1 Introduction

Bargaining is one way to resolve a conflict between two or more parties, chosen when all parties view it more favorably than the alternatives. These include capitulation to the other parties, where tangibles are involved predation and expropriation coupled with protection of the booty, threatening war and destruction upon those those those who do not surrender. Bargaining also has elements of threat, coersion, and acquiesceness. Examples of bargaining situations include negotiations between unions and their employers about wages and working conditions, contract discussions between builders and their clients bargain over the nature and extent of the work, and prenuptial agreements written by partners betrothed to be married.

Unions warrant special mention in discussions of bargaining and industrial relations. They are defined as a continuous associations of wage earners for the purpose of maintaining or improving their remuneration and the conditions of their working lives. In the first half of the 20th century union membership grew from almost nothing to 35% of the labor force, only to decline to less that 15% at the turn of the millennium. Employment in the government sector increased from 5% in the early part of the 20th century to 15% in the 1980s, and then stabilized. Union membership in this sector jumped from about 10% to about 40% between 1960 and 1975. Employment in agriculture declined from 20% to 3% in the same period. This sector was not unionized at the turn of the 20th century. Unionization in the nonagricultural private sector has reflected the aggregate trend, declining to about 10% of the workforce down from 35%. Within the U.S. membership is highest in the industrial belt connecting New York with Chicago though Pittsburgh and Detroit (20 – 30%), lower in upper New England and the west (10 – 20%), and lowest in the South and Southwest (10% or less). Males are 50% more likely to be union members than females, mainly reflecting their occupational choices. Union membership differs greatly across countries. For the most part unions are the province of developed countries, but even within that group range from over 80 percent in Sweden to the levels we see in the United States. Unions are often mentioned in the context of industrial breakdown and strikes. Strikes are dramatic and newsworthy, but they are also quite rare: Less than 5% of union members go on strike within a typical work year. Less than 1% of potential working hours of union members are lost from strikes, before accounting for compensating overtime. About 90% of all collective agreements are renewed without a strike, but the threat of a strike affects more than 10%.

This chpater begins with some general remarks about bargaining and the importance of unions analyze the (two person) ultimatum game, extend the game to treat repeated offers, show what happens as we change the number of bargaining parties, broaden the discussion to assignment problems where players match with each other, turn to bargaining games where the players have incomplete information, discuss the role of signaling in such games.

When trade is so personalized that negotiations between the parties directly determine the terms of each individual exchange, the trading mechanism is best described as a bazaar or a bargaining game. This chapter uses the tools we have developed for noncooperative games to understand strategic behavior in extracting the gains from trade and exchange. We shall focus on three dimensions of bargaining: How many parties are involved, and what is being traded or shared? What are the bargaining rules and/or how do the parties communicate their messages to each other? How much information do the bargaining parties have about their partners? Answering these questions helps us to predict the outcome of the negotiations.

First are the bargaining process or rules of conduct. We begin with some bargaining models for two players, that is bilateral negotiations, starting out with the very simplest, called an ultatmatum game. Then we lay out increasingly flexible mechanisms for sending messages. In this way we demonstrate how the solution to bilateral bargaining games change as more complicated bargaining strategies are entertained.

The second issue we focus on is the information players have when they bargain with each other. The first games in this chapter presumes that the bargaining parties are fully informed about the preferences of all the other parties. The second topic investigates how the solution varies when some relevant information is hidden from one of teh parties.

Third is the number of players, and the main distinction we draw here is between bilateral and multilateral negotiations. In bilateral negotiations we argue little is lost by assuming the game is zero sum, which occurs when one person's loss exactly offsets another person's gain. Incidentally, we are not suggesting here that are no gains from interactions between two parties, only that the distribution of those gains is independent of the gains. by way of contrast, in multilateral negotiations, the potential gains from trade may differ depending on how the players are partition eissue changes as parties sometimes must decide who they will deal with. Thus the assignment or matching of players to activity and relationships assumes paramount importance in multilateral negotiations.

