1 Introduction

Bargaining is one way to resolve a conflict between two or more parties, chosen when all parties view it more favorably than the alternatives. These include capitulation to the other parties, where tangibles are involved predation and expropriation coupled with protection of the booty, threatening war and destruction upon those those those who do not surrender. Bargaining also has elements of threat, coersion, and acquiesceness. Examples of bargaining situations include negotiations between unions and their employers about wages and working conditions, contract discussions between builders and their clients bargain over the nature and extent of the work, and prenuptial agreements written by partners betrothed to be married.

Unions warrant special mention in discussions of bargaining and industrial relations. They are defined as a continuous associations of wage earners for the purpose of maintaining or improving their remuneration and the conditions of their working lives. In the first half of the 20th century union membership grew from almost nothing to 35% of the labor force, only to decline to less that 15% at the turn of the millennium. Employment in the government sector increased from 5% in the early part of the 20th century to 15% in the 1980s, and then stabilized. Union membership in this sector jumped from about 10% to about 40% between 1960 and 1975. Employment in agriculture declined from 20% to 3% in the same period. This sector was not unionized at the turn of the 20th century. Unionization in the nonagricultural private sector has reflected the aggregate trend, declining to about 10% of the workforce down from 35%. Within the U.S. membership is highest in the industrial belt connecting New York with Chicago though Pittsburgh and Detroit (20 – 30%), lower in upper New England and the west (10 – 20%), and lowest in the South and Southwest (10% or less). Males are 50% more likely to be union members than females, mainly reflecting their occupational choices. Union membership differs greatly across countries. For the most part unions are the province of developed countries, but even within that group range from over 80 percent in Sweden to the levels we see in the United States. Unions are often mentioned in the context of industrial breakdown and strikes. Strikes are dramatic and newsworthy, but they are also quite rare: Less than 5% of union members go on strike within a typical work year. Less than 1% of potential working hours of union members are lost from strikes, before accounting for compensating overtime. About 90% of all collective agreements are renewed without a strike, but the threat of a strike affects more than 10%.

This chpater begins with some general remarks about bargaining and the importance of unions analyze the (two person) ultimatum game, extend the game to treat repeated offers, show what happens as we change the number of bargaining parties, broaden the discussion to assignment problems where players match with each other, turn to bargaining games where the players have incomplete information, discuss the role of signaling in such games.

When trade is so personalized that negotiations between the parties directly determine the terms of each individual exchange, the trading mechanism is best described as a bazaar or a bargaining game. This chapter uses the tools we have developed for noncooperative games to understand strategic behavior in extracting the gains from trade and exchange. We shall focus on three dimensions of bargaining: How many parties are involved, and what is being traded or shared? What are the bargaining rules and/or how do the parties communicate their messages to each other? How much information do the bargaining parties have about their partners? Answering these questions helps us to predict the outcome of the negotiations.

First are the bargaining process or rules of conduct. We begin with some bargaining models for two players, that is bilateral negotiations, starting out with the very simplest, called an ultatmatum game. Then we lay out increasingly flexible mechanisms for sending messages. In this way we demonstrate how the solution to bilateral bargaining games change as more complicated bargaining strategies are entertained.

The second issue we focus on is the information players have when they bargain with each other. The first games in this chapter presumes that the bargaining parties are fully informed about the preferences of all the other parties. The second topic investigates how the solution varies when some relevant information is hidden from one of teh parties.

Third is the number of players, and the main distinction we draw here is between bilateral and multilateral negotiations. In bilateral negotiations we argue little is lost by assuming the game is zero sum, which occurs when one person’s loss exactly offsets another person’s gain. Incidentally, we are not suggesting here that are no gains from interactions between two parties, only that the distribution of those gains is independent of the gains. by way of contrast, in multilateral negotiations, the potential gains from trade may differ depending on how the players are partition eissue changes as parties sometimes must decide who they will deal with. Thus the assignment or matching of players to activity and relationships assumes paramount importance in multilateral negotiations.

This question is taken up in Section 5 where we investigate the revelation principle. This states that the outcome of every game of incomplete information can be recast as revelation game, in which players are asked to reveal their private information, and in the solution of the game, they respond truthfully. This considerably simplifies the task of finding an optimal contract when there are not a full set of contingencies, since it proves that we can restrict ourselves to revelation games. In particular the principle shows that long drawn out discussions cannot yield an outcome more highly valued to the principal if the agents are rational. In a direct revelation game, each person has the opportunity to truthfully report his private information or lie about it, payoffs to the players are allocated according to the reports submitted by everybody, and an equilibrium strategy profile is for everyone to submit honest reports. One can show that the equilibrium outcomes of every game of incomplete information can be mimicked by a suitably constructed direct revelation game. This useful result shows that when contemplating which game best serves his own interests, a principal with great discretion over the game rules, can restrict his search to direct revelation games.

2 Ultimatum Game

The simplest version of the ultimatum game is a perfect information game for two players that takes its name from the asymmetric position of the bargaining parties. One player, called a proposer, offers to split a fixed amount of resources between the two players in a proportional share of his choice, and then the other player, the responder, either accepts or declines the proposal. If the responder accepts the offer, the resources are divided according to the offer. If not, the assets are confiscated or destroyed. The game terminates after the responder has chosen to accept or decline the offer.

For example suppose a nuclear power such as North Korea threatens one of its neighbors, South Korea or Japan, with a nuclear strike, unless the neighbor makes a substantial contribution to the wealth of North Korea. Further assume that if the amount it demands is moderate by some international norm, no third power such as the United States will intervene to prevent such threats. How much should a proposer such as North Korea contributes by to a nonnuclear power, a mugger to his victim,

2.1 Extensive form

When only a finite number of offers are possible, the ultimatum game reduces to a finite game of perfect foresight for two players with a unique solution that can be derived suing the backwards inductions methods described in Chapter 2. The extensive form for the game is displayed in Figure 13.1

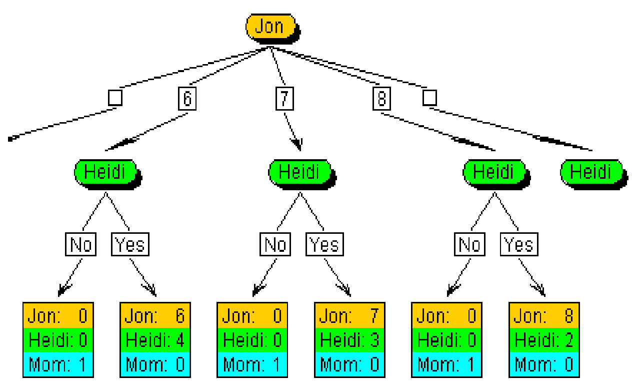

Figure 13.1

Extensive Form of Ultimatum Game with Finite Offers

The solution is for the North to exploit the South by demanding the largest possible share, in this example 8 units out of 10. Since the alternative of annihilation is even more devastating to the South, the South acquiesces by submitting to the North's demands.

