Incomplete information

Up until now we have only developed methods for solving complete information games, those where players about to move know at least as much about the prior history of play as those who have already moved. Games with incomplete information lack this property. In such a game there is at least one decision node assigned to a player who does not have a piece of information that another player had when she moved earlier in the game, at a predecessor decision node. Incompleteness provides a rationale for the informed player to signal her information to the uninformed player, or alternatively hide it, through her choice of moves. These are topics for analysis in this chapter

We had remarked in Chapter 3 that every game has either complete information or incomplete information. This comment warrants repetition because, although the games we analyze here have a different information structure than those we solved in the earlier chapters, the methods for seeking a solution are not. The reasons for this are straightforward. Appealing to subgame perfection and backwards induction methods also applies to games with incomplete information. Moreover the strategic form representation does not exploit the information set structure beyond the definition of each player's strategy space. Hence the tools of dominance and Nash equilibrium are just as important in games of incomplete information.

Incomplete information can arise from two sources, knowledge about a player's past move, and knowledge about an earlier move by nature. In a incomplete information game, player Alpha, say, might be more informed about a certain event (apart from his own choice) than another player Beta, say, even though Alpha moves before Beta does. This would be the case if, for example Gamma moves first, followed by Alpha and then Beta, where Alpha sees Gamma's move, Beta sees Alpha's move, but Beta does not see Gamma's move. Gamma might be the name of another player, or the name of a chance event. In many cases of interest we will be able to model the information asymmetry by specifiying that each player knows her own payoff function, but that she is uncertain about what her opponents' payoff functions are.

In the next section we analyze several examples of games in which the actions of another player induce incomplete information. Then we investigate the solutions of games in which a chance event is the source of incomplete information. This leads us to a general characterization of the solution algorithm to any finite game, while the last sections in this chapter study the value of information in incomplete information games, before wrapping it up with a summary and recommendations for further readings.

Incomplete information induced by the actions of players

Incomplete information about the actions of players can arise in any game for which there is a possible history passing through at least three information sets, through which Figure 11.1 shows how incomplete information might arise in games for two players. Imagine a town in which every intersection is a T junction. A visitor asks a local guide for directions to the local hotel, who responds that to get there you drive straight until you reach the end of the current they are traveling on now, and then turn

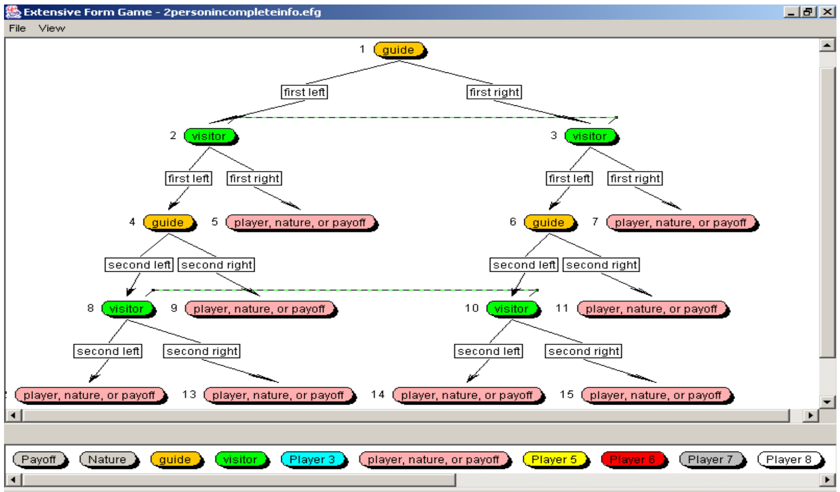


Figure 10.1
Directions

First we shall analyze several games in which one player knows more about a second player's previous move than a third player.

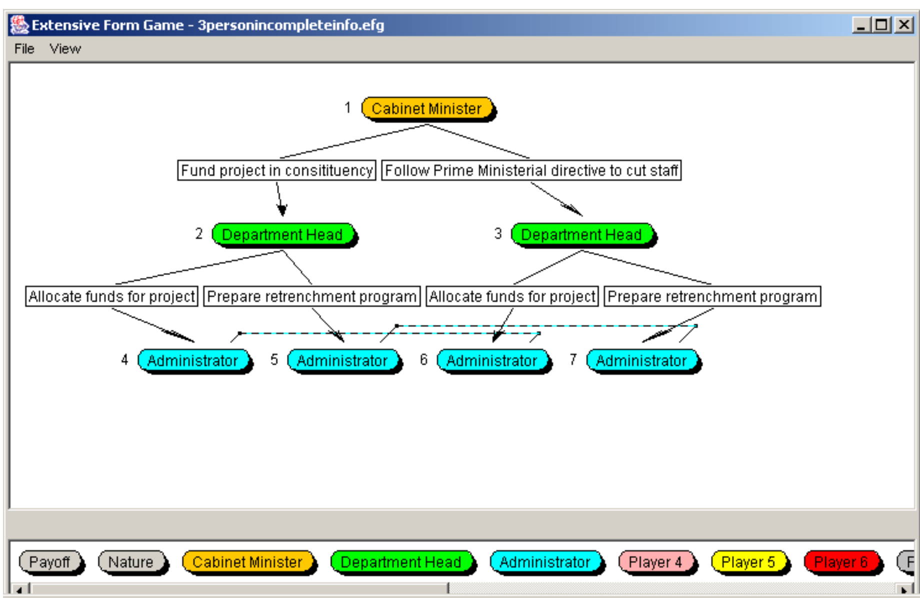


Figure 10.2
Accountability in government

In each of these examples

Armageddon

Three centuries from now, the northern hemisphere is inhospitable for large mammals, because of radioactive deposits from previous nuclear conflicts, atmospheric pollution, debilitating viral diseases, and low fertility rates. Although mutant species genetically related to humans are believed to inhabit North America and possibly Eurasia, the bulk of the world's human population of 400 million live in South America and Australasia. There are two world powers, Australia and Brazil, which compete for hegemony over Antarctica, a cold wasteland once covered with ice, rich in mineral deposits, but still sparsely populated.

The game inspired by this bleak scenario of a future world has only two players, Australia and Brazil. Incomplete information arises because at Node 4, Brazil is aware it chose military expedition at Node 1, but at Nodes 8 and 9, which are successors to Node 4, Australia cannot know this since they belong to the same information set as Node 6.

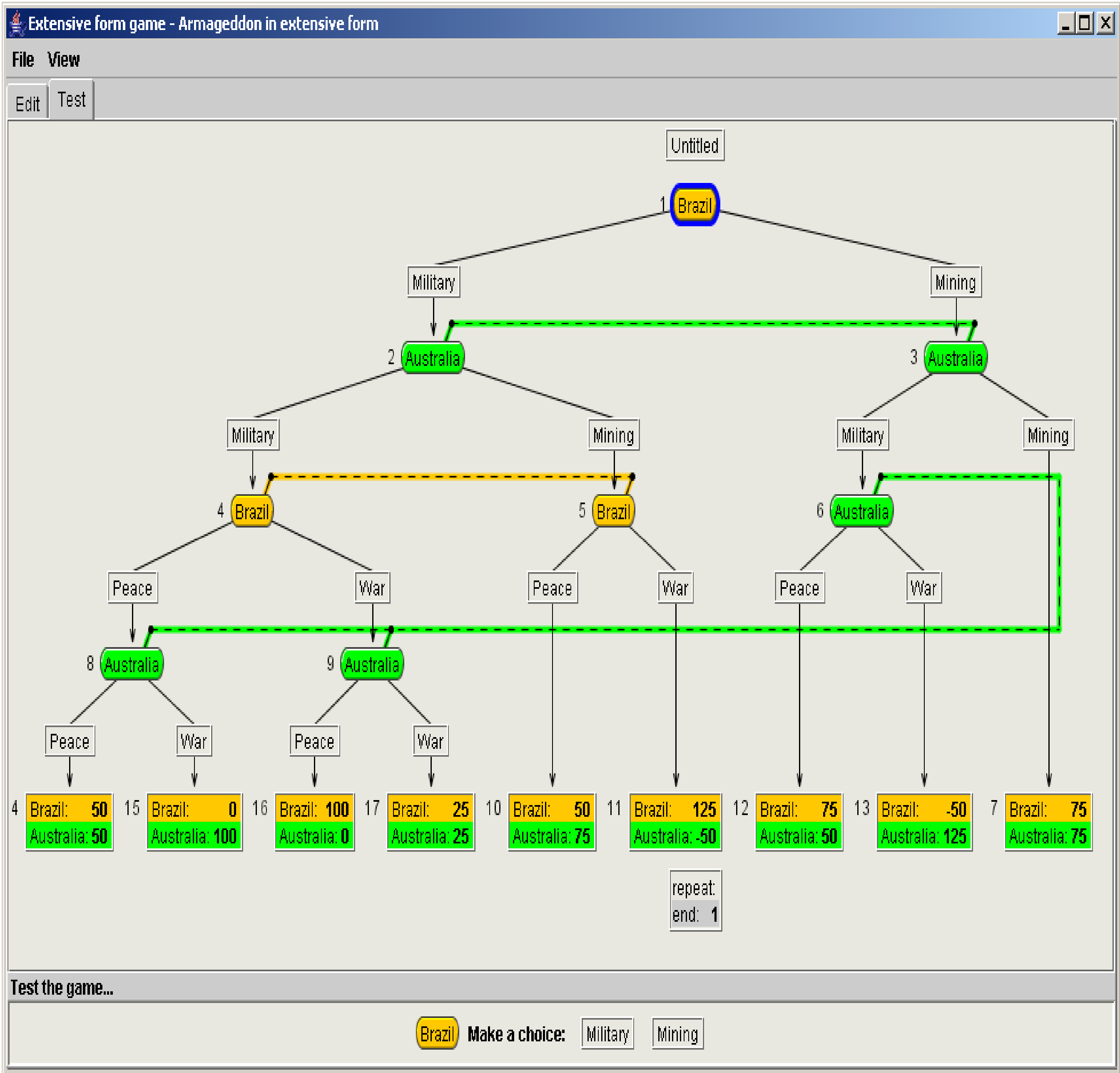


Figure 10.3
Armageddon

This is a symmetric game, and the both the strategy space and the payoff matrices given in Figure 11.2 reflect that. Each player has three strategies, mining, choosing to mount a military expedition and but then withdrawing, or mounting a military decision and declaring war.