This question is taken up in Section 5 where we investigate the revelation principle. This states that the outcome of every game of incomplete information can be recast as revelation game, in which players are asked to reveal their private information, and in the solution of the game, they respond truthfully. This considerably simplifies the task of finding an optimal contract when there are not a full set of contingencies, since it proves that we can restrict ourselves to revelation games. In particular the principle shows that long drawn out discussions cannot yield an outcome more highly valued to the principal if the agents are rational. In a direct revelation game, each person has the opportunity to truthfully report his private information or lie about it, payoffs to the players are allocated according to the reports submitted by everybody, and an equilibrium strategy profile is for everyone to submit honest reports. One can show that the equilibrium outcomes of every game of incomplete information can be mimicked by a suitably constructed direct revelation game. This useful result shows that when contemplating which game best serves his own interests, a principal with great discretion over the game rules, can restrict his search to direct revelation games.

2 Breakdown

All the bargaining games we have considered so far end in agreement after just one period. Two striking features characterize all the solutions of the bargaining games that we have considered so far. An agreement is always reached. Negotiations end after one round. This occurs because nothing is learned from continuing negotiations, yet a cost is sustained because the opportunity to reach an agreement is put at risk from delaying it. To summarize, the preceding discussion shows that when there is complete information and the value of the match is positive and constant throughout the bargaining phase, and is known by both parties, then that it will be formed immediately, if delay entails some risk of losing the match.

Bargaining might take time if the current value of the match rises throughout the bargaining phase as the players gather new information together, and in the next section we will relax the assumption that all the bargaining parties are fully informed. Another reason why agreements are not reached immediatealy is that there is little to be gained from trading. This could explain why negotiation is sometimes time consuming.

2.1 Blame game

Consider the following experiment in a multi-round bargaining game called BLAME. There are two players, called BBC and a GOVT. At the beginning of the game BBC makes a statement, which is a number between zero and one, denoted N. (Interpret N as a proportion of blame BBC is prepared to accept.) The GOVT can agree with the BBC statement N or refute it. If the GOVT agrees with the statement then the BBC forfeits £N billion funding, and the GOVT loses 1 - N proportion of the vote next election. If the GOVT refutes the statement, there is a 20 percent chance that no one at all will be blamed, because a more newsworthy issue drowns out the conflict between BBC and GOVT. If the GOVT refutes the statement, and the issue remains newsworthy (this happens with probability 0.8), the GOVT issues its own statement P, also a number between zero and one. (Interpret P as a proportion of blame the GOVT offers to accept.) Should the BBC agree with the statement issued by the GOVT, the GOVT loses P proportion of the vote in the next election, and the BBC loses £5(1-P) billion in funds. Otherwise the BBC refutes the statement of the GOVT, an arbitrator called HUTTON draws a random variable from a uniform distribution with support [0,2] denoted H, the BBC is fined £H billion, and the GOVT loses H/5 proportion of the vote next election.

Using the principle of backwards induction, we can predict both the timing of the agreement (including whether an agreement is reached at all), and the amount. The mean of H is 1. If the BBC refutes the statement of the GOVT, then the expected loss to the BBC is £1 billion. Therefore the GOVT must offer P = 0.8 to avoid the BBC refuting the GOVT offer (since 5(1-P) < 1 only if P < 0.8). The value of HUTTON to the GOVT is a loss of 20 percent of the vote, much lower than 80 percent. Therefore the GOVT would choose to accept any blame less than that. The expected value to the GOVT from rejecting the BBC initial offer is thus 16 percent of the vote (0.8 of 20 percent), and the expected value to the BBC at that time is £0.8 billion. The BBC anticipates this in the first round. The BBC must offer at least 84 percent of the blame for the GOVT to accept, which is unacceptable to the BBC (since it implies a loss of £0.84 billion). Therefore the BBC should make an unacceptable apology offer to the GOVT of less than 84 percent culpability, the GOVT should counter offer with some number less than 80 percent, and BBC should reject the final offer of the GOVT.

The confidential data depicted in Table 1.1 were obtained by the SUN. They purportedly show the results of experiments ALISTAIR conducted some months ago on the BLAME game. We can see that the ALISTAIR results conform to our theoretical predictions, and that the model is not rejected by the data obtained. Every number falls within the acceptable bounds, and every offer is rationally rejected.