It is in the interests of the proposer to play a game in which a larger share can be extracted, because the . Analogously the responder would prefer to play a game in which fine partions of the total wealth are infeasible. When coarse partitions of the total wealth are possible, it is more difficult for the proposer to extract rent from the relationship. Notice that the proposer would like to be able to have the choice of offering the smallest possible share. This point is evident from extending the game so that North Korea can offer South Korea any share in its own wealth.

We provide the following limiting result. Let $s \in [0,1]$ denote the share the proposer offers the responder, and let $d \in \{0,1\}$ denote an indicator function by the responder whether she declines ($d = 0$) or accepts ($d = 1$). Thus the strategy space of the proposer is the closed interval $[0,1]$, the strategy space of the responder is the set of functions which map $[0,1]$ into $\{0,1\}$, the payoff to the the proposer is $d(1-s)$. and the payoff to the responder is ds. Figure 13.3 provides a schematic representation of the game.

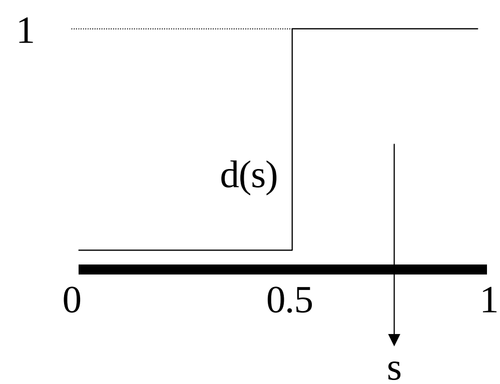

Figure 13.3

Ultimatum Game with continuous offer space

The bold line represents the item to be divided, the first player slides the arrow to a point between 0 and 1, and in the strategic form of this game, the second player draws a line $d(s)$ which shows, for each point s, whether he will accept the offer or not. In this graph he first player is offering three quarters of the cake and the responder would accept anything more than one half. Notice that this strategy is also admissible in the modified ultimatum game.

The ultimatum game has a unique solution which is found by backwards induction. The responder should not refuse any offer the proposer makes. Denoting by $d^o(s)$ the solution response, this means that $d^o(s) = 1$ for all $s \in [0,1]$. Anticipating this response, the proposer should not offer anything. Denoting by s^0 the optimal proposal, the standard backwards induction arguments imply $s^0 = 0$.

We remark that $d^o(s) = 1$ for all $s \in [0,1]$ is not a unique best response. The responder is no worse off by rejecting a zero offer and accepting anything positive.

Technically this amounts to setting $d(0) = 0$, and $(s) = 1$ for all $s \in (0,1]$. However there is no solution strategy profile in which the proposer offers the accepter a positive share! The unique equilibrium outcome is for the proposer receives all the pie.

2.2 Strategic form

Suppose South Korea is restricted to strategies of the following form. the leaders can set a reservation share, accepting any proportion of the total wealth that is at least as big as its reservation share, and rejecting anything less. This modified game rules out many strategies of the original game, such as accepting anything below a quarter of the cake, rejecting any portion between a quarter and three quarters of the cake, and accepting anything larger than that. However it is difficult to rationalize strategies that do not have the reservation share property. Thus the modified game captures the essence of the original ultimatum game, and also corresponds to the idea that representatives from both countries are bound by their own constituencies to choose a particular strategy at the negotiation table.

In the strategic form of the modified ultimatum game, both players $n \in \{1,2\}$ pick shares form their choice respective sets $\{s_{n1}, \ldots, s_{nK}\}$. If $s_1 \geq s_2$ then the first player receives $(1 - s_1)$ and the second player receives s_2. Otherwise $s_1 < s_2$ and both players receive nothing. The strategic form of this modified ultimatum game is portrayed in Figure 7.2 for the case in which:

$$\{s_{n1}, \ldots, s_{nK}\} = \left\{0, \frac{1}{10}, \frac{3}{10}, \frac{5}{10}, \frac{7}{10}, \frac{9}{10}, 1\right\}$$

For example if North Korea demands only 0.1, then any response by South Korea gives it 0.9, but if North Korea demands 0.7, then war will ensue if South Korea requires more than 0.3. The strategic form of this simultaneous move game is depicted in Figure 13.2.

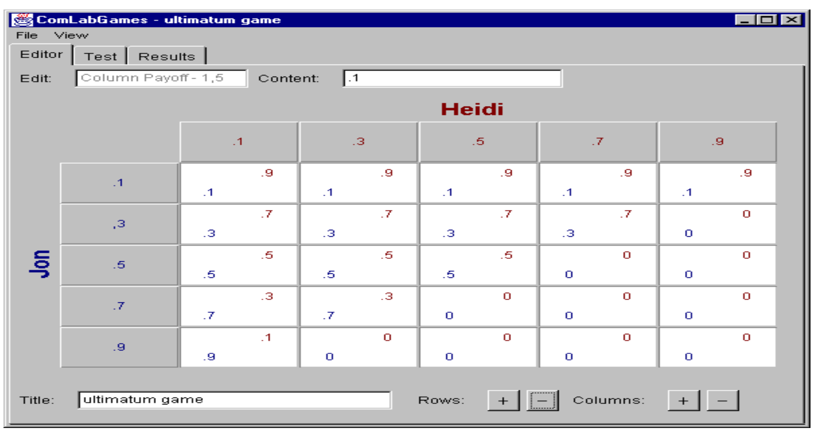

Figure 13.2
Strategic Form of Modified Ultimatum Game

The game is solved using the principle of iterated dominance. Note that it is a weakly dominant strategy for South Korea to accept any size portion of the cake that North Korea offers. Therefore the South should choose $s_2^e = s_{21}$ or $\frac{1}{10}$ in the example.

