1 Introduction

Typically the exchange of goods and services is underwritten by a written or an oral contract specifying how property rights, ownership and service obligations are being codified to reflect the trading agreement. The details defining these contracts help determine the nature of the trade, and indeed, whether any trade is conducted. In this chapter we analyze contracting, or the problem of mechanism design. This chapter analyzes how those who create and administer organizations design the incentives and institutional rules that best serve their ends. Starting out with the simplest of contract design problems, we progress to more complicated environments, where the principal writing the contract knows less than the agents who accept or reject it.

The revelation principle enunciated in the previous chapter states that the outcome of every game of incomplete information can be recast as direct revelation game, in which players are asked to reveal their private information, and in the solution of the game, they respond truthfully. This considerably simplifies the task of finding an optimal contract, since it proves that we can restrict ourselves to revelation games. In particular the principle shows that long drawn out discussions cannot yield an outcome more highly valued to the principal if the agents are rational. In a direct revelation game, each person has the opportunity to truthfully report his private information or lie about it, payoffs to the players are allocated according to the reports submitted by everybody, and an equilibrium strategy profile is for everyone to submit honest reports. One can show that the equilibrium outcomes of every game of incomplete information can be mimicked by a suitably constructed direct revelation game. This useful result shows that when contemplating which game best serves his own interests, a principal with great discretion over the game rules, can restrict his search to direct revelation games.

In the next section we consider contracting environments where there is full information about the the potential gains from reaching an agreement, and where all the details characterizing the contract and its implementation can be perfectly monitored by the players. We consider several examples. Section 3 focuses on contract design when a full set of contingencies is permitted, that is when each histories of the first mover game are uniquely identified by their payoffs. We discuss labor, capital procurement and service provider contracts, before deriving the optimal contract in a general setting. The optimal contract should extract all the rent for the contract designer. This rule has an intuitive interpretation. The principal should design a contract that maximizes the joint surplus that he can achieve with his agents, and then adjust the net benefit to them with compensating side payments that leaves them (almost) indifferent between accepting or rejecting the contract.

The latter parts of this chapter investigate contract design when at least one of the players is not fully informed about some payoff relevant features of the contracting

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environment until the agreement has been selected, and signed or rejected. Uncertainty over the outcome of a contracting game can arise from two sources. First, the players might be differentially informed about the future outcome of the contract at the time it is written. Uncertainty that arises from this source is called hidden information. If their information is not verifiable in a settling up phase of the contract, there might be opportunities for informed players to misrepresent the situation to increase their payoffs. For example an repair firm might claim some damage is worse than it really is. Second, the contract might specify certain contractual obligations that are unobserved and cannot be directly verified by the other players. This creates a situation of moral hazard, where players might choose to neglect their contractual obligations with impunity. For example a person renting a car increases its wear and tear through aggressive driving.

2 Contracts for Goods and Services

Studying a model where there are only three choices and three outcomes provides a useful introduction to the three concepts of rent extraction, the participation constraint, and the incentive compatibility constraint. But the variety of contract types can only be appreciated by looking at richer models.

2.1 Procurement

A manufacturer designs a contract to present to a construction company for building a new factory. If the contract is not signed, then neither player receives any payoff. Upon signing the contract, the builder would decide whether to complete the factory on time or late. The net present value to the manufacturer from starting operations on time is v_1, which is strictly positive, whereas the value to the manufacturer from starting later is only v_2, where $v_2 < v_1$. However the builder incurs costs of c_1 to complete the building on time but incurs only c_2 to complete it later, where $c_2 < c_1$. The contract is a pair of payments (w_1, w_2), where w_1 is what the manufacturer pays the builder if the contract is signed and the builder completes the factory on time, and w_2 is the payent if the factory is completed late. The difference $w_1 - w_2$ is therefore the penalty incurred for being tardy. We assume the manaufacturer knows the parameters (v_1, v_2, c_1, c_2) and chooses (w_1, w_2), thus setting in motion the game of perfect information diplayed in Figure 16.1. The problem confronting manufacturer is how to set (w_1, w_2).