TABLE 1.1

SUMMARY OF ALISTAIR'S EXPERIMENTS

///////////////	BBC Offer: N	GOVT response: P	BBC Response to counteroffer
Session:			
1	65	No, 35	no
2	73	No, 12	no
3	14	No,56	no
4	67	No, 3	no
5	34	No, 44	no
6	0	No, 77	no
7	56	No, 30	no
8	38	No, 62	no
9	9	No, 56	no
10	18	No, 1	no

2.2 Private Valuations

A hallmark feature of all the bargaining games that we have studied up until now is that all the gains from trade are exhausted in the solution, and that the negotiations are completed in minimal time. As we shall see, this arises because the bargaining parties know all the parameters that determine the payoffs for all the parties before the negotiations begins. Playing a solution to the game does not yield any new information to any of the players. In contrast, when players are not privy to all the relevant information that affects the payoffs, then bargaining does not necessarily reveal an efficient outcome, and costly resources might be expended in the bargaining process itself.

To study how incomplete information might affect the ultimatum game, we consider the sale of an object. Call the proposer a seller, the responder a buyer, and suppose the object for sale is more highly valued by the buyer than the seller. Denoting the valuations of the seller and buyer v_1 and v_2 respectively, the potential gain from transacting is then $v_2 - v_1$. The seller sets a price p for the object, and the buyer purchases it by setting an indicator variable $d = 1$, or rejects the offer by setting $d = 0$. Accordingly the payoff to the seller is $(1 - d)v_1 + dp$ and the payoff to the buyer is $d(v_2 - p)$. The unique solution to this ultimatum game is for the seller to demand $p = \max\{v_1, v_2\}$ for the item, and for the buyer to accept the offer if $v_2 \geq p$.

Differential information about the valuations can arise naturally in two ways. The seller may have more information than the buyer about v_1, or the buyer may have more information than the seller about v_2. The former case is straightforward. Since the solution strategy profile does not depend on the seller's valuation, neither the seller's offer nor the buyer's response changes. Consequently the outcome is the same as when there is perfect foresight.

In the latter case, however, the solution must change since the seller does not know the buyer's valuation, and therefore cannot offer the object for v_2. Consider first the following example. The seller does not value the object at all, and the buyer's valuation is equally likely to amount between \$5 and \$9. In terms of the notation above, $v_1 = 0$ and v_2 is a random variable with a uniform distribution over the support $[5, 9]$. The seller chooses an offer price p, the buyer chooses $d(p)$. The payoffs to the seller is $d(p)p$, while the buyer's payoff is $d(p)(v_2 - p)$. Inspecting the latter, we see that the buyer should purchase the good if and only if the price p does not exceed his valuation v_2. That is:

$$d_0(p) = \begin{cases} 1 \text{ if } v_2 \geq p \\ 0 \text{ if } v_2 < p \end{cases}$$

Assuming the seller is an expected value maximizer, she chooses p to maximize her expected payoff. Substituting the buyer's solution strategy into the seller's expected payoff we obtain

$$pE[d_0(p)] = p\Pr[v_2 \geq p]$$

Because the buyer's valuation lies between \$5 and \$9, the seller charges a price between them. Any amount below \$5 would be accepted for sure, and any bid above \$9 would certainly be rejected. Since v_2 is uniformly distributed:

$$\Pr[v_2 > p] = 1 - \Pr[v_2 \leq p] = 1 - \frac{p - 5}{9 - 5}$$

Substituting this formula into the expected payoff function, we obtain upon simplification:

$$(9p - p^2)/4$$

This is a quadratic function with a single stationary point that maximizes its unconstrained value. Differentiating the quadratic yields the first order condition $2p^* = 9$ that defines the unconstrained stationary point $p^* = 4.5$. Since this price lies below the support solving this equation, marginal losses are incurred from increasing the price throughout the entire support $p \in [5, 9]$. It follows that the optimal offer price is boundary point $p_0 = 5$, where every responder buys the item

Figure 7.8 illustrates the expected payoff function for different values of the maximal support.

