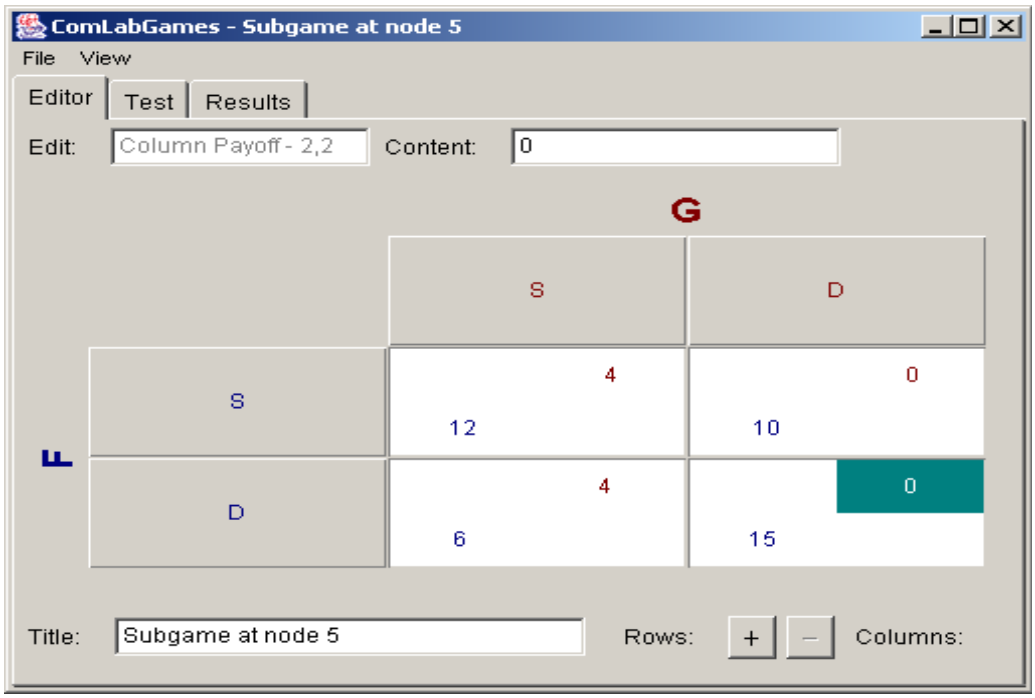
Solution

Subgame at node 4:



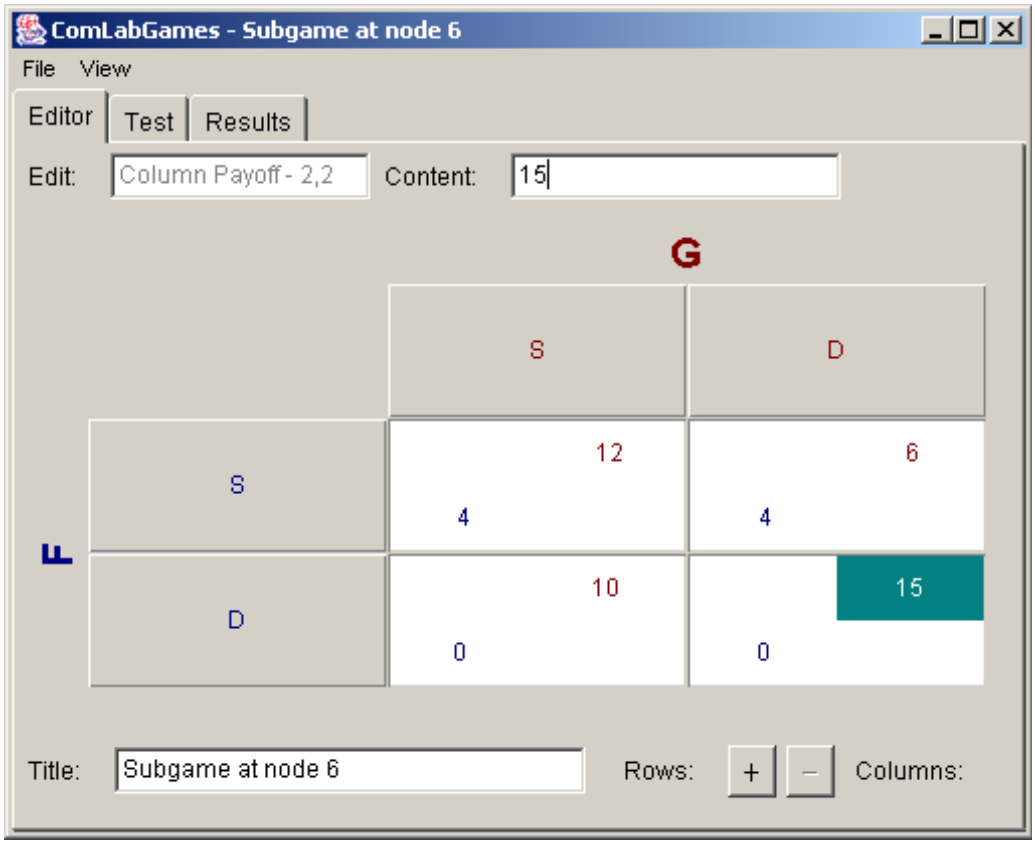
F has a strictly dominant strategy (S) and G's best reply is (S). The payoffs are (6,6)