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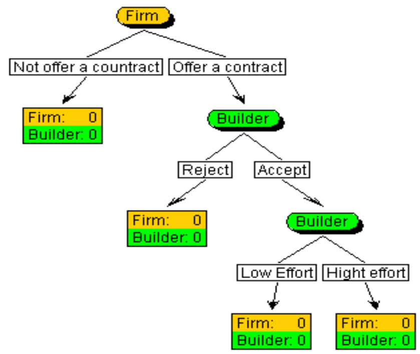

Figure 16.1
Contracting with a Builder

As a first step to solving this problem, we analyze how different coordinate pairs (w_1, w_2) affect the builder's choice in the perfect information game. To cover his costs the builder requires $w_j - c_j$ is positive for some $j \in \{1,2\}$; these are called pariticipation constraints. If $w_1 - c_1$ exceeds $w_2 - c_2$ then the builder is willing to compete the project on time, but if $w_1 - c_1$ is less than $w_2 - c_2$ the builder would prefer to complete the factory late and incur a penalty of $w_1 - w_2$; this is a called incentive compatitbility constraint. The contract space is illustrated in Figure 16.2. The sloping line through $(c_1 - c_2, 0)$ indicates the incentive compatibility constraint. Contracts below that line induce the builder to complete the factory on time whereas contracts above the line result in delays. Similarly the vertical line though $(c_1, 0)$ is the participation constraint for completing the factory on time, while the horizontal line through $(0, c_2)$ marks off the participation constraint for a late completion.

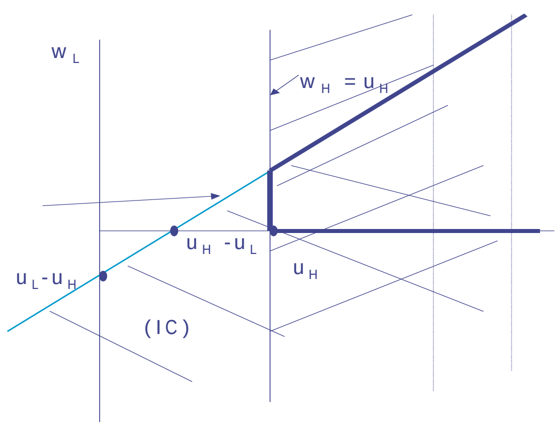

Figure 16.2
Participation and Incentive Compatibility

Next we derive the minimum cost contract for inducing the builder to complete the factory on time, and also the minimum cost contract for inducing the builder undertake construction (and complete the factory late). The minimum cost of achieving a late

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completion is found by minimizing w_2 such that $w_2 \geq c_2$ and $w_2 - c_2 \geq w_1 - c_1$. Since $c_1 > c_2$ the second constraint is automatically satisfied if the contract contains no penalty clause. Therefore a minimum cost contract for a late start is (c_2, c_2). The minimum cost of a timely completion is found by minimizing w_1 such that $w_1 \geq c_1$ and $w_1 - c_1 \geq w_2 - c_2$. Supposing the first inequality is satisfied, then the second inequality is also satisfied if $w_2 \leq c_2$. Hence a minimum cost contract for an on time completion is (c_1, c_2). More generally, any penalty that exeeds the cost differential $c_1 - c_2$ suffices to ensure timely completion.

To find the profit maximizing contract we compare the profits from the two minimum cost contracts derived above. The net present value from a timely completion is $v_1 - c_1$ whereas the value from a late completion is $v_2 - c_2$. If neither of these amounts are positive, the project should be abandoned. Otherwise the manufacturer offers a contract inducing on time completion (c_1, c_2) if the differential value $v_1 - v_2$ exceeds the cost differential $c_1 - c_2$, and (c_2, c_2) otherwise.

2.2 Insurance

A large fraction of the wealth owned by many households is tied up in the house they own. there are several reasons why households do not diversify across a larger spectrum of assets. Taxation provisions for mortgage interest payments subsidize ownership relative to renting. Bankruptcy law allows those filing bankruptcy are permitted to keep their house (and other personal belongings but not other property or stock. Furthermore because people are have a greater interest in protecting, preserving and developing their own assets than managing assets owned by others, owner occupied housing is cheaper to maintain than rented real estate. Moreover in poor countries, financial markets are typically not as well developed, and the opportunities to invest or save are limited . The fact that housing is a literally a cottage industry, owned by largely owned by owner occupied households, rather than large real estate corporations renting or leasing their services to accommodate life cycle and professional changes in demand, has spawned an insurance market to partly protect home owners against catastrophic loss. We now study the nature of an optimal insurance contract.