The assumption that the buyer's valuation is drawn from a uniform distribution is computationally convenient but unnecessary. More generally, suppose the buyer's valuation is drawn from a probability distribution denoted by $F(v_2)$. The final subgame, whether responder chooses to purchase the item, remains unchanged from the example of the uniform distribution above. As above, the seller chooses a price p to maximize expected profit, the multiplicative product of the probability of winning and the profit that she would make if the buyer accepts.

$$p\Pr[v_2 \geq p] = p[1 - F(p)]$$

Changing the price creates both a gain and a loss. Raising the price by Δp lowers probability of making a sale by approximately $F(p + \Delta p) - F(p)$, reducing expected revenue by $p[F(p + \Delta p) - F(p)]$. The gain is the product of the price increment Δp and the probability the probability of being an inframarginal demander $1 - F(p + \Delta p)$. While a lower price raises the probability of making a trade, it also reduces unit profit conditional on a sale. Recall the numerical example introducing this section demonstrated that there might not be an interior solution, and in this case it is optimal to sell the item to everyone. If an interior stationary point for the optimization problem exists, the marginal gain just offsets the marginal loss:

$$p^* F'(p^*) = 1 - F(p^*)$$

The global optimum is found by comparing the expected profit of interior solution(s) p^* with the expected profit from selling the item to everyone by charging the minimal valuation $v_{\min}$. The expected revenue from asking the optimal uniform price of p_0 from any potential buyer responder is:

$$\pi_0 = p_0 \int_{p_0}^{\bar{v}} v dF(v)$$

Of course when the seller knows the valuation of the buyer, she charges v_2. Averaging over buyers of different types, the seller's expected revenue in the full information case is:

$$\pi_1 = \int_{\underline{v}}^{\bar{v}} v dF(v) = \int_{\underline{v}}^{p^o} v dF(v) + \int_{p^o}^{\bar{v}} v dF(v)$$

Taking the difference of π_1 and π_0 reveals the two sources of loss from not knowing the buyer's valuation. Demand from a portion of the population is left unfilled, while the others are charged less than their reservation price:

$$\pi_1 - \pi_0 = \int_{\underline{v}}^{p^o} v dF(v) + \int_{p^o}^{\bar{v}} (v - p^o) dF(v)$$

2.3 Repeated bargaining with private valuations

The solution to the bargaining problem becomes more complex when the possibility of multiple rounds is introduced and the negotiators are uncertain about the valuation of the other party. We consider two cases when v_1 is unknown, and when v_2 is unknown. When v_1 is unknown, and the buyer has an opportunity to make the first proposal, then little changes compared to the full information case.

But the case in which v_2 is unknown is more complicated. For suppose the buyer has an opportunity to make an offer, denoted $p_1,$ and that if his offer is rejected, with probability $(1 - \beta)$ the trading opportunity is squandered, but with the probability β the seller can counter with a final offer p_2.

Management plays the following bargaining game with labor. In the first round management make a wage offer w_1 to labor. If the union accepts the wage offer they receive w_1 and management (representing shareholders) receives $v - w_1.$ If the union rejects the offer, there is some probability $p \in (0,1)$ that negotiations will continue one more round. Otherwise, that is with probability $1 - p,$ both players receive 0 after the initial offer is rejected. (Creditors move in and liquidate the company.) If there is a second bargaining round, then labor makes a (final) demand of $w_2.$ If management accepts labor's demand, then the firm's shareholders receive $v - w_2$ and labor receives $w_2.$ If management rejects labor's proposal then both parties receive 0 (and the company is liquidated). The parameter v is a random variable with cumulative distribution function $F(v).$ Management knows v but labor does not observe it.

We analyze solutions of the following type. There is a theshold value of v denoted by v^* such that for every value of $v \geq v^*$ management offers some $w^* < v^*$ and the union accepts the offer, and that for every value of $v < v^*$ management offers nothing (or more generally a small wage which the union always rejects) so that the bargaining continues to the second round with probability $p.$ Then in the second round the union acts as if the cumulative probability distribution function for v is given by $F(v)/F(v^*)$ and makes an offer that is optimal for a final round bargaining game with this probability distribution.