Subgame at node 5:



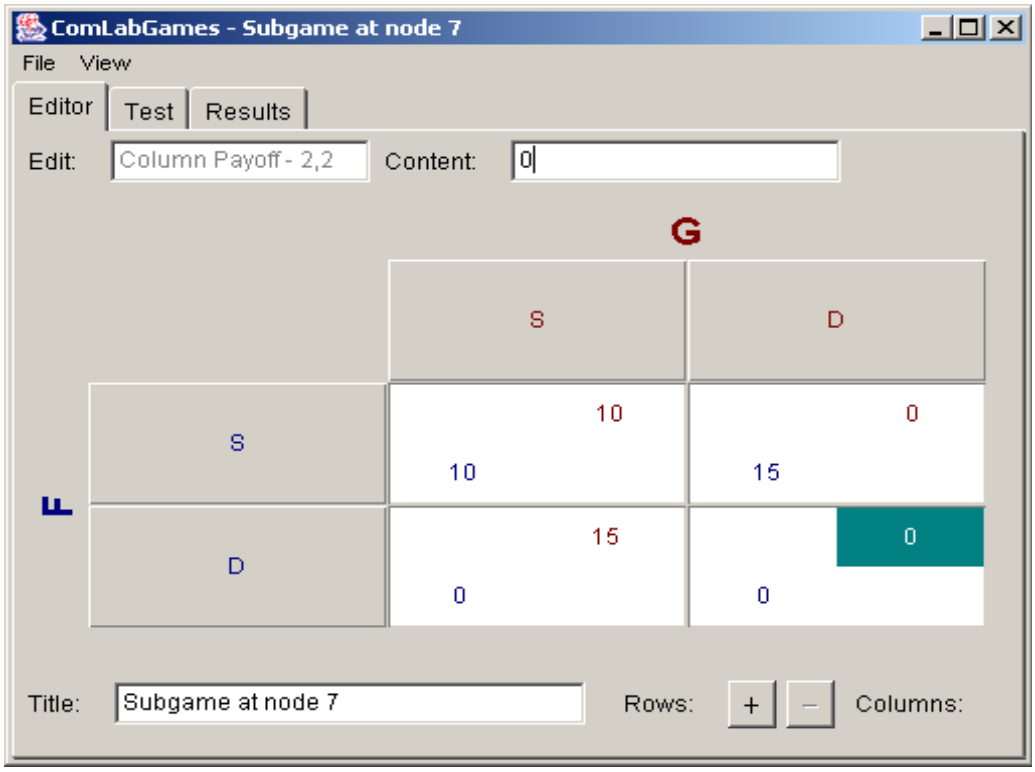
G has a strictly dominant strategy: (S) and F's best reply is (S). The payoffs are (12,4).