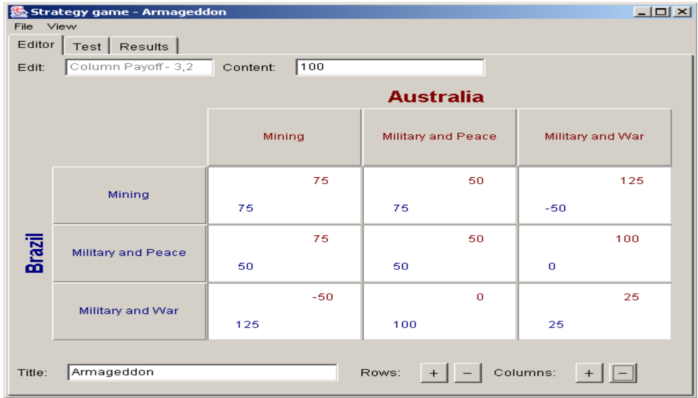


Figure 10.4
Strategic Form of Armageddon

From the strategic form it is clear that there is a dominant strategy for both players, to engage the military and then declare war. Notice that in this example the absence of information has no effect on the outcome of the game. If the game was modified so that every information set was a singleton, and all actions of the players were observed, applying the backwards induction algorithm in the perfect information analogue to this game.

Design of the Experiment

The Armageddon game has a strictly dominant strategy and the principle of dominance can be expoited in solving the games of incomplete information. The experiment conduted to investigate the relationship between the extensive form representation of the game and the strategic form representation of the game.

How to investigate the properties of incomplete information? What do you want to find out?

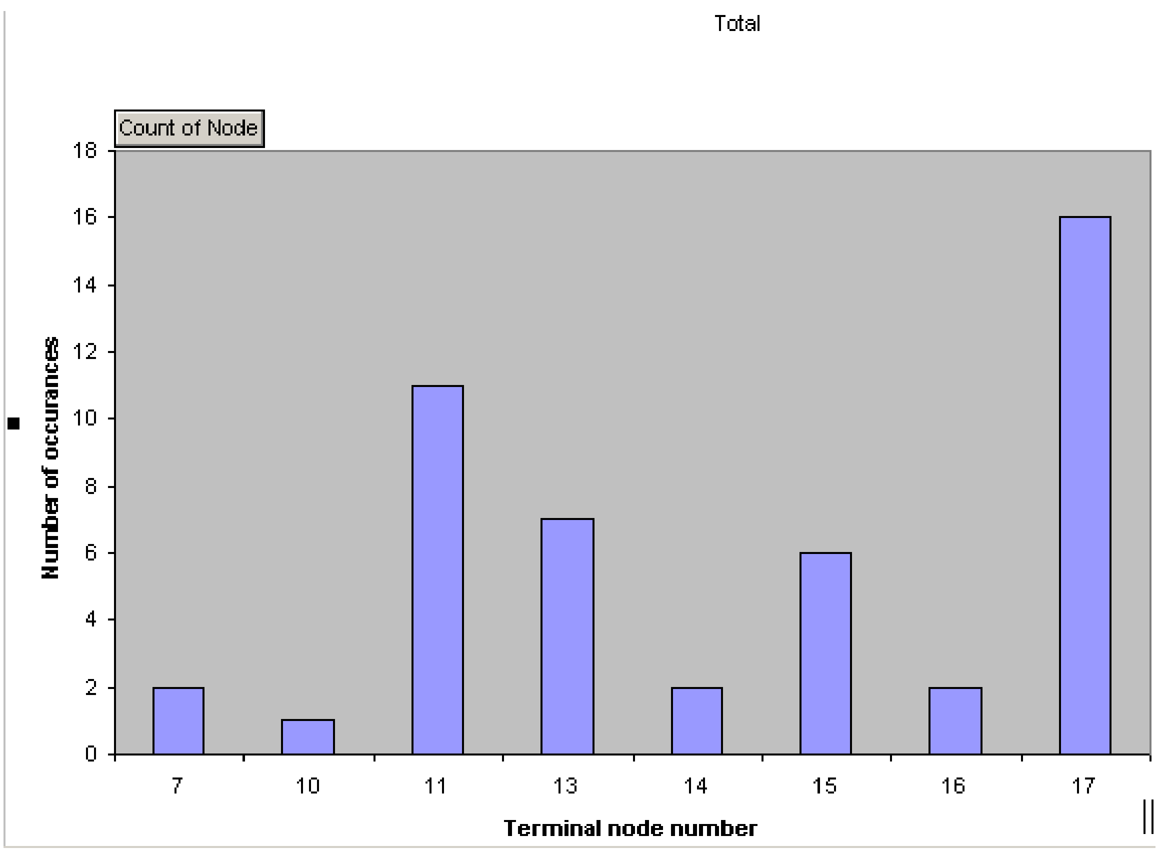
How to construct a model that illustrates the incomplete info? Formulate the hypothesis to be tested .

Design a procedure and list the things needed. What procedure should be followed to test the hypothesis? What measurements and observations are needed to determine whether your hypothesis is correct?

How do we test for the presence of in. info?

Results from the experiment

Nobe Number	Number of Occurances
7	2
10	1
11	11
13	7
14	2
15	6
16	2
17	16
Total number of observations	47



	Mining	Military and Piece	Military and War
Mining			

	Terminal node 7; Mining/Mining	Terminal node 10; Military and Piece/Mining	Terminal node 11; Military and War/Mining	Terminal node 12; Mining/Military and Piece	Termina node 4; Military Piece/M
Extensive form					
Strategic Form					
Total					

Traffic

Illegal trade is big business. The stakes, and the excitement, are high for those playing on both sides of the law. The following example show why random behavior is inherent part of drug trafficing and also of law enforcement authorities' attempts to curtail it. In its war on drugs, the FDA seeks to imprison those who profit most from the drug trade, partly because it directly reduces the supply of drugs, and also because it reduces the incentives of those people who specialize in the trade to enter it in the first place. An important tool of the FDA is to bribe small operators into betraying larger and more dealer . Figure 11.3 displays the extensive form of a game which models some essential features of the conflict between the illegal traders and the agency vested with powers to arrest them. At the initial node the dealer chooses between proposing a job to his drug ring boss, or first approaching the FDA about the possibility of a sting that could lead to his boss's arrest. If the dealer chooses the latter, one response of the FDA is to arrest and convict the dealer on relatively minor charges. Another is to order a sting in the hope of ensuring the boss. Thus when the boss is approached by the dealer, he understands that the plan to enter a new market might be genuine or a ruse.

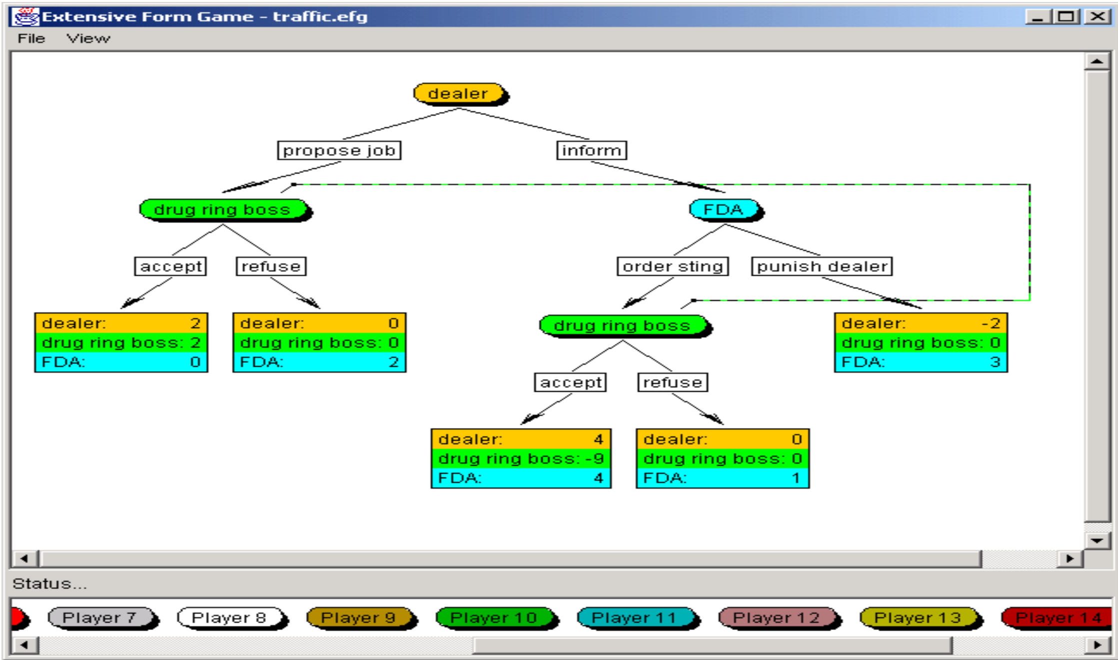


Figure 11.3
Traffic

Since there are no proper subgames in Traffic, we proceed to analyze the strategic form. We first demonstrate there is no pure strategy equilibrium, by showing that the FDA and the dealer must randomize in equilibrium. For suppose that if given the opportunity the FDA would sting. The strategic form of the induced game for the remaining two players is displayed in Figure 11.4. The payoffs in this strategic form are derived from the extensive form in Figure 11.3. For example, if the rejects the proposal, the payoffs at Nodes 5 and 9 in Figure 11.4 to the dealer and the boss are, respectively are both regardless of whether the dealer propose a sting to the FDA or a job to the boss, which explains why all the elements in right column of Figure 11.4 are zero.

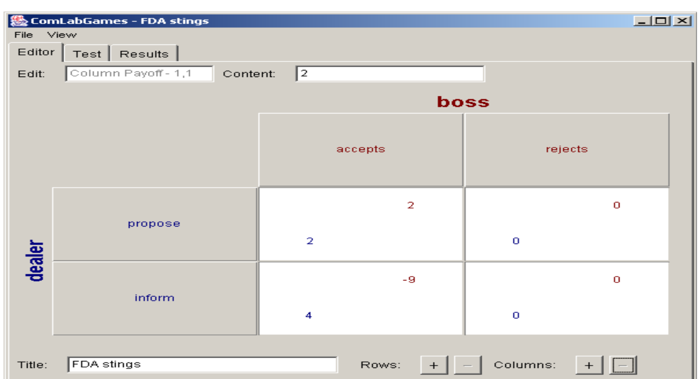


Figure 11.4
FDA would order sting

In this induced game, both players have weakly dominant strategies. The dealer should inform and the boss should refuse. Referring to Figure 11.4, their strategy choices generate a payoff of 1 to the FDA, compared with a payoff of 3 that can be attained from punishing the dealer. This argument proves that in every pure strategy equilibrium the FDA does not order a sting.

Accordingly, suppose the FDA punishes the dealer. Figure 11.5 shows the strategic form of the game induced by this pure strategy. From Node 7 in Figure 11.3, we can see that in the case the payoffs to the dealer and the boss are respectively $(-2, 0)$, and do not depend on the how the boss would have responded if he had been given the opportunity. The other two entries remain unchanged because the outcomes stemming from the dealer not affected by the strategy chosen by the FDA.