Let π denote the premium on the insurance policy due at the beginning of the period, let p denote the probability of catastrophic loss, and x the size of the policy. If the household buys a (π, x) insurance contract we assume it attains an expected utility of

$$(1-p)u(h-\pi) + pu(x-\pi)$$

where h is the housing wealth of the household if there is no catastrophe. If the household rejects the contact, we assume its expected utility is δ. The insurance firm chooses a contract (π, x) to maximize $\pi - px$ subject to the household's individual participation constraint

$$(1-p)u(h-\pi) + pu(x-\pi) \geq \delta$$

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If the inequality above is strict under (π, x), a higher premium could be charged without violating constraint. Therefore the individual participation constraint is binding at the optimal contract (π^o, x^o). Consequently the firm's optimization problem can be expressed as the Lagrangian

$$\pi - p(x + c) + \lambda[(1 - p)u(h - \pi) + pu(x - \pi) - \delta]$$

Differentiating this objective function with respect to x we obtain the first order condition:

$$1 = \lambda u'(x^o - \pi^o)$$

Similarly the first order condition for π is:

$$1 = \lambda[(1 - p)u'(h - \pi^o) + pu'(x^o - \pi^o)]$$

Now equating the right side of both conditions yields:

$$0 = \lambda u'(x^o - \pi^o) - \lambda[(1 - p)u'(h - \pi^o) + pu'(x^o - \pi^o)]$$
$$= \lambda(1 - p)[u'(x^o - \pi^o) - u'(h - \pi^o)]$$

Since $u'(x)$ is a strictly decreasing function, $p < 1$ and $\lambda > 0$, we conclude $x^o = h$. Therefore the household receives a certain utility of δ and the optimal premium π^o is the unique solution to $u(h - \pi^o) = \delta$. To summarize, the firm fully insures the household and extracts the maximal rent that the individual participation constraint permits.

2.3 Service providers

Multipart pricing contracts offered by service providers in the telecommunications industry, for membership to fitness, sporting and social clubs, and schemes to time share exclusive resorts and fractional ownership in a private jet. Typical of a muti-part pricing scheme is telephone service. The duration of certain telephone calls, the location of the receiver, how often a particular number is called, the time of day, and call back features are amongst the myriad of details that determine the costs of subscribing to a telephone service. Although there may be characteristics that are relevant to pricing that the service provider does not know, the (often personal) nature of the services provided in this industry allows the supplier to design a contract that includes many contingencies for extracting maximal rent. For this reason we model the industry as providing fully contingent contracts

Service providers face the following kind of problem, which we have simplified in the interests of clarifying the basic issues. A phone service charges monthly fee for connection charges plus user fees based on quantity used. (Alternatively a club charges annual membership dues plus user fees for its some of its services.) If the user (or member) contracts with the supplier and buys x units of service, her benefit is $u(x)$, an increasing concave function that represents positive but declining marginal benefit from greater usage. The cost of supply the user is a connection cost incurred by the supplier, denoted c_1, plus a variable cost we denoted by c_2. We suppose the service provider knows that the customer would achieve a positive net benefit of δ

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from contracting with another provider if she does not receive at least as favorable a contract with this service.

The problem facing the phone service is to choose the phone contract that maximizes the value of the company constrained only by the fact that the consumer must receive a benefit of at least δ. The most direct way of solving this problem is to choose a phone package that allows the consumer to consume not more than x_0 units for a membership fee of p_0. The service provider chooses (x_0, p_0) to maximize net revenue, subject to an individual participation constraint that the user voluntarily signs on to the contract. Since $u(x)$ is increasing, it follows that a subscriber would consume all x_0 units permitted. Therefore the service provider's problem reduces to choosing (x_0, p_0) to maximizes

$$p - c_0 - c_1 x$$

with respect to (x, p), subject to the inequality

$$u(x) - p \geq \delta$$

which is called an individual participation constraint. The constraint restricts the service provider to contracts that the subscriber would accept. In this problem the incentive compatibility constraint is met with equality at the optimal choice (x_0, p_0). Otherwise the subscription price could be increased without losing the consumer to another provider. Using this fact we substitute out the price p in the service provider's profits to obtain its objective function in the single choice variable x :

$$u(x) - \delta - c_0 - c_1 x$$

Because u is a concave function in x, any strictly positive solution x_0 must uniquely satisfy the first order condition

$$u'(x_0) = c_1$$

In a fully contingent optimal contract: the marginal utility of service unit is equated to its marginal cost. Thus the subscriber is indifferent between accepting versus rejecting the contract, because the provider extracts all the gains from the exchange. Substituting the solution for x_0 into the individual participation constraint now yields for the optimal subscription rate

$$p_0 = u(x_0) - \delta$$

Having determined the optimal contract, the provider only offers the customer the package (x_0, p_0) if the provider makes positive rents from the subscription:

$$u(x_0) - \delta - c_0 - c_1 x_0 \geq 0$$

For example suppose

$$u(x) = x^{\gamma}$$

where $\gamma \in (0, 1)$ is a fixed a parameter to represent declining marginal benefit from greater usage. Then the first order condition reduces to