Recognizing this fact North Korea optimally offers the smallest possible positive portion, that is $s_1^e = s_{11}$.

There are many other strategic form of the modified ultimatum has many Nash equilibrium. For example every set of shares that add up to 1 is a pure strategy Nash equilibrium. Since there is more than one pure strategy Nash equilibrium, there are mixed strategy Nash equilibrium as well. However the plethore or Nash equilibria should not distract us from the main point, which is that this game has a unique dominance solvable solution.

3 Multiple Rounds

If the proposer in an ultimatum game knows how the responder values every possible outcome, in equilibrium the responder is placed in the worst possible position, subject only to the constraint that it is optimal for the responder to capitulate. The proposer extracts almost all the rent from their relationship. If the responder has discretion over which bargaining game the two parties can play, he might prefer different bargaining rules. If, for example, the responder can hide his preferences from the proposer, then it is not possible for the proposer to extract the rent in this way, a game that we shall explore this presently. Another extension of the original ultimatum game which we now explore is that the parties to play repeated bargaining game.

To demonstrate the main ideas, we consider a bargaining game between management and workers in a unionized airline. Suppose a union calls a strike on the company its members work for, and negotiations continue until the shareholders represented by their management, and the employees as represented by their union bosses agree on how the net value of the company will be divided between the two groups. At successive rounds of negotiations the management and the union take turns in proposing a split of the remaining value of the firm, the other party responding to the offer. We suppose that in odd numbered periods the managers make a proposal, and that in even numbered periods the union makes a counterproposal. The strike ends when a responding player accepts the most recent proposal of the other. If however the strike goes beyond some period $T,$ the negotiation phase ends. Depending on the game, parties must submit to binding arbitration in which the shares are announced by a third party, or creditors bankrupt the firm leading to the dismissal of the workforce.

Let v denote the aggregate value to all the bargaining parties if the first proposal is accepted and workers immediately resume work. Thus v is the expected value of the airline plus the net present value of wages to its staff. We suppose this value declines by a factor of $\beta \in (0,1)$ with each additional bargaining round. For example if the strike ends after τ periods the value to the bargaining parties declines to $\beta^\tau v,$ implying that the total loss of value to managers, shareholders and the firm's employees from holding a strike for τ periods is $v - \beta^\tau v.$ The discount factor β captures the costs of continuing the strike another period, representing the cost of maintaining idle equipment, lost wages over the round, income to the firm forgone from cancelled flights, legal costs incurred during negotiations, and the loss of customer goodwill which reduces future income. It clearly depends on the speed with which negotiations can proceed. For example the longer it takes union members to ratify the latest proposal by management, and or reject their proposal and produce a counter offer, the lower is the value of β. Finally we suppose that if an agreement is not reached after T periods, creditors force the firm into bankruptcy, and the bargaining parties lose everything.

We denote by s_t the share of total value proposed to the union members (in the form of higher wages and benefits) in period t, regardless of which player is proposing and which player is responding. Thus s_1 is the manager's first proposal to the union members, s_2 is the union's first demand for its members if it rejects s_1, and so on. Similarly we denote by $d_t \in \{0,1\}$ the responder's reply in period, where $d_t = 1$ means the proposal s_t is accepted, and $d_t = 0$ means the proposal s_t is rejected. The game ends at time $\tau \in \{1,\ldots,T\}$ if $\sum_{t=1}^{\tau-1} d_t = 0$ and $d_\tau = 1$, that is the first time τ a proposal is accepted. Otherwise $\sum_{t=1}^{T} d_t = 0$ and the airline is dissolved. Denoting the final payoff to the union by u, the rules of this game imply

$$u = \begin{cases} s_\tau \beta^{\tau-1} v \text{ if } d_\tau = 1 \text{ and } \sum_{t=1}^{\tau-1} d_t = 0 \text{ for some } \tau \in \{1,\ldots,T\} \\ 0 \text{ if } \sum_{t=1}^{T} d_t = 0 \end{cases}$$

Managers receive a payoff of m defined as

$$m = \begin{cases} s_\tau v - u \text{ if } \sum_{t=1}^{T} d_t \neq 0 \\ 0 \text{ if } \sum_{t=1}^{T} d_t = 0 \end{cases}$$

This framework can be modeled as a Markov game. At any given point before an agreement is reached there is one variable determining the state of play, t. The number of rounds indicates the current size of the bargaining pie, and whose turn it is to propose. The game is illustrated in Figure 13.4.

3.1 Finite number of rounds

To begin the analysis, consider a two period bargaining game, where $T = 2$. In the first period, the penultimate, the manager proposes a division of the value of the firm between the stockholders and the workers, which union either accepts or rejects. If the union accepts the division, a wage contract is written that reflects the shares agreed upon. If union rejects the offer, the total value of the partnership shrinks from v to βv, and then the pilots make a proposal. The manager can either accept or reject the counter offer. If rejected, the company is liquidated.

Figure 13.4 is a schematic that illustrates this two period Markov game.

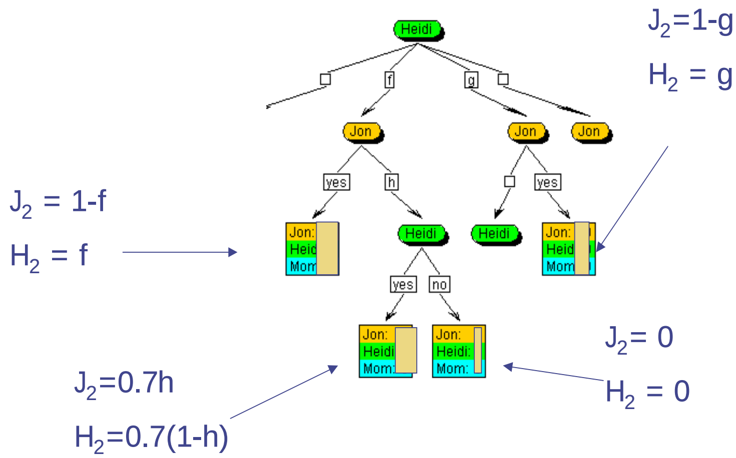

Figure 13.4
Schematic illustrating two period game

In this game, the union recognizes that the management will submit the final offer if their own offer is rejected. The unions's offer will be rejected if it is less than the expected utility of management from playing the continuation game at the end of the first round. Drawing from our discussion of one period ultimatum games, the value of the final round to managers would be the total value of the smaller partnership, that is βv. To avoid capitulating to a counter offer of a negligible share in second period, or a liquidation value of 0, the union must propose at least βv to management. If the union's offer is accepted, its payoff is $s_1 v$. Formally the goal of the union in the reduced game after folding back the last round of bargaining is choose s_1 to maximize $1\{0 \le s_1 \le 1-\beta\}s_1 v$, where $1\{0 \le s_1 \le 1-\beta\}$ is a indicator function taking a value of 1 if $0 \le s_1 \le 1-\beta$ and 0 otherwise. In the unique subgame perfect equilibrium of this game, the union sets $s_1 = 1-\beta$ and receives $(1-\beta)v$, the management accepting the union's proposal.