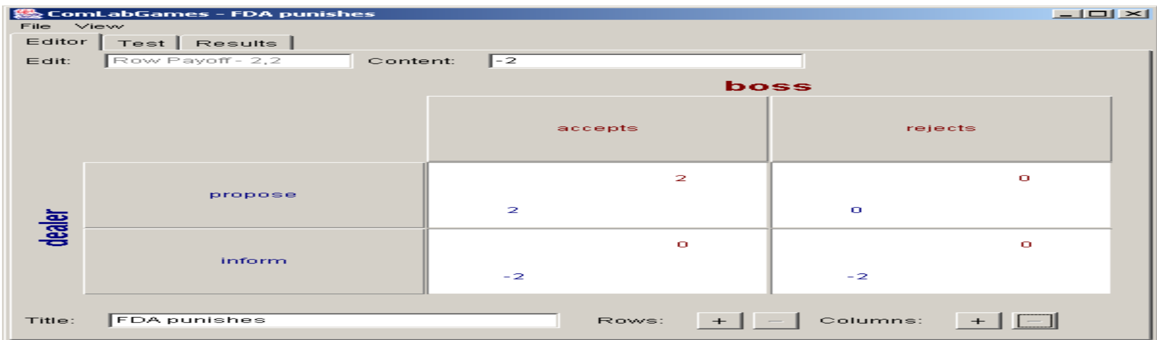


Figure 11.5

FDA would punish dealer

Now the dealer has a strictly dominant strategy of proposing to the dealer, and dealer has a weakly dominant strategy of accepting the job. Therefore the unique solution to this induced game is for the dealer to propose the job, and for the boss to accept it.

This only leaves (propose, punish, accept) as a candidate for an equilibrium profile in which the FDA does not randomize. Using the same approach we now show that (propose, punish, accept) is not a solution. For consider the strategic form of the game induced by the boss accepting, displayed in Figure 11.6.

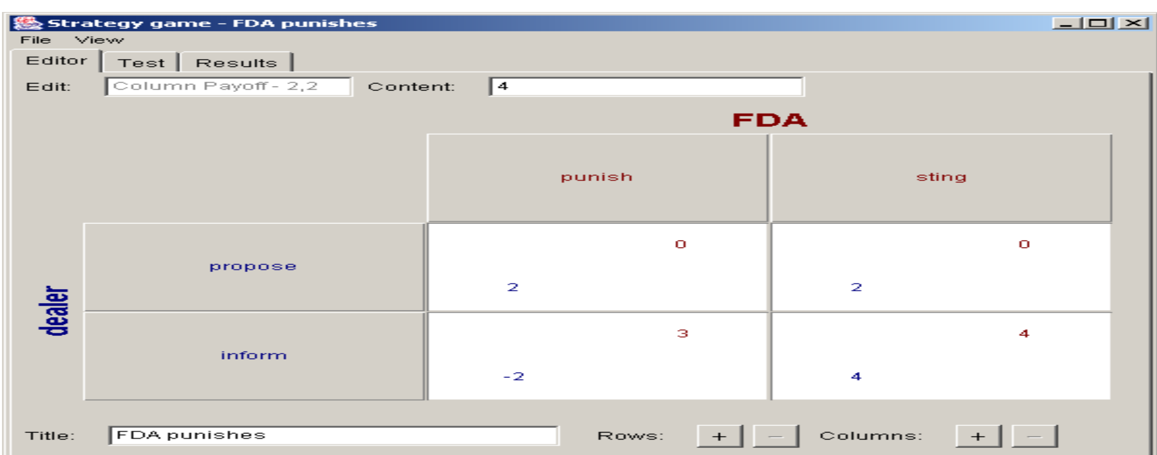


Figure 11.6
Boss accepts

In the solution to this induced game, the FDA orders a sting because it is a weakly dominant strategy (and the dealer's best response is to inform), contradicting the candidate equilibrium choice prescribed for the FDA.

Having established that the FDA mixes in equilibrium, this game can be solved in the standard way. Let p denote the probability that the dealer informs, q the probability that the FDA mixes, and r the probability that the boss accepts. Note that the FDA will not mix unless it is indifferent between ordering a sting and punishing the dealer. Reviewing Nodes 7 through 9 in Figure 11.3, this requires the boss to pick a mixed strategy, so that the expected benefit of ordering a sting, $4r + (1 - r)$, is equated with the sure benefit of punishing the dealer, 3. Solving for r, in equilibrium:

$$4r + (1 - r) = 3 \Rightarrow r = 2/3$$

Naturally the boss will not mix unless he is indifferent between accepting or refusing any proposal that is put to him. Inspecting Nodes 5 and 9 in Figure 11.3, the value of refusing is clearly zero, whereas the value of accepting depends on the probabilities that dealer informs and FDA stings:

$$2p + [(-9)q + 0(1 - q)](1 - p) = 2p + 9q - 9p$$

Equating this expression to zero and solving for q we obtain

$$2p + 9q - 9p = 0 \Rightarrow q = 7p/9$$

Since we have already established that the FDA mixes in equilibrium, both p and q must be strictly positive. The dealer's indifference between informing versus proposing provides the last equation necessary to fully solve the mixed strategy equilibrium. The value to the dealer from proposing is $4/3$ while the value of the dealer from informing is:

$$\frac{2}{3}4q - 2(1 - q) = 2 - \frac{14}{3}q$$

Equating both expressions yields the equilibrium value of q, and hence p :

$$\begin{aligned} 4/3 &= 2 - \frac{14}{3}q \\ &\Rightarrow q = 1/7 \\ &\Rightarrow p = 9/49 \end{aligned}$$

Total quality Management

Producers have the opportunity to compromise the versatility, strength and integrity of their products by using cheaper materials and tolerating poorer workmanship. The advantage of this strategy is that costs are reduced, the disadvantage is that consumers might shun the product, thus reducing its demand. Retailers play a role in the marketing process, preferring to sell brands that consumers are happy with. Because retailers play a monitoring role, producers also benefit from the promotion of their produce through the amount of shelf space and store advertising

In this game the producer chooses between manufacturing a defective or a flawless item, which is then delivered to a retailer. The retailer inspects its quality and decides whether to market the item or not. If the item is returned to the producer as defective, the producer incurs a loss. If the item is marketed by the retailer the producer gains regardless of whether the item sells or not, but would much prefer a sale.

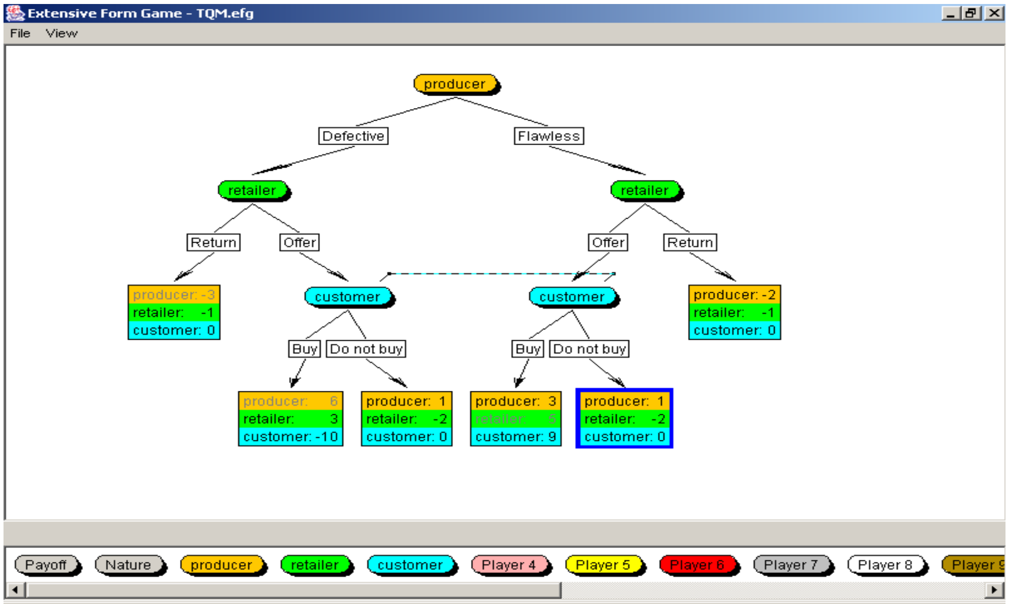


Figure 11.17
TQM

To solve this game we first remark that the consumer plays a mixed strategy in equilibrium. This claim can be established by contradicting the alternative hypothesis that she plays a pure strategy. First suppose that in equilibrium she buys the product. Regardless of whether the producer manufactures a defective or a flawless item, the conditionally dominant strategy is for the retailer to offer the item for sale. Anticipating this will be the case the producer manufactures a defective product. Therefore a pure strategy equilibrium profile in which the consumer buys the product However this is not a best response by the consumer to the strategies of the manufacturer and the reailer. She would be better off by not buying the defective product for sale. Alternatively suppose the consumer chooses the pure strategy of not buying the item. Anticipating that the retailer returns the item to the producer. Recognizing this will happen the producer would rather have the more durable item returned fro uses elsewhere. Hence that candidate equilibrium is for the producer to ship a flawless item to the retailer, the retailer to return it and the shopper not to take it if offered. These strategies are conditionally weakly dominated and therefore cannot be part of a solution.

Let q_1 the probability that the retailer offers a flawless item for sale, q_2 the probability that the retailer offers a defective product item sale and r the probability the shopper buys the item. The arguments in the previous paragraph imply both probabilities are strictly positive. receiving both kinds of products is strictly positive if the shopper mixes between buying and not buying the product, then she must be indifferent between making either choice.

Noting q_1 is strictly positive, if $q_1 < 1$, then the retailer is indifferent between offering a flawless product and returning it. Then

$$5r - 2(1-r) = -1 \Rightarrow r = 1/6$$

Similarly if $0 < q_2 < 1$, then the retailer is indifferent between offering a defective product and returning it. In that case

$$3r - 2(1-r) = -1 \Rightarrow r = 1/4$$

These equations cannot hold simultaneously, it follows that $q_1 = 1$ and/or $q_2 = 1$.

The producer will only mix between defective and flawless items if the benefit from both are equated. Therefore

$$[6r + (1-r)]q_2 - 3(1-q_2) = [3r + (1-r)]q_1 - 2(1-q_1)$$

There are two cases to consider:

1. If $q_1 = 1$, then:

$$[6r + (1-r)]q_2 - 3(1-q_2) = [3r + (1-r)] \Rightarrow 2r - rq_2 + 4q_2 = 4$$

If $q_2 = 1$ as well, then $r = 0$ which contradicts our precious finding that the probability of the shopper purchasing the item is strictly positive. If $q_2 < 1$, then $r = 1/4$ and this implies $q_2 = 14/15$.