Subgame at node 6:



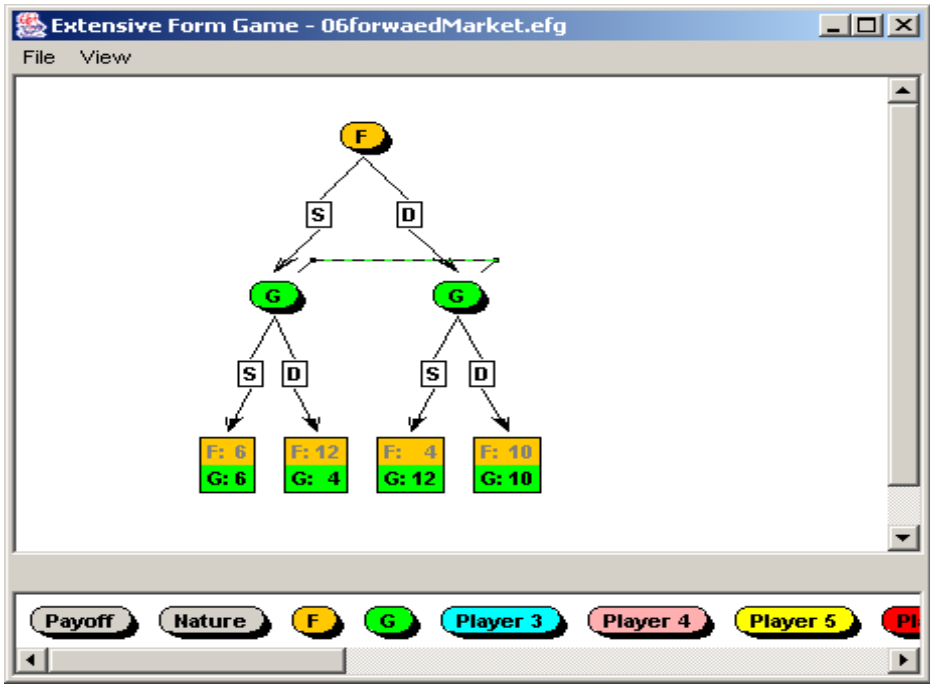
F has a strictly dominant strategy: (S) and F's best reply is (S). The payoffs are (4,12).

Subgame at node 7:



G has a strictly dominant strategy: (S) and F has a strictly dominant strategy: (S). The payoffs are (10,10).

The reduced game is:



F has a dominant strategy: (S) and G has a dominant strategy: (S) that leads to the solution (S,S) and the payoffs of (6,6) for both players. In the original game node 16 is the solution to this game.

Governance

The second example of a Markov solution to state space game that we consider before providing a general treatment of this approach concerns delegation of authority within the firm.

Our simple game of corporate governance has three stages. In the first stage the delegates a managerial task to one player and a subordinate task to the other. Then the delegated manager decides whether to husband the ower's resources, or to propose to fraud the owner by entering into a side contract with the subordinate. If the manager diligent chooses who to manwe suppose an owner. Figure

Figure 13.9
Governance

Teh owner has two strategies, to appoint an unpricipled or a principled person to the job of manager. The other two players have eight strategies each (since they each face three decision nodes wtih two choices in this infinite horizon game of perfect information). Since this is a three player game, we cannot depict its Markov strategic form within a bimatrix. Following our approach taken in Chpater 6 to the strategic form of three player games,