There are two conditions that ensure the existence of an equilibrium with these properties:

1. When an offer of w^* is made, the union believes that $v \geq v^*$ and is a random variable with cumulative probability distribution function for v is given by $[F(v) - F(v^*)]/[1 - F(v^*)].$ An alternative confronting the union is to reject the offer of $w^*,$ and with probability p counter with a proposal that is optimal for a final round bargaining game with this probability distribution. One necessary condition for the solution to have the properties described above is that this alternative is less lucrative than accepting $w^*.$ This is called a signalling condition, because if it is not met, a firm with a high valuation is not willing to release any information about it.

2. If management has valuation v^*, they are indifferent between offering w^* versus stalling negotiations, and the difference between the payoffs to management is increasing in v. This ensures every firm with $v \geq v^*$ prefers making the w^* offer and every firm with $v < v^*$ prefers to risk liquidation and the entertain the counterproposal rather than offering w^*. This is called a monotonicity condition.

Suppose that v is uniformly distributed on the unit interval, that is $F(v) = v$ on support $[0,1]$. We derive the equilibrium for this example. First we exploit the second condition to solve for the union's counter demand in terms of v^*. That is the offer the union will make if management do not make an acceptable offer in the first round. Recall that in a final round bargaining game when $F(v)$ is uniformly distributed between $[\underline{v}, \bar{v}]$ the union optimally demands $\bar{v}/2$ or $\underline{v}$, whichever is more profitable. In this case $\underline{v} = 0$ and $\bar{v} = v$, so an interior solution to the final round bargaining holds. Thus the union would counter with $v^*/2$, which is accepted half the time, and so has expected value of $v^*/4$ to the union.

Next we turn to the monotonicity condition. Management anticipates the counter demand of $v^*/2$. By the definition of v^*, if $v = v^*$ management is indifferent between offering w^* now and receiving $v^* - w^*$ for sure, or receiving $v^* - v^*/2 = v^*/2$ with probability p after next round. We use this fact to solve for w^* as a mapping of v^* and p. Equating the two payoffs

$$v^* - w^* = pv^*/2$$

Therefore

$$w^* = \frac{v^*(2-p)}{2}$$

Consider now the difference in expected payoffs to management when it has any valuation $v \in [0,1]$ from offering w^* versus passing on the first stage. It is:

$$\left[v - \frac{v^*(2-p)}{2}\right] - p(v - v^*/2) = (1-p)v - \frac{v^*}{2}$$

which is increasing in v. Therefore management with valuations higher than v^* prefer to offer w^* and whereas management with lower valuations than v^* prefer negotiations to stall. Consequently the monotonicity condition is met for all values of p and v^*.

This leads us finally to the signalling condition, which identifies a range for v^* as a function of p. If the union rejects w^* it will treat v as coming from a uniform distribution with support $[v^*, 1]$. If it had the opportunity the union could, in this case, demand either v^* or $1/2$, whichever yields higher profits. Note that a necessary condition for the union to demand $1/2$ is that $v^* < 1/2$. If the union demands v^* the management will certainly accept the counteroffer, but if the union demands $1/2$, the probability that management will accept the union's proposal is

$$\frac{1}{2(1-v^*)}$$

in which case its expected revenue is

$$\frac{1}{4(1-v^*)}$$

The difference in the the net revenues from demanding v^* versus $1/2$ is therefore

$$v^* - \frac{1}{4(1-v^*)}$$

By inspection the net revenues are equated at $v^* = 1/2$, and the difference is increasing in v^* for all values of $v^* < 1/2$ since its derivative is

$$1 - \frac{1}{4(1-v^*)^2}$$

Having rejected w^* the union counters with v^* if $v^* \geq 1/2$ and $1/2$ if $v^* < 1/2$. Now the signalling condition requires the union accept the w^* offer, which is equivalent to the inequality

$$w^* = \frac{v^*(2-p)}{2} > p\max\left\{v^*, \frac{1}{4(1-v^*)}\right\}$$

Thus if $v^* \geq 1/2$ the signalling condition requires $p < 2/3$ and if $v^* < 1/2$ the signalling condition requires

$$\frac{v^*(2-p)}{2} > \frac{p}{4(1-v^*)}$$

which can be expressed as

$$v^{*^2} - v^* + \frac{p}{2(2-p)} < 0$$

The quadratic expression on the left has a minimum at $v^* = 1/2$ where it attains a value of

$$\frac{p}{2(2-p)} - \frac{1}{4}$$

A necessary condition for the inequality to be satisfied anywhere is that $p < 2/3$. In this case the roots of the quadratic expression are

$$\frac{1}{2} \pm \sqrt{1 - \frac{2p}{(2-p)}}$$

Since $v^* < 1/2$ in this case, we may discard the larger of the two roots, and focus on the region between the lower root and $1/2$.