2. If $q_2 = 1$, then:

$$[6r + (1-r)] = [3r + (1-r)]q_1 - 2(1-q_1) \Rightarrow r + rq_1 + 3 = 2q_1$$

If $q_1 = 1$ as well, then $r = -1/2$ which contradicts the fact that r is a positive number. But if $q_2 = 1$, then $q_1 < 1$, which implies $r = 1/6$. Substituting the values for q_2 and r into the equation above, and solving for q_1, we obtain $q_1 = 17/11$, which contradicts the fact that q_1 is a probability.

Investigating the cases above shows that in a mixed strategy equilibrium $(q_1, q_2) = (1, 14/15)$. The only unsolved parameter is the equilibrium mixing probability the producer chooses. Accordingly let p denote the probability a flawless item being produced. Since the shopper is indifferent between buying the item versus leaving it on the shelf, there are no expected benefits of acquiring the item :

$$10pq_1 = 9(1-p)q_2$$

$$150p = 9(1-p)14$$

$$(75 + 63)p = 9.7$$

$$p = 63/138$$

Substituting for (q_1, q_2) the we find $p = 63/138$.

It is straightforward to calculate the profits to the three players.

The effects of offering a partial refund are quite dramatic. Suppose the consumer could return a defective item, and partially reimbursed for his losses, say a little more than the extra net benefit from the producer from using a less costly manufacturing process. In this case the extensive form looks like Figure 11.18. In this the producer pays the consumer 4 for

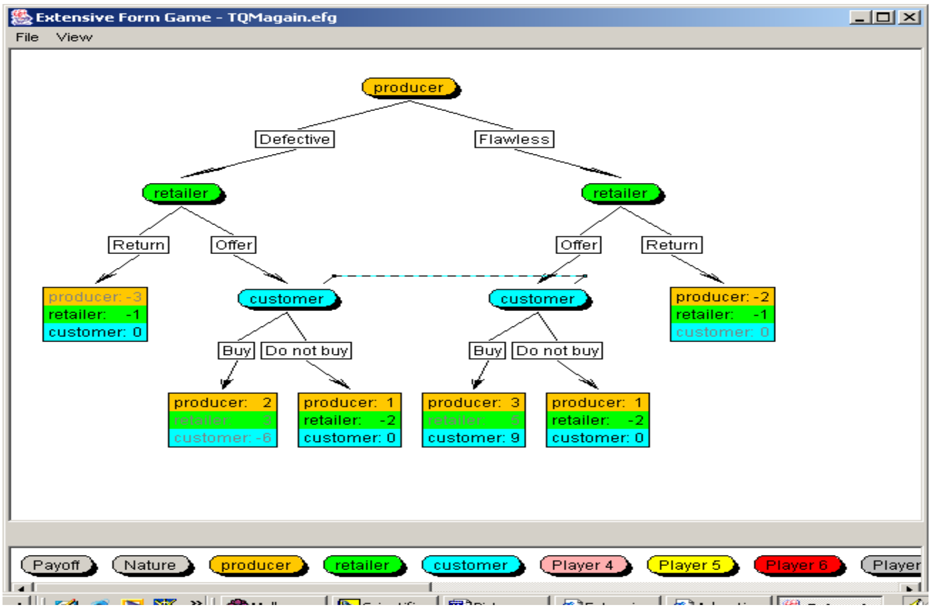


Figure 11.18

Partial Refund

There is now a unique strategy to produce only flawless items. The gains to each party are. The difficulty in implementing this policy is the commitment required to enforce the warranty. Leads to brand name products.

Incomplete information induced by nature

More on incomplete information

We will conclude our analysis of games with incomplete information, by investigating the equilibrium outcomes when nature, rather than a player's move is the source of the difference in information.

More examples of games with incomplete information

A buyer of health insurance, or car insurance, typically has access to information that affects the probability of making a claim that the seller does not have.

A seller has more information about the product he is retailing than the buyer but the buyer makes his choice after the seller has decide to retail the product or not.

Startup

An entrepreneur seeking relatively small amounts of resources to form and run his own company might be more likely to convince his workers to build up the firm by taking stock in the company in lieu of higher wages rather an outside venture capitalist. There are two main reasons why this might be so. First workers and the entrepreneur can more easily monitor each other's efforts, both at the workplace and also the consumption patterns. An outsider providing funds but not otherwise involved must spend effort to ensure the resources are not misused by the manager, and furthermore the entrepreneur also must expend effort monitoring the effort of a straight time worker.

Second, workers are more familiar with the production and marketing of a new product than an outsider, so are more likely to be in a position to contribute to the product development. An outsider must devo

of course they are limited by what workers are willing to participate in the firm as stockholders, and take on more risk than a diversified shareholder would. by forgoing the advantages a well developed capital market can offer, both the manager and the worker are exposing themselves. For this reason one is only likely to observe startup ventures like this firm when there is not much to collateralize and when the moral hazard problems are significant. computer software offers a case in point. The code is practically useless unless well documented, and the main cost of developing the program is in forgone wages, which cannot be collateralized in an economy where there are no slaves.

Even if all the conditions descried are pre, that is two sided moral hazard and little collateral to the company, Nevertheless does not resolve all the incentive problems, since the entrepreneur is likely to have a better idea of the project's success than his own employees, and he might be able to use that knowledge to his benefit and their expense.

The next game models the superior information that the entrepreneur has following game an entrepreneur . Some companies offer their employees to opportunity to take part of their compensation in stock options

and incurring a higher risk of bankruptcy

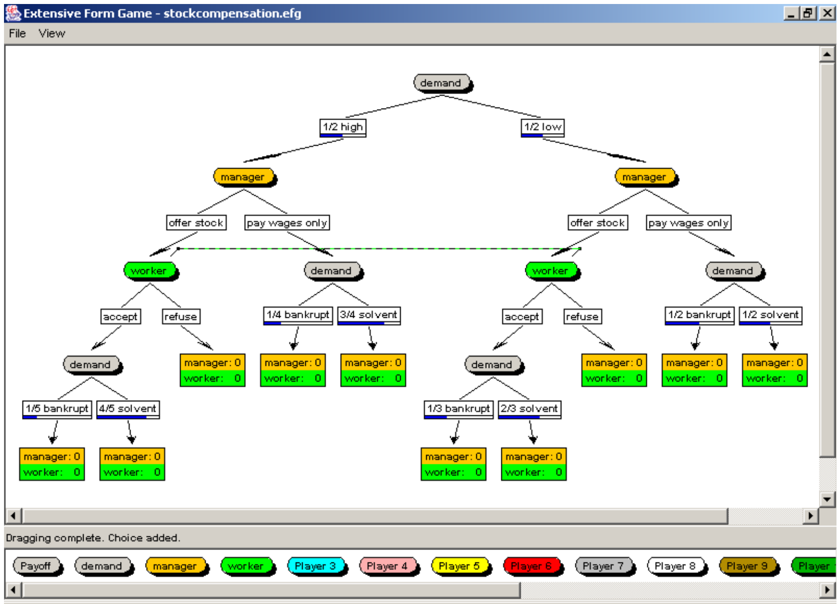


Figure 11.10
Startup

The final determination of demand can be folded into the game to yield a reduced game with an extensive form displayed in Figure 11.11.

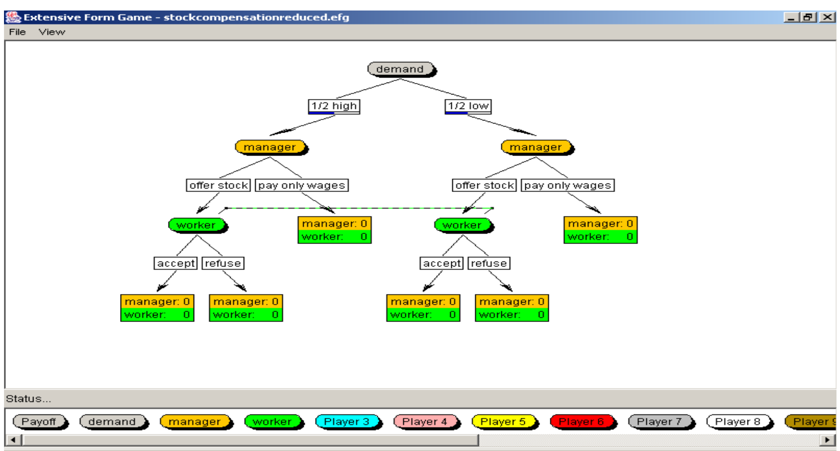


Figure 11.12
Reduced game for Startup

There are no proper subgames for startup, so searching for a solution leads us directly to the strategic form, displayed in Figure 11.13. The worker has only two pure strategies, to accept or reject the compensation package if stocks are offered to him. By way of contrast, the entrepreneur has four pure strategies to choose from, paying wages only regardless of demand conditions, offering the stock regardless of demand conditions, paying wages only if demand is poor, and paying wages only if demand is strong.

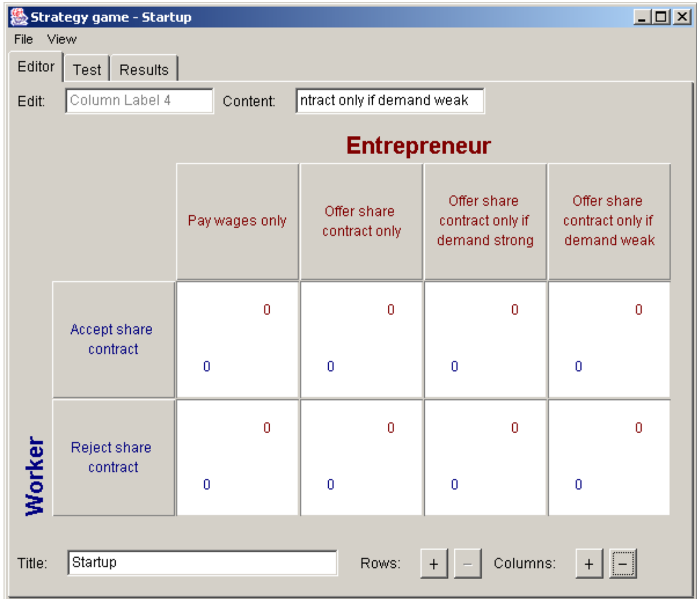


Figure 11.13
Strategic Form of Startup

Health insurance

When choosing between insurance options, it is in the interests of customer clients have an .