We now extend the bargaining horizon one more round to a total of three. The manager makes an initial offer, the union accepts the initial offer or makes a counter offer, and in the latter event, the manager accepts the union's counter offer or makes a final offer, which the union can accept or reject. If all three offers are rejected, then both players receive a payoff of zero. The solution of the two round game is used to solve this game, because it provides the values of the continuation game if the union rejects the manager's initial offer. More specifically, the manager recognizes that unless it initially offers the union a share of at least $(1-\beta)\beta$, it will reject the manager's offer and make a counter offer of $1-\beta$, which we have already established is the unique solution to the continuation game, and yields a present value of $(1-\beta)\beta v$ to the union. Offering any more than that is pointless because it reduces the manager's payoff without affecting the union's behavior. The value of the continuation game to the manager discounted back to the beginning of the game is $\beta^2 v$, but the value of offering $(1-\beta)\beta v$ is $(1-\beta+\beta^2)v$ which exceeds $\beta^2 v$ by $(1-\beta)v$. Following the logic of the previous discussion the unique solution to the the three round game is the manager offers $(1-\beta)\beta$ and the union accepts the initial offer.

Two principles seem to emerge from this discussion. The first is that the initial offer equals the discounted value that the responder would make in the continuation game after rejecting it. Offering less leads to a reduction in the total amount available, and gives the responder an opportunity to put forth his counter offer. Offering more does only reduces the proposer's share without increasing the total amount for distribution. The second feature is that in equilibrium every initial offer is accepted. Recall that if the responder does not always accept the payoff implied by discounted value of the continuation game, then anticipating this response the proposer would offer infinitesimally more. Therefore there is no equilibrium in this perfect foresight game where the responder only accepts a proportion of the time. The second feature shows that when all the parameters of the bargaining situation are known by all the parties there is no scope for bargaining in the equilibrium solution, a point to which we will later return.

Successively applying these two principles to the multi-round bargaining game, in a four round game the union begins negotiations by demanding a share of $1-\beta+\beta^2-\beta^3$, in a five round game the management offers the product of β and that quantity, receiving a share of $1-\beta+\beta^2-\beta^3+\beta^4$ and so on. Table 13.1 displays these results for a five round bargaining game in which v is normalized to unity.

T	v_T	*proposer*	s_T	*manager's payoff*	*union payoff*
1	1	*manager*	0	1	0
2	β	*union*	$1-\beta$	β	$1-\beta$
3	β^2	*manager*	$1-\beta+\beta^2$	$1-\beta+\beta^2$	$\beta-\beta^2$
4	β^3	*union*	$1-\beta+\beta^2-\beta^3$	$\beta-\beta^2+\beta^3$	$1-\beta+\beta^2-\beta^3$
5	β^4	*manager*	$1-\beta+\beta^2-\beta^3+\beta^4$	$1-\beta+\beta^2-\beta^3+\beta^4$	$\beta-\beta^2+\beta^3-\beta^4$

Table 13.1
Outcomes of Multiple round Bargaining Games

More generally we conjecture that if one party proposes a share for itself of s_T at the beginning of a game lasting T rounds, in a game lasting $T+1$ rounds, the other player initiates the bargaining by demanding a share of $1-\beta s_T$ for itself. This implies that the share garnered by the proposer evolves according to the difference equation

$$s_{T+1} = 1-\beta s_T$$

Telescoping this equation

$$s_{T+1} = 1-\beta+\beta s_T = 1-\beta+\ldots+(-\beta)^T$$

the sum of the geometric series is

$$s_T = \frac{1 + (-\beta)^{T-1}}{1 + \beta}$$

in the limit of an infinite round game.

To prove that this formula is indeed correct, we appeal to the principle of mathematical induction. Suppose that the formula holds up to a game of T_1 rounds, and that the proposer's initial offer is accepted. Then the value of a game lasting T_1 rounds to the proposer is $\left[1 + (-\beta)^{T-1}\right]/[1 + \beta]$. Consider extending the game an extra round, then the other bargaining party becomes the proposer, the continuation value to the responder conditional on rejecting the first proposal is $v\beta\left[1 + (-\beta)^{T-1}\right]/[1 + \beta]$, which leaves the proposer in a $T_1 + 1$ round game with

$$\begin{aligned} v - \frac{v\beta\left[1 + (-\beta)^{T-1}\right]}{1 + \beta} &= \frac{v + \beta v - v\beta\left[1 + (-\beta)^{T-1}\right]}{1 + \beta} \\ &= v\frac{1 + (-\beta)^{T}}{1 + \beta} \\ &= s_{T+1} \end{aligned}$$

If the proposer initially demands more than s_{T+1} in a $T + 1$ round game, his proposal is rejected, and appealing to the induction hypothesis he accepts $\beta(1 - s_T)v$ next period. Hence the loss to the proposer from being to greedy at the beginning of the $T + 1$ round game is proportional to

$$s_{T+1} - \beta(1 - s_T) = \frac{1 + (-\beta)^{T}}{1 + \beta} - \beta\left(1 - \frac{1 + (-\beta)^{T-1}}{1 + \beta}\right) = 1 - \beta$$

upon simplification. Since $\beta < 1$, the proposer prefers demanding s_{T+1} to some greater share, and by arguments we are familiar with, has no incentive to offer more. Therefore the proposer optimally offers $1 - s_{T+1}$ and the responder accepts. The principle of induction now implies that the unique solution to any finite round bargaining game of length T, is for the proposer to initially demand a share of s_T, and for the responder to accept the initial offer.