To summarize the uniform $[0,1]$ case, the monotonicity condition is automatically satisfied, but the signalling condition is only satisfied if the probability of continuation is less than $2/3$. In addition we require

$$v^* > \frac{1}{2} - \sqrt{1 - \frac{2p}{(2-p)}}$$

In such an equilibrium all managers with $v \in [v^*, 1]$ offer the union

$$w^* = \frac{v^*(2-p)}{2}$$

which accepts the offer. If, however, management draw $v \in [0, v^*)$ then they make only a nominal offer (of nothing say) which the union rejects. In this event the union solves the one period ultimatum game when the manager has a private value drawn from a uniform distribuiton with support $[0, v^*]$ and accordingly counters with a wage demand of $v^*/2$ which, from the definition of w^* and p, is less than $w^*.$ Finally note that if $v < v^*/2,$ then it knows from the outset that negotiations are doomed.

3 The Revelation Principle

We explained in the introduction why contracting problems can be analyzed as bargaining games with first movers. the revelation principle explains what kinds of games are worth considering.

The second two sources of restricted to direct mechanisms, where players truthfully report their and honestly fulfil the demands made upon them precisely because the contract provides the appropriate incentives to do so. This naturally raises the question whether a more favorable contract for the principal could be written if it did not confine itself to contracts which induced truth revealing behavior. We prove the answer to this question is no. The solution to every contract is also a solution to a truth revealing direct revelation game. This proposition, called the truthful revelation principle, simplifies the search for optimal contracts, by providing reassurance that only direct mechanisms need be considered.

3.1 Direct mechanisms

We now turn to games with incomplete information where there is a first mover

A fundamental result in mechanism design theory is that all games of incomplete information can be condensed into games. Several games that we have reviewed exhibit incomplete information. Depending on the types of moves available to the first mover, the outcomes of the games Is there a sense in which the games are equivalent. The revelation principle is a result that An important

The revelation principle implies that if the principle has the power to set the

When the responder has a private value, the ultimatum game is a game of perfect information. This is because finite games can be defined by a game tree where every information set is a singleton. First the principal proposes a share, then the agent draws his private value from his probability distribution, and once informed responds by accepting or rejecting the offer. Our analysis of this game in Chapter 13 established that the principal

If the principal had a choice over which bargaining game he would play with the agent, he might conceivably require the agent to first declare his type, or provide some other information about himself and then make an offer contingent on the agent's declaration. The agent would then have the opportunity to accept or reject the contingent offer. Would this yield a greater share of the rent to the principal?

Notice that this new bargaining game has incomplete information.

Consider a bargaining game where the proposer makes an all or nothing offer to a responder. We now consider a different game in which the responder simply announces his type and on the basis of the that both parties are allocated a reward. The question at hand is how to replicate the solution found in the ultimatum game where there is incomplete information.

This is an application of the revelation principle.

As before denote the valuations of the seller and buyer v_1 and v_2 respectively, and the potential gain from transacting by $v \equiv v_2 - v_1$. How the shares from the gains from trade are assigned to the traders becomes the object of the bargaining game. Accordingly the payoff to the seller is now written as:

$$u_1 = \begin{cases} v_1 + (1 - s_1)(v_2 - v_1) \text{ if } s_1 \leq s_2 \\ v_1 \text{ if } s_1 > s_2 \end{cases}$$

and

$$u_2 = \begin{cases} s_1(v_2 - v_1) \text{ if } s_1 \leq s_2 \\ 0 \text{ if } s_1 > s_2 \end{cases}$$

Note that these equation imply that the sum of the valuations is v_2 if the item is sold, and v_1 if not:

$$u_1 + u_2 = \begin{cases} v_2 \text{ if } s_1 \leq s_2 \\ v_1 \text{ if } s_1 > s_2 \end{cases}$$

Interpreted in this way, the solution to the ultimatum game is for the seller to demand v_2 for the item, and for the buyer to accept the offer.