Not all the information can be easily hidden. For example, agents use actuarial tables, by age, gender and other factors (may ask for voluntary histories and so on. If the agents know the sources of information available to the client, they can simply request them. Those who refuse to comply with the request are treated has poorly as those whose records show the worst possible case, repeat accidents,

Not all information is verifiable, and the agent may not think of how the a patient formed his own view about health, or whether it is well founded. In this example there are two kinds of clients, those whose risk of death is 5 percent within , and those whose risk of is 10 percent. Three quarters of them are in the low risk category, each person knows to which category he belongs, but the insurance agency does not. It offers two policies, full and partial, and also rejects some. which policy a person gets is determined jointly be the client and the agent. person selects his preferred policy, there is some probability he receives the policy he has requested, but also will be downgraded

are distributed throughout the population in the ratio of 3 to 1

Figure 11.10 displays the extensive form. Genetics determines whether a client is high risk or not. Based on this information the client request a policy, understanding that he might be given something less than he requested, the agent decides which policy to

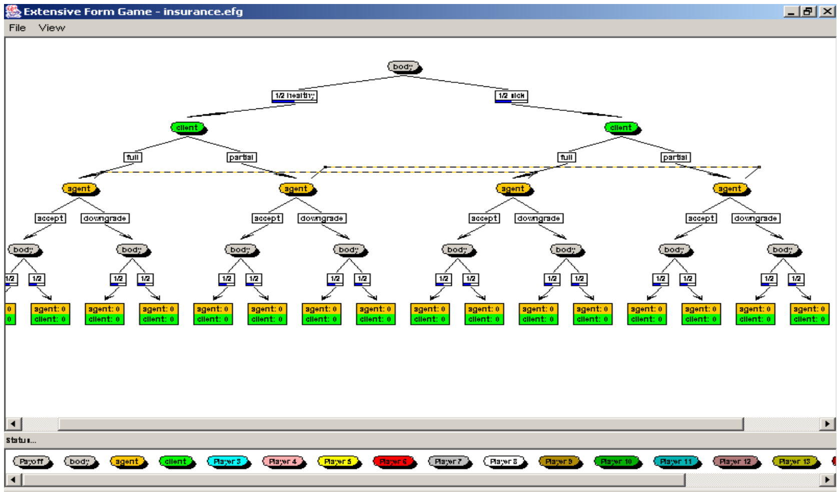


Figure 11.10
Health Insurance

The first step in solving this game is to fold back the natures final move in the game. Figure 11.11 displays the results.

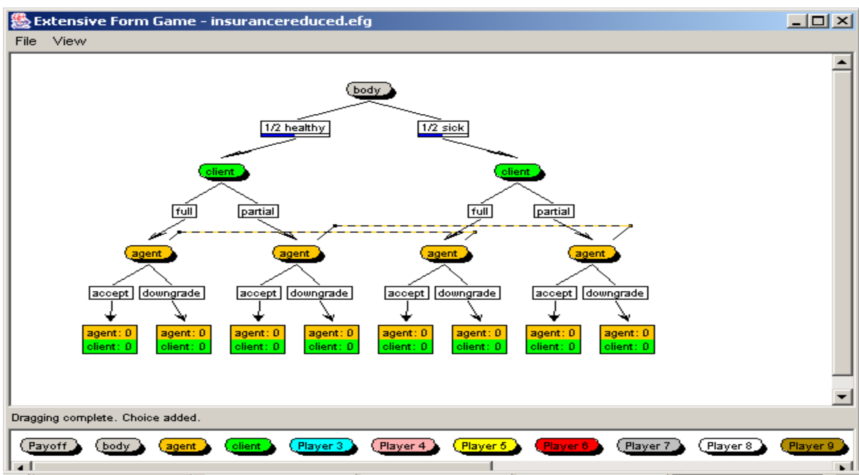


Figure 11.11
Reduced game for Health Insurance

One can see from the extensive form of this reduced game that there are no proper subgames, so we proceed by solving the strategic form. Each player has four strategies, which are listed on the left side of the rows and the top of the columns in Figure 11.12.

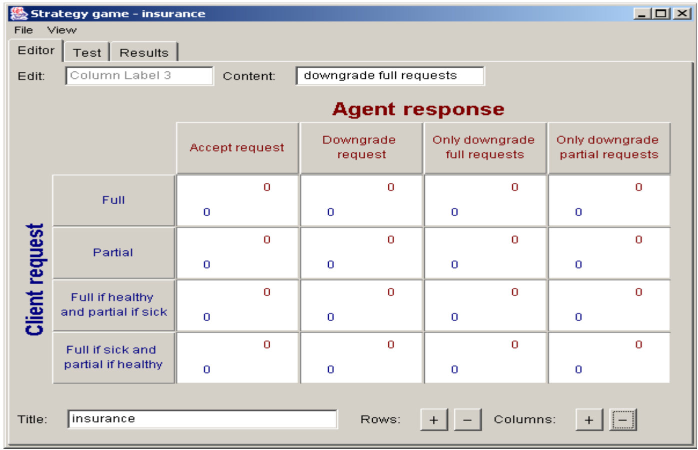


Figure 11.12
Insurance

Mechanic

The problem of getting truthful professional advice is illustrated by the stereotypical experience that car and house owners face when dealing with tradesmen and mechanics. Suppose the owner detects an noise that never occurred before and occurs intermittently. Past experience with previous vehicles and those of her friends suggests that there is an even chance that this signals a severe problem that will grow worse if remedial action is not taken, versus a minor problem that can be ignored with any repercosion If the problem is sever the car will break down and cost $2,000, in which case the mechanic has 20 percent chance of getting the job. If he does not get the job, he will be otherwise occupied with jobs from other clients. Her first choice is whether to visit the mechanic, a visit that would cost $200 for his diagnosis. For this price the mechanic can indeed tell what the nature of the problem is and how to fix it. He chooses to explain what is wrong with the car, or . At this point the car owner decides whether to have the car repaired or not.

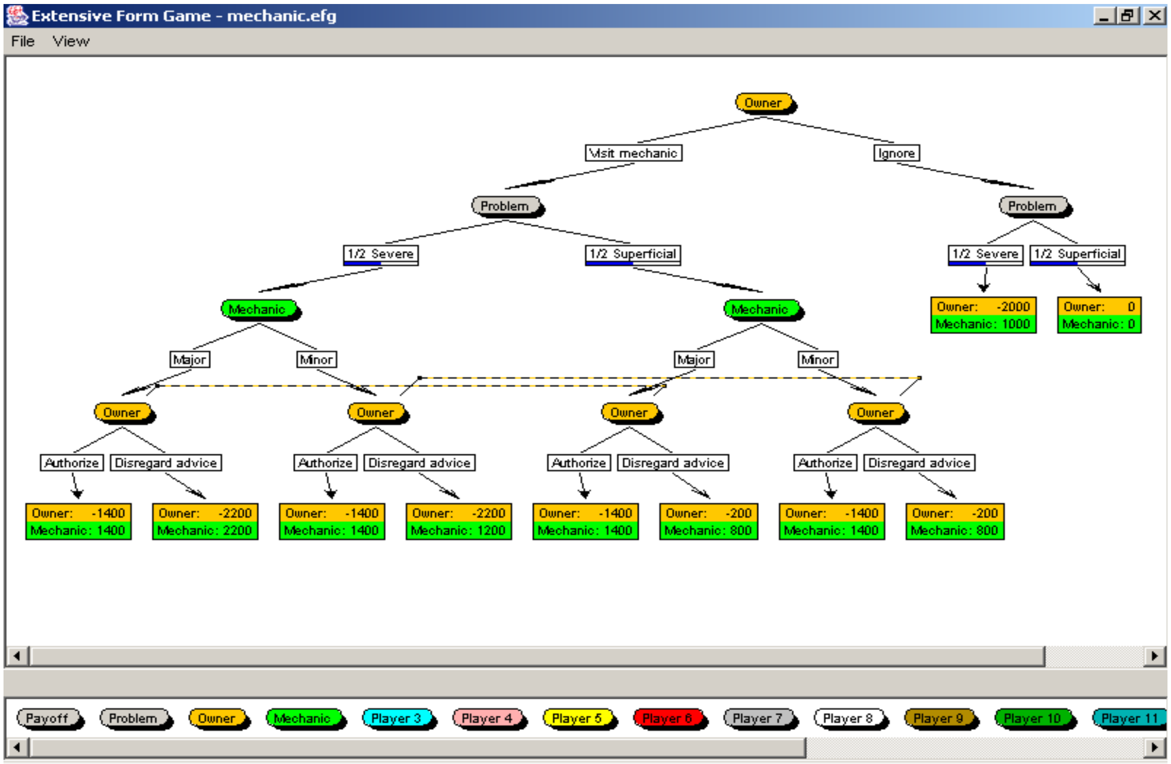
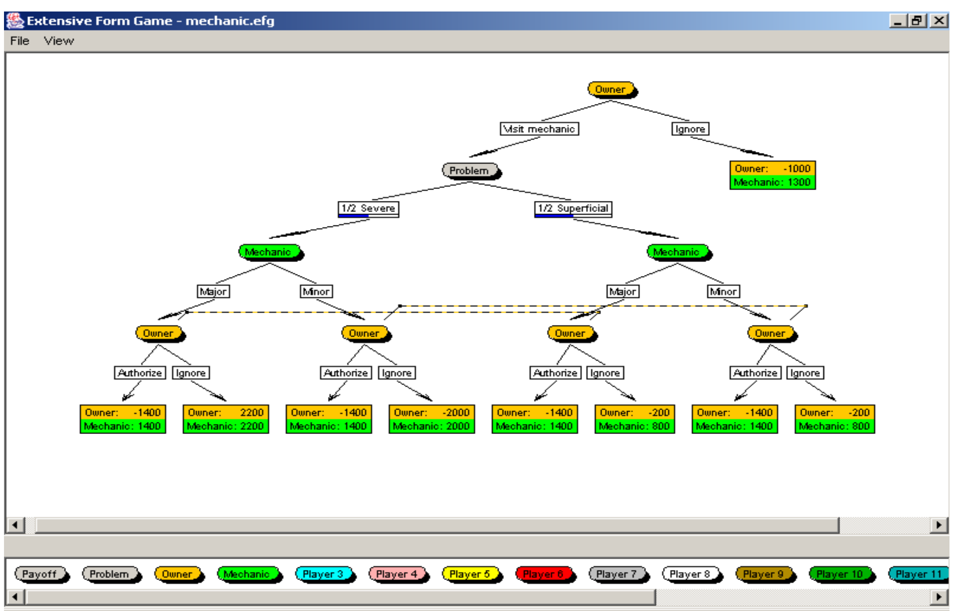
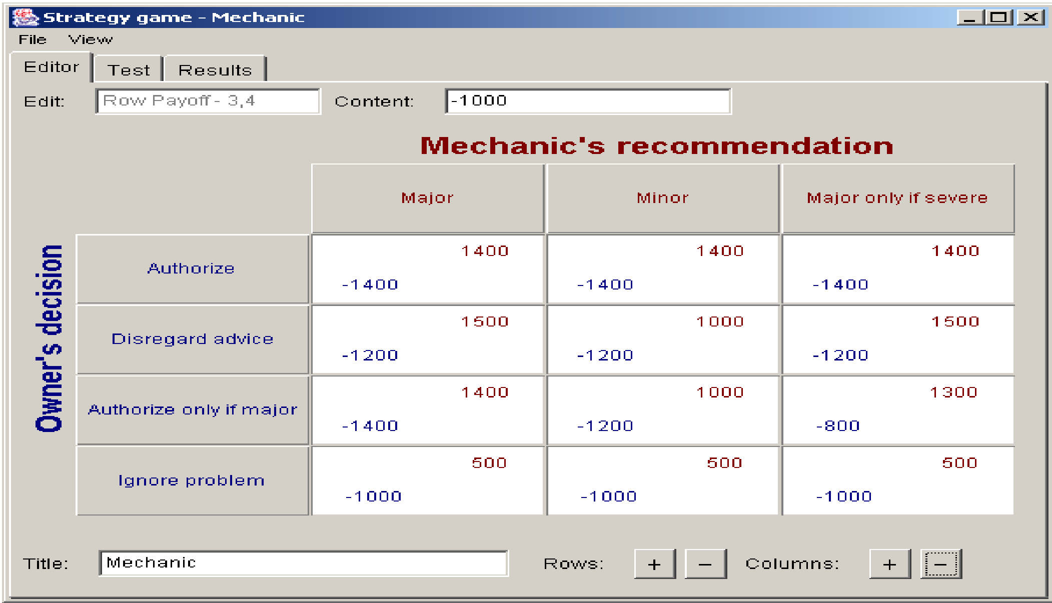