3.2 Infinite number of rounds

Many games do not end after a fixed number of turns, but only finish when both players agree to stop, or when nature steps in. For example marriage ends in divorce or death of one of the spouses, and working partnerships end when one of the partners finds better employment, or retires, or becomes disabled. Typically the timing of these events are hard to predict perfectly. In these cases we can not strictly apply the role of backwards induction or rollback because, as in any infinite horizon game, there is no fixed endpoint.

Following the procedures we developed in Chapter 12 on Markov games, we take the limit of the solutions to the finite horizon games, and then verify this limit is indeed a solution to the bargaining game when there are an infinite number of rounds. As

$T \to \infty$, the solution $s_T \to s_\infty$ defined as $s_\infty = (1+\beta)^{-1}$. This implies that the value of the game to the proposer is $v(1+\beta)^{-1}$ while the value of the game to the responder is $v\beta(1+\beta)^{-1}$. Notice that the value of the continuation game to the responder upon rejecting the initial proposer is also $v\beta(1+\beta)^{-1}$ if this decision rule is used, hence ensuring that the two principles characterizing the solution to finite round bargaining games are also obeyed here. Using the contraction mapping theorem mentioned in Chapter 4 on investment, one can show that s_∞ is indeed the unique solution to the infinite round bargaining game.

4 Bargaining Roles

The practice of rotating offers between the players leads in a bargaining game that continues indefinitely yields an outcome that approaches equal sharing as the frequency of rounds increases and the costs of adding rounds decreases. Whether convergence to equal sharing is a consequence of less discounting or alternating roles is hard to discern without further experimental design. We now analyze two variations on the ultimatum game that separate these effects.

4.1 Fixed roles in repeated bargaining

First consider a horizon bargaining with T rounds in which the management makes a sequence of offers until the union accepts its most recent one, or a creditor liquidates the company. As before we assume there is a discount factor of β that reflects the costs of forgone income and the probability of liquidation, and let v_t denote the offer made at round $T-t+1$. Note that when $T=1$, this game collapses to the ultimatum game, with its unique solution that management demands all the surplus from the union, and that the union acquiesces, accepting the offer. In a two period game, forward looking union leadership recognizes that next period it will accept any offer made to it, and therefore has a continuation value of zero. The union will therefore accept any offer made to it in the first period as well. Anticipating the union's rational response the management should off a negligible share in the first period as well. Indeed there is a unique equilibrium in which the manager sets $v_1 = v_2 = 0$, and the union accepts every offer. This result can be generalized using the backwards induction principle and the contraction mapping theorem discussed in Chapter 11. If the manager makes all the proposals and the union is always a responder there is a unique solution for a bargaining game with any number of rounds, including the infinite horizon case. The manager demands all the gains from trade, and the union accepts every proposal.

There are several instances in history where repeated use of the ultimatum threat has been used quite successfully. Nazi Germany is one example, the recent episode with North Korea might be another. With each successive round the proposer captures a larger booty, and strategically attempts to achieve world domination. Note that since death is inevitable, the fact that the empire does not survive might be of less concern to a dictator than is commonly believed, providing a memory of his own legacy lives on in one form or another. Consider the Hitler gambit.

4.2 Random assignment

We have concluded that increasing the number of rounds is not by itself sufficient to yield a more egalitarian allocation of the scarce resource. Another possibility is that the assignment of the proposer and responder roles might be key to determining the shares. To explore this possibility, suppose at the beginning of each of T rounds of a repeated bargaining game, a player is randomly selected each period with equal probability to submit a sharing proposal, while the other player is delegated to respond. As before the last round of this game is identical to an ultimatum game, where the management and union have an equal chance at putting forward their own proposal (and subsequently collecting all the rent). At the end of the second last period each player recognizes that in the last period he has an equal chance of being pick as proposer. Hence the expected value of the game to each player at the end of that period is $\frac{\beta}{2}$. Appealing to the same arguments as above leads us to the unique solution that the proposer offers the responder $\frac{\beta}{2}$ in the second last period, and that the responder accepts everything at least as good as that offer. Hence the proposer receives a share of

$$1 - \frac{\beta}{2} = \frac{2-\beta}{2}$$

Consider now a bargaining game in which there are T rounds, where a proposer is picked at random at the beginning of each round. We conjecture that the solution to this problem is for the proposer to offer $\frac{\beta}{2}$ in the first period and for the responder to accept it. For suppose that his conjecture is right in a game of $T-1$ rounds. Then we would conclude that the value of the game after one round in a game of T bargaining rounds would be $\frac{\beta}{2}$ if the proposal was rejected, identical to its value in a two period game. Appealing to the same reasons given above for the two period model, the conjecture now follows from principle of mathemantical induction.

To conclude this section we compare the method of rotating the proposer with the method of randomly assiging one.

$$1 - \frac{\beta}{2} - \frac{1+(-\beta)^{T-1}}{1+\beta} = \frac{2(1+\beta) - \beta(1+\beta) - 2 - 2(-\beta)^{T-1}}{2(1+\beta)}$$

$$= \frac{\beta - \beta^2 - 2(-\beta)^{T-1}}{2(1+\beta)}$$

which si positive for all even integers $T,$ converges to a positive number. We conclude that a proposer would prefer to have a system in which proposers are randomly assigned in future periods to one in which they rotate. The figure below graphs the (β, T).

5 Multilateral Bargaining

Bargaining between pairs is often undertaken when there are alternatives available to one or both partners. These alternatives may influence the course of the negotiations. This section analyzes several bargaining games between three or more players to show how changes in the set of interests affect the ways in which the gains from trade are split, and explores how competition or rivalry from other players can affect the nature of the agreement reached between the bargaining parties.

First we consider a game in which a dictator seeks a mandate from a concensus of the population to support his regime. the basic question revolves around whom he should subsidize whom he should tax, and how much.

Then we turn to an ultamatium game in which two proposers make competitive offers to one recipient, and show another dimension of the ultamatum game. Our investigations analyze how solution agreements are affected in games involving three or more players when they compete for the role of proposer or responder. This leads us to models where there are multiple proposers and recipients. We conclude that when all proposers are identical to each other and all respnders are identical to each other, the difference between the number of proposers and responders fully determines which party receives the gains from the relationship. If there are more proposers than responders, then each proposer receives all the gains from the relationship, and vice versa.