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$$\gamma x_0^{\gamma-1} = c_1$$

Solving the first order condition for x_0 we obtain

$$x_0 = \left(\frac{c_1}{\gamma}\right)^{1/(\gamma-1)}$$

It is optimal for the provider offer the (x_0, p_0) package providing it yields postive rents, that is if

$$\left(\frac{c_1}{\gamma}\right)^{\gamma/(\gamma-1)} - \delta - c_0 - c_1\left(\frac{c_1}{\gamma}\right)^{1/(\gamma-1)} \geq 0$$

Otherwise the consumer attains a utility of δ from subscribing to another provider.

The optimal contract can be also be implemented by a two part tariff, which we denote by (p_1, p_2), where the subscriber pays a membership fee of p_1 to join the club, and is then charged for her usage at the rate of p_2. Once the subscriber has purchased a contract (p_1, p_2), she chooses x to maximize

$$u(x) - p_1 - p_2 x$$

Differentiating this expression yields the first order condition for subscriber usage $u'(x^*) = p_2$.The marginal benefit of the service unit is equated to its price. Setting $p_2 = c_2$ ensures the subscriber consumes the same number of units in both schemes, or $x^* = x_0$. Setting membership fees at:

$$p_1 = p_0 - c_1 x_0$$

now ensures the consumer's participation constraint is met with equality, a fact which can be verified by substituting the expressions for (p_1, p_2) into the constraint. Therefore the two contract forms (x_0, p_0) and (p_1, p_2) yield the same solution outcome.

2.4 Charging a uniform price

Continubg with the example of a service provider we now contrast the two schemes described above with an analysis of the rent obtained from selling the service for a uniform price per unit. This is equivalent to restricitng $p_1 = 0$ in the two part tariff scehme and choosing p_2 to maximize

$$(p_2 - c_1)x - c_0$$

subject to consumer demand. If the consumer buys from the provider then $x > 0$ and her first order condition satisfies the equation:

$$u'(x) = p_2$$

We now substitute for p_2 in the net revenue function to obtain an expression in x alone:

$$u'(x)x - c_1 x - c_0$$

In this way we reformulate the provider's optimization problem: anticipating the consumer's demand to its price offer the provider implicitly chooses usage x when it picks p_2 to maximize its rent.

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Let x_m denote consumer usage if she subscribes ot the provider. If

$$u(x_m) > \delta + p_2 x_m$$

then the consumer reaps some surplus from her purchase. Maximizing the firms objective function with respect to x an interior first order condition pertains, where $x_m = x_1$ defined:

$$u'(x_1) + u''(x_1)x_1 = c_1$$

Otherwise the consumer is indifferent between purchasing from the provider versus an alternative supplier. In that case $x_m = x_2$ defined:

$$u(x_2) = \delta + u'(x_2)x_2$$

Since $u(x)$ is strictly increasing in x, it follows that $x_m = x_1$ if $x_1 > x_2$ and $x_m = x_2$ if $x_1 < x_2$. Summarizing

$$x_m = \max\{x_1, x_2\}$$

All that remains to determine is whether serving the consumer is profitable for the provider or not. If

$$[u'(x_m) - c_1]x_m \geq c_0$$

then the provider will serve the consumer but otherwise will not.

To cover its fixed costs a uniform price operator must price above marginal cost, meaning $u'(x_m) > c_1$. Since $u'(x)$ is declining in x, and $u'(x_0) = c_1$ it immediately follows that $x_m < x_0$. In words usage is higher under an optimal two part tariff than under an optimal uniform price. The difference between the rent extracted from an optimal two part tariff versus an optimal uniform price is:

$$u(x_0) - c_1 x_0 - u(x_m) - c_1 x_m = \int_{x_m}^{x_0} [u'(x) - c_1]dx$$

which is strictly positive since $u'(x) > c_1$ for all $x < x_0$.Therefore supplementing marginal cost pricing with an optimal connection fee is more profitable than charging a higher usage price and connecting the consumer for free.