Figure 11.7

Mechanic

The incomplete information structure of this game arises from the fact that





Mechanic's recommendation

Owner's decision	Major	Minor	Major only if severe
Authorize	1400, -1400	1400, -1400	1400, -1400
Disregard advice	1500, -1200	1000, -1200	1500, -1200
Authorize only if major	1400, -1400	1000, -1200	1300, -800
Ignore problem	500, -1000	500, -1000	500, -1000

Corporate Governance

The last example of incomplete information that is induced by players is a problem in the strategic roles played by auditors and shareholders in providing incentives to managers to act in the interests of the firm.

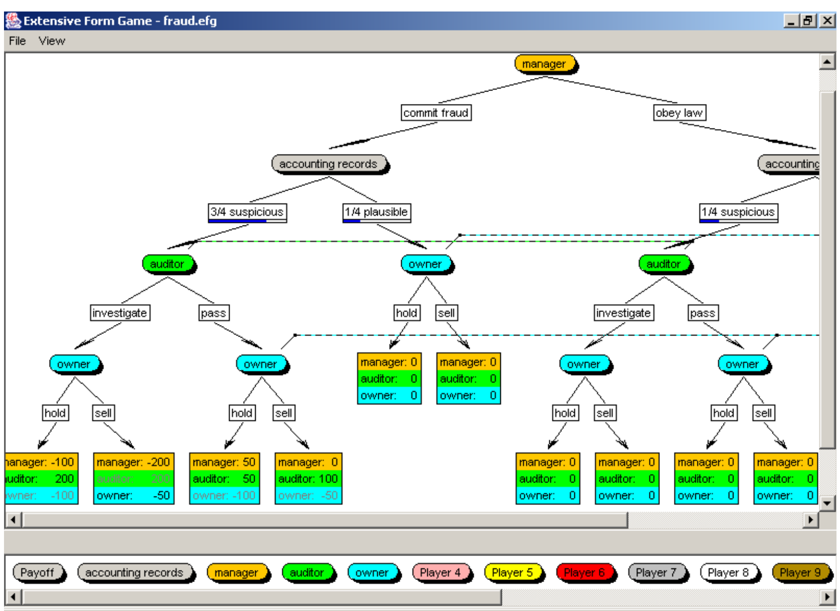


Figure 11.25
Corporate governance

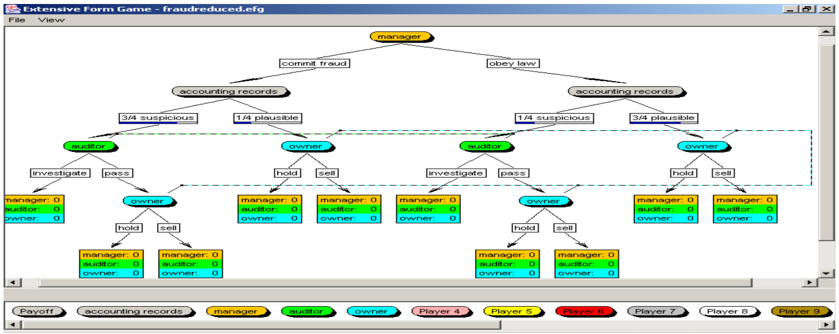


Figure 11.26

Reduce game of Corporate governance

In the reduced game, each player has two pure strategies. The manager decides between committing fraud or obeying the law, the auditor decides between conducting a full audit or an overview, and the owner decides between selling or holding his shares.

Supposing the manager chooses the pure strategy of obeying the law, then the auditor's best strategy is to scan the books. The best response of the manager to the auditor scanning the books is to commit fraud. Therefore choosing a pure strategy of obeying the law cannot be a solution strategy for the manager. Similar arguments to this one establish that both the auditor and the manager play mixed strategies in equilibrium.

The strategic value of information

Having completed the game theoretic parts of the course, this lecture uses the tools for solving games that we have developed to analyze the value of information in strategic contexts.

Information is power

We have all heard this phrase.

Is it true?

This lecture seeks to analyze the value of information in strategic contexts.

Espionage

When a country is planning to invade another, the tactical advantage of surprise partly compensates for the disadvantages confronting foreign troops seeking to overwhelm an army defending its own land. Consequently the defending force has a great incentive to neutralize the surprise element by infiltrating the intelligence network of the invaders, and communicating the invasion plans to the defenders.

In this example the United States is planning to topple the current regime in Iraq and replace it with one which has interests that are more closely tied to North America's. At the initial node it chooses between two main strategies, a land assault undertaken by a large number of ground troops, versus air strikes that are coupled with the strategic use of a relatively small force whose goal is to orchestrate and coordinate internal rebellion by opposition groups within Iraq. A spy working for the Iraqis has infiltrated U.S. intelligence. After a decision has been reached about the form of the invasion, the spy sends a message to the government in Iraq, and the Iraqis prepare for a land or air attack.

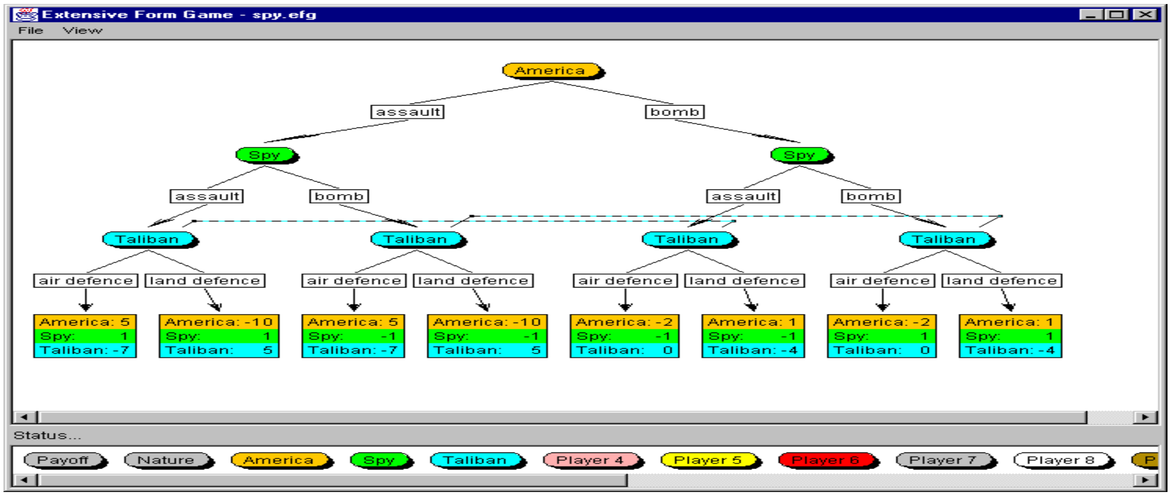


Figure 11.15
Espionage

Perhaps the easiest way of solving the game of espionage is to first focus on the spy's strategy set, which contains four elements:

1. Send assault message when the U.S. is planning an assault, and bomb message when the U.S. is planning to bomb
2. Send bomb message when the U.S. is planning an assault, and assault message when the U.S. is planning to bomb
3. Send assault message regardless of what the U.S. is planning
4. Send bomb message regardless of what the U.S. is planning

Reviewing the payoffs of the extensive form, one can see that regardless of the Iraqi reaction to the message, the first strategy of truthfully reporting is a dominant strategy for the spy. Therefore the Iraqis receive accurate information about the Americans' war plans in equilibrium. Comparing Nodes . . . Consequently their best response is to mount a land defence if they receive the assault message, and an air defence if they receive the bomb message. Recognizing that their security has been breached, that the spy will truthfully reveal their plans to the Iraqis, and that the defenders will react appropriately, the Americans choose to bomb.

An ambitious approach would be to explicitly model the choice of whether the Iraqis should employ a spy or not and the value to the U.S. of engaging in counterespionage. However some insight about the value of espionage and counterespionage can be gleaned without going to such lengths, by investigating this framework. Whether spying is permitted depends on the value to their masters and the costs of preventing them.

What is the value of spying?

There are really two different questions:

1. What is the penalty the U.S. pays from being infiltrated?
2. What are the benefits from spying to the Taliban?

Retaliation without spies:

If there were no spies, the extensive form of the game would become:

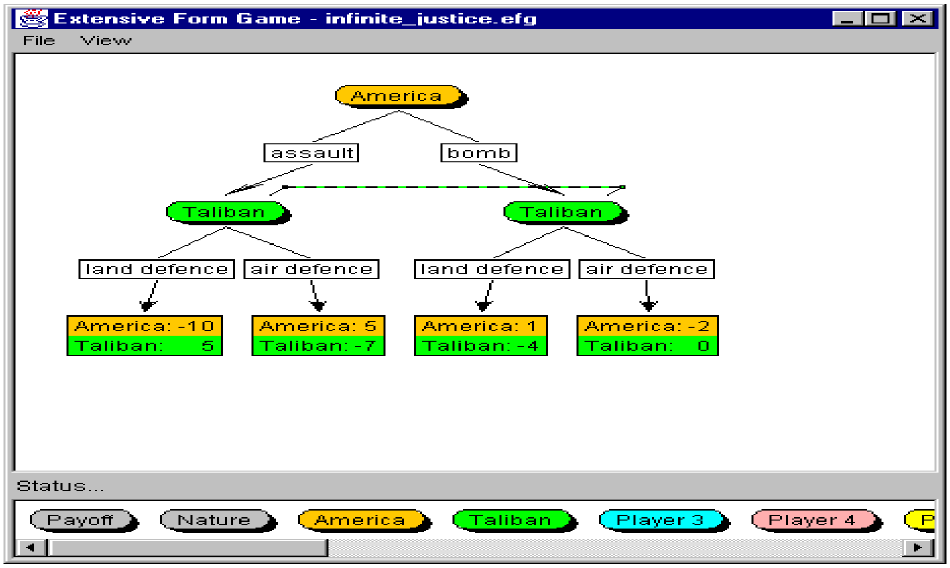


Figure 11.16
Infinite Justice

Solution to infinite justice

If there are no spies, there is a unique mixed strategy solution. The U.S. mounts an assault by land with probability 1/4, and the Taliban prepares for a land defense with probability 7/18.