This stark conclusion derives from the assumptions that all proposers are identical to each other, and that there are no differences between responders either. We relax these assumptions in the latter parts of this section when we index proposers and responders by a productivity parameter, and explore equilibrium matching relationships that depend on the productivity of both players.

5.1 Dictator

Each of N players or constituents is initially allocated a random endowment, which everyone observes. The proposer or dictator proposes a system of taxes and subsidies to everyone. If at least $J < N - 1$ of the responders accept the proposal, then the tax subsidy system is put in place. Otherwise the resources are not reallocated, and the players consume their initial endowments.

The solution to multiplayer ultimatum game is easy to confirm. Rank the endowments from the poorest responder to the richest one. Let w_n denote the endowment of the n^{th} poorest responder. The proposer offers the J poorest responders their initial endowment (or very little more) and then expropriate the entire wealth of the $N - J$ remaining responders. In equilibrium the J poorest responders accept the proposal, the remaining responders reject the proposal, and it is implemented.

5.2 Owner manager

The value of being first mover is evident from the ultimatum game, in which the proposer garners all the rent. This finding extends to the principal agent problems we solved above. In each case we compared the optimal utility that the principal and agent together can gain from entering a relationship with what their utility is apart, they principal.

The algebraic expression . Let x denote a vector of choices or parameters that affect both the benefit. we write X for the set of feasible contracts, and assume $0 \in X$ is the null contract in which the principal and agent do not engage in any relationship. Thus $x \in X$ represents the contract chosen by the principal. Let p_n denote the transfer payment between the principal and the n^{th} agent, where we interpret positive values of p_n as a payment from the agent to the principal, and negative values as a payment from the principal to the agent. We assume the utility function of the principal is

$$u_0(x) + \sum_{n=1}^{N} p_n$$

and the net benefits for the n^{th} agent is

$$u_n(x) - p_n$$

If no contract is written then the principal receives $u_0(0)$ and the n^{th} agent respectively receives $u_n(0)$. Thus the participation constraint for the n^{th} agent is

$$u_n(x) - p_n \geq u_n(0)$$

The optimal rule of maximal rent extraction says that the principal should choose x^o to solve the optimization problem

$$\max_{x \in X} \left[\sum_{n=0}^{N} u_n(x) \right]$$

and set the payment to each agent $n \in \{1, \ldots, N\}$ as

$$p_n = u_n(x^o) - u_n(0)$$

Each example above is solved using this rule. To prove the rule solves the contracting problem note that under this rule the principal obtains a net benefit of

$$\begin{aligned} u_0(x^o) + \sum_{n=1}^{N} p_n &= u_0(x^o) + \sum_{n=1}^{N} [u_n(x^o) - u_n(0)] \\ &= \sum_{n=0}^{N} u_n(x^o) - \sum_{n=1}^{N} u_n(0) \\ &= \max_{x \in X} \left[\sum_{n=0}^{N} u_n(x) \right] - \sum_{n=1}^{N} u_n(0) \end{aligned}$$

Thus x^o maximizes $u_0(x)$ subject to the N participation constraints. Notice this rule implicitly includes the possibility that only a subset of the agents will be dealt with. In particular if

$$\sum_{n=0}^{N} u_n(0) \geq \sum_{n=0}^{N} u_n(x)$$

for all $x \in X$ then null contract $x = 0$ is optimal.

5.3 Competitive bargaining

We now modify the ultimatum game by assuming that there are two identical foreign investor firms seeking a local partner instead of just one. The international firms simultaneously submit the share they would relinquish of an asset they control in return for the single local distributor, currently a monopolist in the region. Then the local partner accepts at most one proposal. Let b_1 denote the first offer, and b_2 the second. If a proposal is accepted, the proposer and the responder receive the allocation specified in the terms of the proposal. If the local firm accepts the first offer, she receives b_1, the first interna receives $(1 - b_1)$ and the second player receives nothing. An analogous outcome applies if the responder accepts the second player's offer, but if both offers are rejected, nobody receives anything.

There is a unique solution to this game. Noting that it the responder optimally accept the best offer greater than zero, the best response of the first player is to submit a slightly more attractive offer than second offer if the second offer is less than one, and likewise for the second player. This rules out pure strategy Nash equilibrium in which either player submits an offer less than one. If a proposer makes an offer that does not give the entire surplus to the responder, then the other proposer could make a slightly more attractive offer and still recieve positive rents. This argument leads to the (unique) Nash equilibrium at $b_1 = b_2 = 1$, a fact which can simply be verified by noting that neither proposer can extract any rent by deviating from the strategy which gives the responder all the rent. Therefore the solution to this bargaining game is for both proposers to offer the entire gains from trade to the responder, and for the responder to pick either one. What a contrast to the solution of the original ultimatum game, where a single proposer captures all the rent from the relationship!

More generally, imagine a game for N players, where each one of I proposers make an ultamatum offer for sharing the proceeds from a partnership unit to one of J responders, who accept at most one offer. Using the same logic as above one can prove that if $I > J$, meaning there are strictly more proposer than responders, then all the responders receive the full benefit of the partnerships formed with J of teh proposers, but if $I \leq J$ then the I proposers capture all the rents in the solutiaons to this game.

This simple multilateral bargaining game can be extended in other ways. For example suppose the international firms competing for the local concern place different values on it because of they would integrate the acquisition in different ways. As before, the proposers make simultaneous offers to the responder, the responder accepts at most one proposal, if a proposal is rejected, the proposer receives nothing, if a proposal is accepted, the proposer and the responder receive the allocation specified in the terms of the proposal, and if both proposals are rejected, nobody receives anything. But let us now suppose that the proposers have different valuations for the item, say v_1 and v_2 respectively, where $v_1 < v_2$.

It is not a best response of either proposer to offer less than the other proposer if the other proposer is offering less than both valuations. Furthrermore the n^{th} firm is unwilling to offer more than v_n, because this strategy is dominated by offering nothing for the local company and failing to acquire it. Consequently the first proposer offers v_1 or less. Therefore the solution of this game is for the second proposer to offer (marginally more than) v_1 and for the responder to always accept the offer of the second proposer.