Returning to our example of $u(x) = x^\gamma$, the quanitites x_1 and x_2 are respectively defined as:

$$x_1 = \left(\frac{c_1}{1+\gamma}\right)^{1/(\gamma-1)}$$

and

$$x_2 = \left(\frac{\delta}{(1-\gamma)}\right)^{1/\gamma}$$

which implies

$$x_m = \max\left\{\left(\frac{c_1}{1+\gamma}\right)^{1/(\gamma-1)}, \left(\frac{\delta}{(1-\gamma)}\right)^{1/\gamma}\right\}$$

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In this case a uniform price provider will serve the consumer if

$$\gamma x_m^{\gamma} - c_1 x_m \geq c_0$$

3 Employment

Just over 10% of the workforce is self-employed. The remaining 90% of workers receive wages, tips and other compensation from their employers. Thus, most demand for labor comes from private firms (75%) and the government sector (15%).

Suppose one of the inputs is a continuous variable. For example let y denote the income (or total payment) the worker (or supplier) receives for her labor services (or components). Let h denote her hours of labor supplied to the firm (or the quality of her components), if she is employed at (contracts with) the firm. We assume she receives a utility (or the component suppliers receives profits) of $u(y,h)$ where u is a concave function, increasing in y and decreasing in h. We suppose the firm receives $f(h)$ in revenues as a function of h, where f is positive, concave, and increasing in h. Thus more labor (or higher quality inputs) increases profits, but at a decreasing rate. Finally let v denote the utility (or value) of the laborer pursuing a different alternative; v is called the worker's (component supplier's) reservation value.

3.1 Franchise

Imagine a retail chain store which distributes its outlets across a geographical market. Given its location decisions, the objective of the chain is to choose h and y to maximize $f(h)$ subject to the participation constraint by a store manager that $u(y,h) \geq v$. We denote by λ the Lagrange multiplier associated with this constraint. The firm maximizes:

$$f(h) - w + \lambda[u(y,h) - v]$$

Denote the solution to this optimization problem by (y^o, h^o), and let $u_j(y,h)$ denote the derivative of $u(y,h)$ with respect to the j^{th} argument. An interior solution (in which $h^o > 0$ so that a contract is written and accepted) satisfies the first order conditions (found by setting the derivatives of the Lagrangian to zero). So far as hours (or quality) is concerned:

$$f'(h^o) + \lambda u_2(y^o, h^o) = 0$$

where $f'(h)$ is the derivative of $f(h)$ with respect to $h.$ With regards w :

$$-1 + \lambda u_1(y^o, h^o) = 0$$

Substituting for λ we obtain the marginal valuation condition:

$$f'(h^o) u_1(y^o, h^o) = -u_2(y^o, h^o)$$

This condition states that at an interior optimum the marginal decline in the worker's utility (or component manufacturer's profits) from supplying more hours (producing a higher quality input) is just offset by the product of marginal increase in firm's revenues and the marginal increase in utility to the worker.

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Noting that the firm's objective function is strictly decreasing in y, the firm equates the utility of the inside and outside options:

$$u(y^o, h^o) = v$$

We now have two equations in the two unknowns (y^o, h^o), which can be solved once we specify the utility from alternative employment v, the production function $f(h)$, and the utility function $u(y, h)$.

For the purposes of illustration consider a firm who retails each unit of the service for price p. In this example $f(h) = ph$ and $f'(h) = p$. Let a denote the worker's non-wage wealth, and assume the worker's utility function takes the form

$$u(y, h) = \log(a + y) + k\log(24 - h)$$

where k is a positive constant that reflects the value of leisure to the worker. We assume that if the worker does not accept the contract her utility is derived purely from her non-wage wealth and her nonworking time.

$$v = \log(a) + k\log(24)$$

Differentiating we see that the worker's marginal utility of wealth is

$$u_1(y, h) = (a + y)^{-1}$$

and her (negative) marginal utility loss from working more hours is

$$u_2(y, h) = k(h - 24)^{-1}$$

Thus the interior opitmality equation for determining wages and hours at the margin is

$$p(a + y^o)^{-1} = k(24 - h^o)^{-1}$$

which can be expressed as

$$y^o = \frac{p}{k}(24 - h^o) - a$$

Optimal compensation balances leisure $24 - h^o$ with income y^o, so that as the utility of the worker is increased both rise proportionately. To satisfy the participation constraint we require

$$\log(a + y^o) + k\log(24 - h^o) = \log(a) + k\log(24)$$

Substituting the equation we just derived for y^o into the participation condition, and solving for h^o yields the optimal amount of work offered to the employee:

$$h^o = 24 - p^{\frac{-1}{1+k}}(ak)^{\frac{1}{1+k}}24^{\frac{k}{1+k}}$$

Raising the service price induces more hours worked, but the wealthier the worker is from other sources of income, the less she is prepared to work. and optimal hours worked are lower for those who attach high utility to their leisure.