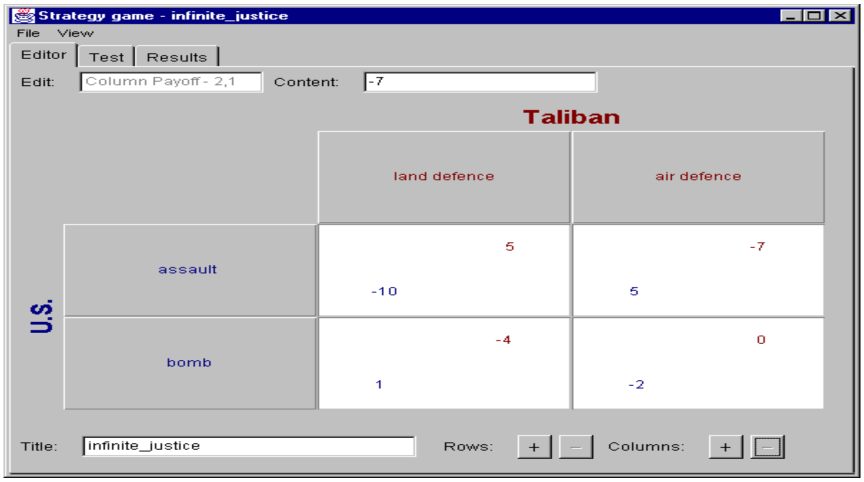


Figure 11.17
Strategic solution to infinite justice

Value of spying to the Taliban

The value of the espionage game to the Taliban is 0, whereas the value of infinite justice is:

(5*7 - 7*11 - 4*7*3)/4*18 = -7/4

Therefore if presented with the opportunity to choose which game they played, the Taliban would be willing to pay up to 1.75 units to infiltrate the U.S. military establishment.

Value of counterespionage to the U.S.

Alternatively, the value of preventing infiltration to the U.S. is:

(-10*7 + 5*11 + 7*3 - 2*11*3)/4*18 = -15/18

whereas the value of the espionage game is –2. Therefore if presented with the opportunity to choose which game they played, the U.S. would be willing to pay up to 1.67 units to prevent an informant infiltrating its military establishment, and thus preserve the element of a surprise attack.

Medical malpractice

One problem health insurance providers face is fraudulent behavior by doctors who prescribe treatment for healthy clients. Consider the following extensive form game:

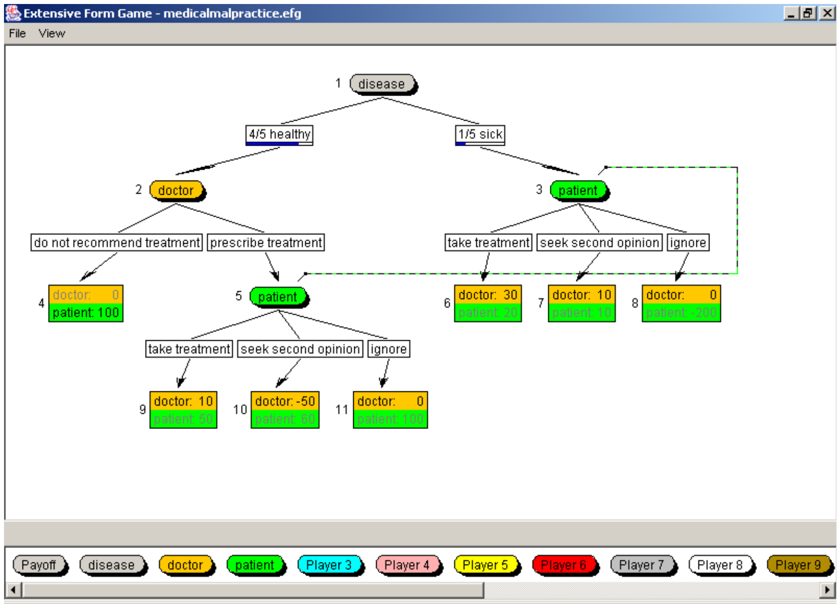
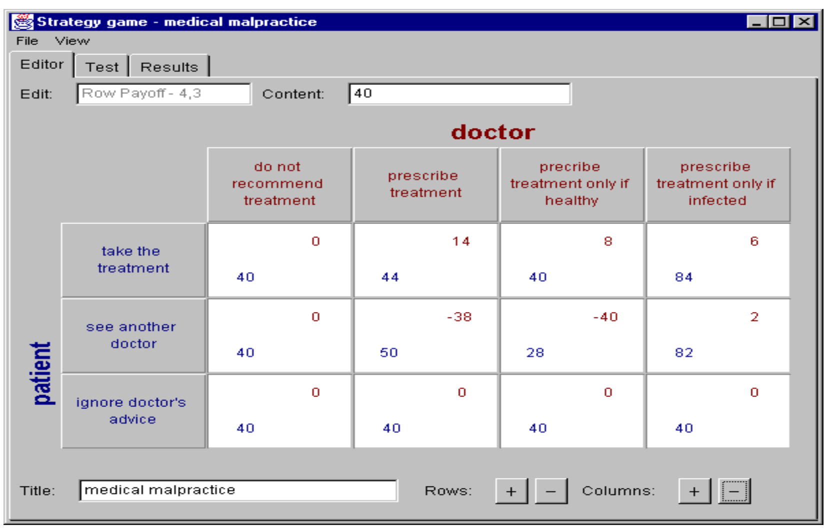


Figure 11.20
Medical malpractice

Strategic form of medical malpractice
This game has the strategic form:



patient \ doctor	do not recommend treatment	prescribe treatment	precribe treatment only if healthy	prescribe treatment only if infected
take the treatment	0 / 40	14 / 44	8 / 40	6 / 84
see another doctor	0 / 40	-38 / 50	-40 / 28	2 / 82
ignore doctor's advice	0 / 40	0 / 40	0 / 40	0 / 40

Figure 11.21
Strategic Form of Medical Malpractice

Strategic form of reduced game

After eliminating 2 weakly dominated strategies for the doctor, and iteratively removing one dominated strategy for the patient, we are left with:

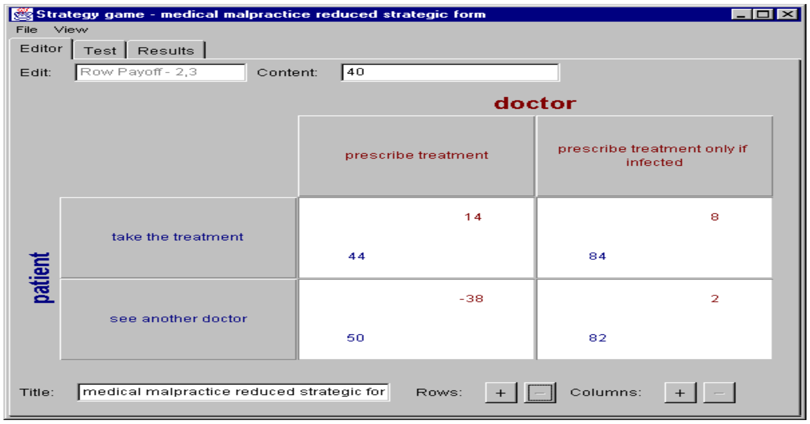


Figure 11.22
Reduced Strategic Form

Solution to malpractice

There is a unique mixed strategy solution, in which the patient takes the treatment with probability 9/20, getting a second opinion with probability with 11/20 while the doctor prescribes treatment 1/4 of the time regardless of whether the patient is sick, and 3/4 of the time correctly diagnoses the patient. This implies that the patient is incorrectly diagnosed with the disease 1/5 of the time, and healthy patients receive the treatment with probability 9/100, less than 10% of the time.

Verification

Is it profitable to invent a serum that verifies whether someone is ill or not? Consider the game after the truth serum (for doctors) is invented:

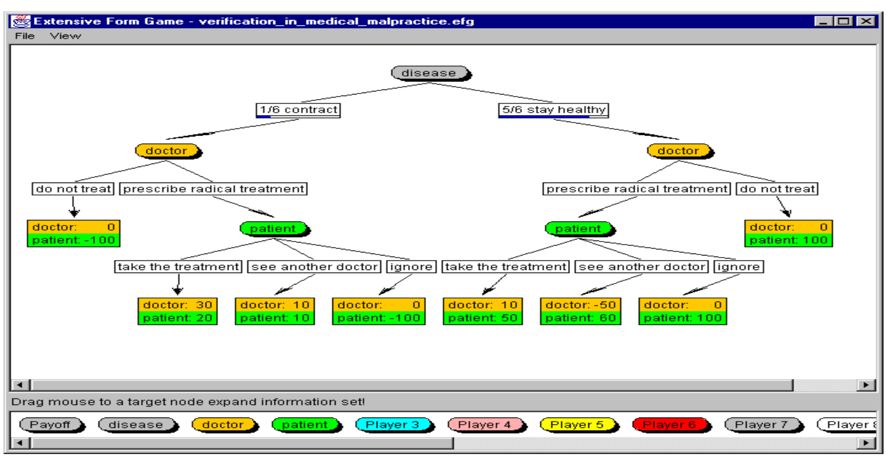


Figure 11.23
Perfect Test

Solving the perfect information game, the doctor will only prescribe the treatment to sick patients, and the patient will accept the treatment when it is prescribed.

Gains from test

The solution to the perfect information game yields an expected benefit of 84 to the patient and 8 to the doctor. In the malpractice game, the expected benefit to the patient is:

(9*44 + 9*3*84 + 11*50 + 9*3*82)/20*4 = 67.85

while the expected gain to the doctor is:

(9*14 + 9*3*8 - 11*38 + 9*3*2)/20*4 = 10.175

Perhaps there is an opportunity to develop a test, depending on its research and development costs.

What about a test that is right most of the time? How would the game play out then?

A Solution Approach

This chapter essentially concludes our general analysis of games. The remaining chapters in this book apply the techniques developed for solving games in the extensive and strategic form to more specialized structures. Thus the conclusion to this chapter is an appropriate place to take stock of what is an One final chapter in this section Summarizing the theory

The general theory for analyzing noncooperative games is based on two components, first classifying games in an abstract way in order to solve them, and second developing solution techniques. we have developed a somewhat complicated for describing the information structure of different games, which have heavily drawn upon.