5.4 Matching

The bargaining process is often complicated by an assignnment problem, how to decide who trades with whom. To illustrate this problem we consider I authors, each of whom has written one manuscript, and J publishers, each of whom will publish at most one textbook. We suppose the authors are ranked by fame, denoted by the index a_i for $i \in \{1,\ldots,I\}$, and publishers are ranked by reputation, denoted by the index p_j for $j \in \{1,\ldots,J\}$. Total sales from the j^{th} publisher producing the work of the i^{th} author are denoted by $f(a_i,p_j)$. The question is how pairs are assigned and rents are allocated to players in the solution to a bargaining game. More generally, how does matching and rent depend on the production function $f(a_i,p_j)$ and the bargaining rules or conventions?

Ordering both rankings from best to worst, one would think that $f(a_{i+1},p_{j+1}) \leq f(a_i,p_j)$ for all $i \in \{1,\ldots,I-1\}$ and $\{1,\ldots,J-1\}$. This inequality implies that to maximize industry revenue the manuscripts of the least known $I-J$ authors would not be published if $I > J$, whereas the $J-I$ least reputable publishers would lie idle if $J > I$. In the application we consider below we assume that $f(a_i,p_j) = a_i p_j$. Noting:

$$(a_1 - a_I)(p_1 - p_J) > 0$$

we obtain the inequality:

$$a_1 p_1 + a_I p_J > a_I p_1 + a_1 p_J$$

upon expanding terms. It states that the total sales value to both publishers from matching the most famous author with the most respectable publisher and least known author with most disreputable publisher exceeds the proceeds from matching the other way around. Repeating the same argument for any two pairs of authors and publishers proves that total sales revenue to the industry is maximized by matching the k^{th} most respectable publisher with the k^{th} most famous author for all $k \leq \min\{I,J\}$. In fact this result, called positive assortive matching, extends to any twice differentiable function $f(a,p)$ with a positive cross partial derivative $f_{12}(a,p)$, or production index complementarity, because, for all $\Delta a > 0$ and $\Delta p > 0$, the revenue difference between matching $a+\Delta a$ with $p+\Delta p$ and a with p, compared to $a+\Delta a$ with p and a with $p+\Delta p$ is by definition the integral of the cross partial derivative, implying:

$$\begin{aligned} 0 &< \int_0^{\Delta p}\int_0^{\Delta a} f_{12}(a+s,p+t)\,ds\,dt \\ &= [f(a+\Delta a,p+\Delta p) + f(a,p)] - [f(a+\Delta a,p) + f(a,p+\Delta p)] \end{aligned}$$

Intuitively if signing on to a better firm increases your own productivity, then output is maximized by matching the best workers with the best firms.

The assumption of production index complementarity is not always valid. Teaming experienced with inexperienced workers might promote learning for example. In urban housing markets, residences might be indexed by their location from the business district (such as commuting time), tenants might place different values on their proximity to work (such as their wage rate), and the total amenity value might be the negative product of the two characteristics, meaning $f(a_i, p_j) = -a_i p_j$. Another example is where coal fired electricity power station and mining firms owning coal deposits. We might index the cost by the absolute difference between the two utility and the miner, such as $|a_i - p_j|$. Alternatively firms selling similar products might prefer their outlets to be located in regions where they are not competing with each other. In the case where both characteristics perfect substitutes for each other, consumers only care about their sum, so $f(a_i, p_j) = a_i + p_j$. In this additive case, matching is arbitrary. More generally, the definition of $f(a_i, p_j)$ is a critical factor explaining how sorting should occur to maximize total industry surplus.

Whether the solution to the bargaining game yields outcomes that maximize industry surplus is another matter. We analyze two bargaining games based on the market for textbooks described above, where there are only two authors, with indices a_1 and a_2, and two publishers with publicity indices of p_1 and p_2. The first author is unknown, the second one is famous, and the other is unknown. Similarly the second publisher is respectable, the first is disreputable. The publishers sequentially approach the authors, and make an offer to at most one author of a royalty payment in exchange for publishing the author's manuscript. Then the authors individually review whatever offers they have received and accept at most one offer. In one game the disreputable publisher makes the first offer; in the other game the respectable publisher is the first mover.

First suppose the respectable publisher approaches one of the authors with an offer. Which author should be approached, and what royalty should be offered? This is a perfect information game that can be solved using backwards induction. Both authors will accept any two positive royalty, the larger of the two if given a choice between them. The respectable publisher would garner $a_2 p_1$ from approaching the unknown author and offering (almost) nothing for his manuscript, because in that case the disreputable publisher would gain more from contracting with the famous author, $a_1 p_2$, than the unknown one, a little less than $p_2 a_2$. If, on the other hand, the respectable publisher makes an offer to the famous author, it must be sufficiently generous to deter the disreputable publisher from undercutting him. Now the disreputable publisher would make $a_2 p_2$ from offering a contract to the unknown author, so the respectable publisher must offer the famous author a sufficiently high royalty, denoted r_1, to deter the disreputable publisher from making more attractive offer to the famous author. We require

$$a_1p_2 - r_1 \leq a_2p_2$$

to prevent undercutting. Therefore the respectable author makes an offer of r_2 to the famous author if

$$a_1p_1 - r_1 - a_2p_1 > 0$$

for some r_1 satisfying both inequalities. Substituting for r_1 verifies this inequality is true:

$$\begin{aligned} a_1p_1 - r_1 - a_2p_1 &\geq a_1p_1 - a_1p_2 + a_2p_2 - a_2p_1 \\ &= (a_1 - a_2)(p_1 - p_2) \\ &> 0 \end{aligned}$$

To summarize the respectable publisher offers (a little more than) $a_1p_2 - a_2p_2$ to the famous author, the disreputable publisher offers (almost) nothing to the unknown author, and both authors accept their respective contracts.

The equilibrium rents, but not the assignment, changes if the disreputable publisher makes the first offer. As in the previous game, should the disreputable publisher approaches the the unknown author he can extract the full gains of the partnership, which are a_2p_2, because in that case the respectable publisher would approach the famous author and garner all the rents from that partnership, namely a_1p_1, since $a_1p_1 > a_2p_1$, where a_2p_1 is the maximal profits the respectable publisher extract from publishing the unknown author. If the disreputable publisher offers the famous author r_2, then r_2 must be sufficiently attractive to discourage the respectable publisher from undercutting him. The respectable publisher will approach the unknown author and undercut the disreputable publisher only if a_2p_1 is less than $a_1p_1 - r_2$, or

$$r_2 < a_1p_1 - a_2p_1$$

Looking ahead, the disreputable publisher compares a_2p_2 (rent from publishing the unknown author) with $a_1p_2 - r_2$ (rent from publishing the famous author), where

$$r_2 \geq a_1p_1 - a_2p_1$$

Differencing the rents from the two offers

$$\begin{aligned} a_2p_2 - (a_1p_2 - r_2) &\geq a_2p_2 - a_1p_2 + a_1p_1 - a_2p_1 \\ &= (a_1 - a_2)(p_1 - p_2) \\ &> 0 \end{aligned}$$

This proves the disreputable publisher approaches the unknown author, the respectable publisher approaches the famous author, and both authors accept their offers of (almost) zero.