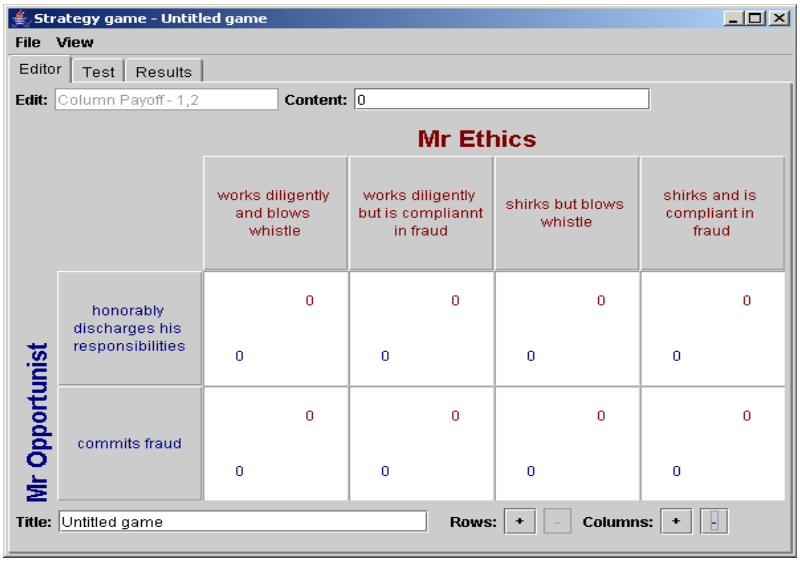


Figure 13.10

Mr Oppotunist is assigned the managerial position

Following the same approach we can compute the equilibrium if Mr Ethics is assigned the role of manager.

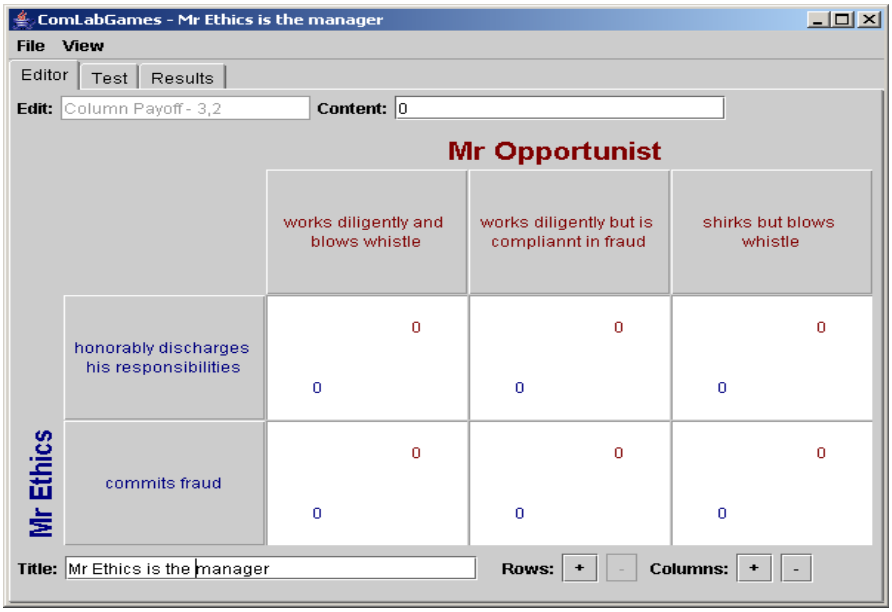


Figure 13.11

Mr Ethics is assigned the managerial position

Intuitively the owner is able to benefit from the principles of Mr Ethics by using him to enforce good behavior by Mr Opportunist.

Change payoffs for whistle blowing? If they are reduced, then there might be a shift in job assignment, but we run teh risk of either fraud or poor effort.

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Brandts, Jordi, and Charles A. Holt (1992) “Forward Induction: Experimental Evidence from Two-stage Games with Complete Information,” in Research in Experimental Economics, Vol. 5, edited by R. M. Isaac, Greenwich, Conn.: JAI Press, 119-136. Abstract Forward induction formalizes the idea that actions taken in an early stage of a game can restrict reasonable beliefs about play off of the equilibrium path, which in turn can rule out equilibria that satisfy backward induction rationality. Observed play tends to converge to the equilibrium that is not ruled out by forward induction arguments.

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prior moves being unobserved. An analysis of individual decisions shows a tendency for decisions to shift in the direction predicted by virtual observability.