Differential information about the valuations can arise naturally in two ways. The seller may have more information than the buyer about v_1, or the buyer may have more information than the seller about v_2. The former case is very straightforward. Since the solution strategy profile does not depend on the seller's valuation, neither the seller's offer nor the buyer's response changes. Consequently the outcome is the same as when there is perfect foresight. In the latter case, however, the solution must change since the seller does not know the buyer's valuation, and therefore cannot offer the object for v_2.

We remark that to maximize her expected profit, the seller should not necessarily maximize revenue conditional on a sure acceptance the probability of getting the seller to accept your bid. Her expected profit is the multiplicative product of the probability of winning and the profit that she would make if the buyer accepts. While decreasing the offer would increase the probability of making a trade, it would also reduces her profit conditional on a sale, and both factors are important. If the support of the buyer's valuation, the optimal bid involves taking some risk of losing the deal, in order to make the deal more profitable if you get it.

The assumption that the buyer's valuation is drawn from a uniform distribution is a computational convenient one, but by no means necessary. More generally, let us now suppose that the buyer's valuation is drawn from a probability distribution denoted by $F(v_2)$. The final subgame in which buyer chooses to purchase the item or not remains unchanged and the seller maximizes

$$\begin{aligned} E[u_1] &= p\Pr[v_2 \geq p] \\ &= p[1 - F(p)] \end{aligned}$$

Exercise *Compare the outcomes of the following ultimatum games. The seller makes a take-it-or-leave-it offer. The buyer announces his type, and then the seller makes a final offer. The seller makes an initial offer, the buyer either accepts the offer or announces his type, and the seller makes a final offer.*

1. *Plot the offers as a function of the support.*

2. *Solve for the solution offer as a function of the maximal value of the support.*

3. *Plot the offers as a function of the support. Are they distributed about the solution*

3.2 The main result

In principal agent games, one player has the power to set many of the parameters that determine the nature of the subgames which follow. One issue that arises in games of incomplete information is what kind of structure supports the payoffs from the possible games. To what extent does this discretion give the player power over the other ones? In bargaining games we have already seen the advantage a first mover has.

A revelation game is one in which each person has certain information which they are asked to reveal, and the payoffs that depend on how they respond. For example suppose that each of N players could be one of J possible types drawn from the set $\Theta \equiv \{\theta_1, \ldots, \theta_J\}$. We denote the n^{th} player's type by $\theta_n \in \Theta$. The strategy space for the n^{th} player is to declare he is type $\theta_{n'} \in \Theta$. For convenience we assume that a player's strategy only depends upon his type, although the revelation principle we derive below does not depend on this assumption. Accordingly we denote a strategy for the n^{th} player with type θ_n as $s(\theta_{n'}, \theta_n)$. The payoffs to all the n^{th} player depend on the types of all the players, and also what they declare about themselves. We write the utility or payoff to the n^{th} player as

$$u_n(s(\theta_{1'},\theta_1),\ldots,s(\theta_{N'},\theta_N))$$

A revelation game is called truth revealing if there is a solution to the games in which each player responds by truthfully revealing his type regardless of which type he draws. In this game, telling the truth means choosing $s(\theta_n,\theta_n) \equiv s_n$. If all the other players truthfully declare their type, then the expected utility of the n^{th} player is

$$u_n(s_1,\ldots s_{n-1},s(\theta_{n'},\theta_n),s_{n+1},\ldots,s_N)$$

Therefore revelation game is truth revealing if and only if for each $n \in \{1,\ldots,N\}$ and each $\theta_n \in \Theta$,

$$u(s_1,\ldots,s_N) \geq u(s_1,\ldots s_{n-1},s(\theta_{n'},\theta_n),s_{n+1},\ldots s_N)$$

From its definition we can see that a revelation game is very specialized because it only has a very small strategy space (each player sending a single message announcing his type), and because it supports a solution in which honesty is the best response by each player. A remarkable feature is that the solution payoffs to every game of incomplete information can be attained by a truth revealing revelation game.