Since the disreputable publisher's rent are not affected by which publisher moves first, but the respectable publisher loses rent if forced to make the first offer, the disreputable publisher can be enticed to move first in any game where the timing of offers is endogenous and there is a cost of delaying publication. By symmetry, similar results apply if the game is changed so that the authors approach the publishers with all or nothing offers, the authors collecting all the rents if the unknown author makes the first offer. The games can be extended in other directions as well. Expanding the number of players does not affect the basic intuition about assignment and rent, because the logic we pursued can be applied to any two pairs of authors and publishers in bargaining games for more than four players, forming the basis of an induction argument. Generalizing the amenity value of the match between i^{th} author transacting with the j^{th} publisher from multiplicative match value to other functions $f(a_i, p_j)$ that have the positive sorting property does not affect the main results either. Finally, allowing counteroffers leads to sharing in the gains from each partnership, as we discovered in Section 3.

As a second example, consider a real estate rental market in which the amenity value of a property can be summarized by its proximity to the city. This measure is roughly the inverse of distance, and might also account for access to a commuter railway line, and could be weighted to allow for proximity to the airport as well. We suppress other features of properties such as size and condition that affect the desirability of a house, and/or consider a sub-market of apartments with a given size and condition. We suppose there are I properties for rent and J potential tenants. Let v_i denote the proximity of the i^{th} apartment from the city, measured by the amount of time saved from living closer to the city. Define w_j as the j^{th} potential tenant's wage rate. Hence tenant j would gain $v_i w_j - r_j$ from renting property i for rent r_j. As before we consider bargaining games where there is a fixed order of offers and responses, each offer being made from landlord to a tenant or vice versa. If no bids are received, the landlord loses the rent for that period, and if a potential tenant does not make a contract then his utility is normalized to zero. We rank properties from the most desirable to the least, so that $w_1 > w_2 > \ldots > w_J$, and also rank the shadow values of potential tenants in decreasing order, meaning $v_1 > v_2 > \ldots > v_I$. The arguments we have given above imply that the apartments and tenants will be assigned to maximize efficiency. If $I > J$, so there are more properties than prospective tenants, only the N most valuable properties should be occupied, whereas if $J > I$ and there are more tenants than properties, the I tenants placing the highest value on proximity should be allocated apartments. Thus the maximum social surplus from the apartment accommodation is achieved by pairing v_i with w_j for all $i \leq \min\{I, J\}$. Similarly the solution rents are determined using the same principles as in the publishing games. Competition that affects the prices can only come from the possibilty of the responder have the opportunity to make a counter offer, or an inferior rival moving later. For example if all offers are made by landlords, and those furthest from the city are made first, then all the rents are acrue to the landlords. If the i^{th} landlord knows that some k^{th} landlord will make an offer after he does, his offer of rent to the i^{th} tenant is bounded above by $v_k w_j - v_k w_k$, so in the special case where $I > J$ rent for the apartment falls to $v_{J+1} w_j$. Depending on the order in which offers are made, even distant dwellings can affect the rental on prime property.

6 Summary

This chapter has studied bargaining games. The ultimatum game was the first one we analyzed, because its solution is easy to derive, and its structure lies at the heart of many other bargaining games. In the case where there are only a restricted number of shares the first mover demands the smallest allowable amount, and in the case where any positive amount at all is possible the proposer offers nothing. The reason for this result is not at all attributable to first mover advantage. If the responder can announce a reservation strategy without conditioning on the offer of the proposer, then there are a many Nash equilibrium solutions to the game, including an even split. But the solution is the same. The stark one sided prediction of the ultimatum game is however sensitive to the assumption that the response of the responder is so limited. If the role is switched round by round . We noted that the number of rounds is not the key here: if the same player is designated as proposer each round, the outcome of all the gains going to the proposer holds. We also investigated what happens as chance to make a proposal increased, and found that.

All the other games that we analyzed in this chapter generalize and extend the ultimatum game in various dimensions. Instead of If the there are a small number of rounds, and negotiation costs are low, there is an advantage of being the last person to make an offer.

If there are a large number of rounds and there are high costs of conducting each round, there is an advantage of being the first person to make an offer.

There is value from preventing the other party from ever making its own proposal, and restricting to them to responding to your side.

Bargaining is rarely undertaken in a social vacuum. The players typically have other opportunities to explore. We examined how explicitly accounting for these opportunities can affect the nature of the solution strategy. For example we showed that increasing the number of proposer shifted the entire rent of the match from a proposer to the responder. More generally the incidence of the match rent depends on the relative numbers of each type. Then we turned to situations where proposers are not identical to each other, and neither are responders. In this case, an optimal assignment maximizing the social surplus can be maximized by optimally assigning partners to each other, rather than simply randomly forming matches. We showed how an extension of the ultimatum game achieves this optimal assignment, and how the effect of competition from imperfect substitute partners mitigates against the proposer from extracting all the rent from their respective relationships. Nevertheless the opportunity to bargain repeatedly is sometimes a more effective way of achieving an equitable distribution, depending on the costs of bargaining additional rounds versus the how substitutable partners are for one another.

When the bargaining parties have the same information as each other, there is no reason to delay reaching an agreement, and in the strategic solution to the game, the social surplus is maximized. These two results do not always pertain to situations where one or more of the bargaining parties has more information about the value of the transaction than the other. In a game where the proposer has private information about his own valuation, the results of the full information game are unaffected, but when the valuation of the responder is hidden from the proposer, the proposer picks a (uniform) price, and consequently not all matches that yield positive rents to both parties are consummated. We also investigated how the solution is affected when there is more than one round of bargaining. Given the opportunity, a player who sees there is some chance that he will be made an ultimatum offer next period or the value of the match being fully dissipated, will signal this by making a more attractive offer to counterpart thus revealing something about the value of the match to him.