Figure 11.30 shows the various classifications and their relationship to one another. The figures shows there are several ways of partitioning games by their information structure. Games have either perfect (the top half circle) or imperfect (the rest) information. Moves can be simultaneous (the lower half of the circle) or sequential (the rest). The information can have complete (left half of the rectangle) or incomplete (the right half of the rectangle). Finally games of perfect information can be partitioned into whether there is perfect foresight (left top quadrant of circle) or whether uncertainty plays an independent role in helping determine the outcomes (right top quadrant of circle).

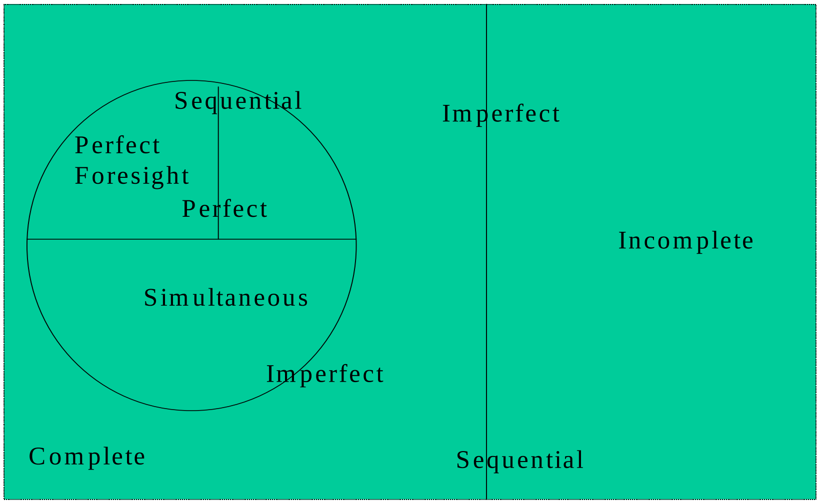


Figure 11.30
Information structure of games

We have developed five rules for seeking solutions to noncooperative games. They are:

1. Iteratively look ahead, find the shortest sub-games, and fold back.
2. Choose weakly dominant strategies.
3. Iteratively remove strongly dominated strategies.
4. Discard remaining weakly dominated strategies.
5. Find the Nash equilibrium in pure and mixed strategies for the reduced game.

These rules are the basis for an algorithm or heuristic to use when solving games. An outer loop uses the extensive form to guide the order in which the various parts of a game are solved. An inner loop uses the strategic form to solve subgames, reduced games, and conditional games.

Outer loop

The extensive form is used to:

1. Identify the largest set of sub-games partitioning the terminal nodes
2. Solve the reduced games and sub-games using the inner loop
3. Fold back and form the reduced games
4. Repeat until the initial node is reached in the only remaining reduced game

Inner loop

The strategic form is used to:

1. Select weakly dominant strategies

2. Iteratively eliminate strongly dominated strategies

3. Discard remaining weakly dominated strategies

4. Derive the pure and mixed strategy Nash equilibria for the reduced game

Conditioning on choices

There is one other tool we have used to help find solutions in sequential games of imperfect information, only useful when there are more than two information sets assigned to players within an inner loop of a proper subgame or reduced game. We condition on the various choices available at one or more information sets to analyze the potential for conditional dominance players amongst the other choices, and also the set of strategies that are conditionally rationalizable.

Summary

Consider again the introduction. Supposing Gamma is another player, who picks a pure strategy in equilibrium, then Beta would not change his behavior if he actually observed Gamma, since by the definition of equilibrium, his own choice must be a best response to Gamma. Can revealing Gamma choice to Alpha have any effect on the equilibrium?

Further Readings

There is a more advanced literature that imposes additional restrictions on the solution concept, in order to make tighter predictions. While these are outside the scope of this text, curious readers are invited to pursue this topic by referring to the references provided in the section on further readings.

Bibliography

Anderson, Chris, Colin Camerer, and Tech H. Ho (1998) “Learning and Logic-Based Refinements in Signaling Games,” University of Pennsylvania, Discussion Paper, presented at the Fall 1998 ESA Meeting. Abstract: Experience-weighted attraction models of learning are adapted to explain patterns of adjustment in signaling games with many periods.

Banks, Jeffrey S., Colin Camerer, and David P. Porter (1994) “An Experimental Analysis of Nash Refinements in Signaling Games,” Games and Economic Behavior, 6:1 (January), 1-31. Abstract: Subjects play a series of randomly ordered signaling games. These games have multiple equilibria, one of which is "ruled out" by a refinement of the sequential Nash concept. For the games considered, the refinements predict well, at least up to the Divinity refinement ("refine to divine" as one coauthor remarked).

Blume, Andreas, Douglas DeJong, Y.-G. Kim, and G. Sprinkle (1998) “Experimental Evidence on the Evolution of Meaning of Messages in Sender-Receiver Games,” American Economic Review, 88:5 (December), 1323-1340.

Bolle, Friedel, and P. Ockenfels (1990) “Prisoners’ Dilemma as a Game with Incomplete Information,” Journal of Economic Psychology, 11:1 (March), 69-84.

Brandts, Jordi, Jacob K. Goeree, and Charles A. Holt (1996, revised 1999) “Naive Bayesian Learning and Adjustment to Equilibrium in Signaling Games,” University of Virginia, Discussion Paper. Abstract: A logit learning model is used to explain dynamic adjustment patterns in signaling games. Simulations with the estimated learning and error parameters reproduce the qualitative behavior patterns in the choices made by human subjects. For some games, these adjustment paths lead to equilibria that are ruled out by the intuitive criterion and other refinements.

Brandts, Jordi, and Charles A. Holt (1992) “An Experimental Test of Equilibrium Dominance in Signaling Games,” American Economic Review, 82:5 (December), 1350-1365. Abstract: Behavior in sender-receiver signaling games converges to the "more refined" equilibrium in games that are based on the "beer-quiche" example. However, behavior does not converge to the more refined equilibrium when the payoffs are manipulated in a way so that decisions in initial matchings conform to "out of equilibrium" beliefs that are ruled out by the refinements. The conclusion is that beliefs are determined by experience in the process of adjustment, not by deductive logic that begins with players in equilibrium and then considers what can be inferred if a player deviates.

Brandts, Jordi, and Charles A. Holt (1993) “Adjustment Patterns and Equilibrium Selection in Experimental Signaling Games,” International Journal of Game Theory, 22:3 279-302. Abstract: Signaling games with "reverse type dependence" lead to behavior in the laboratory that violates all of the standard refinements of the Nash equilibrium (intuitive criterion, divinity, strategic stability, etc.) The history of the adjustment path seems to determine out-of-equilibrium beliefs that are ruled out by these refinements.

Cason, Timothy N. (1994) “The Strategic Value of Asymmetric Information Access for Cournot Competitors,” Information Economics and Policy, 6:1 (March), 3-24.

Cason, Timothy N., and Lata Gangadharan (1999) “Environmental Labeling and Incomplete Consumer Information in Laboratory Markets,” Purdue University, Discussion Paper, presented at the Summer 1999 ESA Meeting. Abstract: Producers in a laboratory posted offer market choose a level of environmental quality for the goods they sell. This quality is valued by consumers but cannot be observed by prior to purchase. The informational asymmetry results in inefficiently low levels of environmental quality that is only imperfectly corrected by reputations and signaling.

Cooper, David J., Susan Garvin, and John H. Kagel (1997) “Signalling and Adaptive Learning in an Entry Limit Pricing Game,” RAND Journal of Economics, 28:4 (Winter), 662-683.

Davis, Douglas D. (1994) “Equilibrium Cooperation in Three-Person, Choice-of-Partners Games,” Games and Economic Behavior, 739-53. Abstract: The game begins with a buyer choosing to purchase from one of the two sellers, who can then either deliver high quality or low quality (at a lower cost to the seller). Longer repetitions of this game with the same three subjects results in a higher incidence of cooperative, high-quality outcomes, supported by buyer switching as a punishment for the delivery of low quality.

Diekmann, A. (1993) “Cooperation in Asymmetric Volunteer’s Dilemma Game: Theory and Experimental Evidence,” International Journal of Game Theory, 22:1 75-85.

Holt, Charles A., and Roger Sherman (1986) “Quality Uncertainty and Bundling,” in Empirical Approaches to Consumer Protection Economics, edited by P. M. Ippolito and D. T. Scheffman, Washington, D.C.: Bureau of Economics, Federal Trade Association, 221-250. Abstract This paper investigates the performance of laboratory markets in which sellers can sell units in bundles.

Holt, Charles A., and Roger Sherman (1990) “Advertising and Product Quality in Posted-Offer Experiments,” Economic Inquiry, 28:3 (January), 39-56. Abstract: The experimental markets with asymmetric information allow an analysis of professional association rules that prohibit price and or quality advertising.

Huck, Steffen, and Wieland Mueller (1998) “Perfect versus Imperfect Observability – An Experimental Test of Bagwell’s Result,” Humboldt University, Discussion Paper, presented at the Summer 1998 ESA Meetings. Abstract: The experiment provides little support for Bagwell’s claim that the first-mover advantage vanishes if this action is only imperfectly observed by the second-mover. The first-mover advantage is not always fully exploited when it is perfectly observable, and the Stackelberg outcome has a lot of drawing power even when the first move is not perfectly observed.

Meidinger, Claude, Stephane Robin, and Bernard Ruffeiux (1998) “Repeated Game, Incomplete Information and Coordination: Experimental Results in the Investment Game,” Ecole Nationale Superieure de Genie Industriel, Discussion Paper, presented at the Summer 1998 ESA Meeting. Abstract: The paper uses Rabin’s fairness model as a basis for explaining results of a standard trust game.

Muller, Andrew, and Asha Sadanand (1998) “Virtual Observability in Two Player Games,” University of Guelph, Discussion Paper, presented at the Summer 1998 ESA Meeting. Abstract: "Virtual observability" is the principle that players in sequential games with imperfect information act as if they observed earlier player’s choices. Several standard games are played under three conditions: simultaneous play, sequential play with others’ moves being observed, and seqential play with others’ prior moves being unobserved. An analysis of individual decisions shows a tendency for decisions to shift in the direction predicted by virtual observability.

Peterson, Steven P. (1996) “Some Experimental Evidence on the Efficiency of Dividend Signaling in Resolving Information Asymmetries,” Journal of Economic Behavior and Organization, 29:3 (May), 373-388.

Potters, Jan, and Frans van Winden (1996) “Comparative Statics of a Signaling Game: An Experimental Study,” International Journal of Game Theory, 25:3 329-353.

Rapoport, Amnon, and James A. Sundali (1997) “Induction vs. Deterrence in the Chain Store Game: How Many Potential Entrants Are Needed to Deter Entry?,” in Understanding Strategic Behavior: Essays in Honor of Reinhard Selten, edited by W. Albers, Werner Gu*th, P. Hammerstein, B. Moldovanu and E. van Damme, Berlin: Springer-Verlag.