To establish this proposition we consider the strategic form of any game in which one or more players is privy to payoff relevant information that the other players do not have. The strategic form of any finite game can be written as a set $S \equiv \{s_1,\ldots,s_K\}$ for some positive integer K. Suppose that in the solution to the game, the n^{th} player picks $s_n(\theta_j) \in S$ when he observes $\theta_j \in \{\theta_1,\ldots,\theta_J\}$. The expected payoff to him is then

$$U(\theta_j) \equiv E[u(s_1(\theta_{1j}),\ldots,s_N(\theta_j))|\theta_j]$$

We now define a revelation game in which each person is asked about their type and the payoffs are structured as

$$v_n(\theta_j,\theta_k) \equiv E[u(s_1(\theta_{1j}),\ldots,s_{n-1}(\theta_{1j}),s_n(\theta_{1k}),s_{n+1}(\theta_{1j}),\ldots,s_N(\theta_j))|\theta_j]$$

Then it optimal

$$v_n(\theta_j,\theta_j) \geq v_n(\theta_j,\theta_k)$$

and the result is proved.

Applying the revelation principle

The value of knowing the revelation principle is that it sharply reduces the space of contracts that a principal must search over to find the one that is most consistent with his own interests.

4 Summary

This chapter has studied bargaining games. The ultimatum game was the first one we analyzed, because its solution is easy to derive, and its structure lies at the heart of many other bargaining games. In the case where there are only a restricted number of shares the first mover demands the smallest allowable amount, and in the case where any positive amount at all is possible the proposer offers nothing. The reason for this result is not at all attributable to first mover advantage. If the responder can announce a reservation strategy without conditioning on the offer of the proposer, then there are a many Nash equilibrium solutions to the game, including an even split. But the solution is the same. The stark one sided prediction of the ultimatum game is however sensitive to the assumption that the response of the responder is so limited. If the role is switched round by round . We noted that the number of rounds is not the key here: if the same player is designated as proposer each round, the outcome of all the gains going to the proposer holds. We also investigated what happens as chance to make a proposal increased, and found that.

All the other games that we analyzed in this chapter generalize and extend the ultimatum game in various dimensions. Instead of If the there are a small number of rounds, and negotiation costs are low, there is an advantage of being the last person to make an offer.

If there are a large number of rounds and there are high costs of conducting each round, there is an advantage of being the first person to make an offer.

There is value from preventing the other party from ever making its own proposal, and restricting to them to responding to your side.

Bargaining is rarely undertaken in a social vacuum. The players typically have other opportunities to explore. We examined how explicitly accounting for these opportunities can affect the nature of the solution strategy. For example we showed that increasing the number of proposer shifted the entire rent of the match from a proposer to the responder. More generally the incidence of the match rent depends on the relative numbers of each type. Then we turned to situations where proposers are not identical to each other, and neither are responders. In this case, an optimal assignment maximizing the social surplus can be maximized by optimally assigning partners to each other, rather than simply randomly forming matches. We showed how an extension of the ultimatum game achieves this optimal assignment, and how the effect of competition from imperfect substitute partners mitigates against the proposer from extracting all the rent from their respective relationships. Nevertheless the opportunity to bargain repeatedly is sometimes a more effective way of achieving an equitable distribution, depending on the costs of bargaining additional rounds versus the how substitutable partners are for one another.

When the bargaining parties have the same information as each other, there is no reason to delay reaching an agreement, and in the strategic solution to the game, the social surplus is maximized. These two results do not always pertain to situations where one or more of the bargaining parties has more information about the value of the transaction than the other. In a game where the proposer has private information about his own valuation, the results of the full information game are unaffected, but when the valuation of the responder is hidden from the proposer, the proposer picks a (uniform) price, and consequently not all matches that yield positive rents to both parties are consummated. We also investigated how the solution is affected when there is more than one round of bargaining. Given the opportunity, a player who sees there is some chance that he will be made an ultimatum offer next period or the value of the match being fully dissipated, will signal this by making a more attractive offer to counterpart thus revealing something about the value of the match to him.