3.2 Sales Commission

This solution to this example can be compared with the outcome of a sales commission. Here the retailer sets a unit service payment s but the laborer chooses

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how much to supply. The worker chooses h to maximize

$$\log(a + sh) + k\log(24 - h)$$

The first derivative is

$$\frac{s}{a + sh} - \frac{k}{24 - h}$$

The objective function is strictly concave in h, it therefore has a unique stationary point. Hence the optimal service choice of the salesman, denoted $h(s)$, is:

$$h(s) = \begin{cases} \frac{24s - ka}{s(k+1)} \text{ if } \frac{s}{a} > \frac{k}{24} \\ 0 \text{ if } \frac{s}{a} > \frac{k}{24} \end{cases}$$

Clearly the worker prefers a sales commission to the optimal contract for the firm, where she only attains a utility of v.

We derive the firm's optimal commission by substituting $h(s)$ into the net revenue function, and maximizing

$$(p - s)h(s) = (p - s)\frac{(24s - ka)}{s(k + 1)}$$

with respect to s. The first order condition for the firm's problem is

$$(p - s)\frac{24}{s(k + 1)} - \frac{(24s - ka)}{s^2(k + 1)} = \frac{(24s - ka)}{s(k + 1)}$$

which can be solved for s^o. Since the worker's utility his higher in a commission contract than a two part optimal contract solved in the previous section, we conclude that firm profits are lower in this case.

3.3 Freelance

Yet a third scenario is for the professional to proposes a contract, which the outlet it can either accept or reject. In this case the worker chooses hours h and the payment by the firm y to maximize her utility $u(y, h)$ subject to the constrain that the firm makes positive profits, $f(h) \geq y$. From the professional's perspective permitting the firm to make strictly positive profits yields less utility than extracting all the firm's value. Consequently $f(h) = y$, so the professional's problem simplifies to choosing the hours h that maximizes $u[f(h), h]$. The first order condition is the same as for the original problem. However the participation condition is not, and this affects the equilibrium contract.

In the example discussed above, the professional maximizes

$$\log(a + ph) + k\log(24 - h)$$

Expressed as a function of p, the solution $h(p)$ is found by substituting p for s in the sales commission contract derived for $h(s)$ above. the professional prefers the third contract to the other two. This contract yields less output than the optimal contract for the firm, solely attributable to wealth affects increasing the leisure taken by the

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professional.

4.2 Financial Partners

New firms have a low probability of success, most failing within two years. Most entrepreneurs starting new firms use up their own time and wealth to no avail (apart from the experience itself). Of the remainder, many new firms reward their founders with much toil for modest wages. If founders were rational, we could infer that a relatively small proportion of new firms prove extremely lucrative for their founders. That is, entrepreneurship entails a huge gamble with the founder's time, and sometimes his or her initial wealth, for the prospect of very large rewards. The entrepreneur often knows more about the expected value of his risky project than everyone else. Our work on bargaining and contracts explains why it is hard for entrepreneurs have difficulty funding their projects. There is no self financing, efficient bargaining mechanism. Thus the entrepreneur sells the project for less than its true expected value, or owns some of the project himself, accepting its inherent risks.

By definition newly created firms are the brainchild of one individual or a very small group of coworkers. When seeking to sell their idea, or attract outside funding in return for partial ownership, they must prove to potential buyers or investors that their project is valuable (hidden information), simultaneously protect their idea or invention from theft by rivals with a lower cost of capital or some other advantage in development (adverse selection), and prove they are motivated to ensure the project's success (moral hazard). Not surprisingly reliable data are difficult to find on how likely venture capitalists are to support a typical start up, but it is a safe bet that venture capitalists are besieged with countless business plans from entrepreneurs seeking funding and only a tiny proportion of new firms incorporated annually are financed by professionally managed venture capital pools.

Because raising outside funds is very costly, entrepreneurs might exchange shares in their projects for labor and capital inputs to known acquaintances, called insiders. Marriage, kinship and friendship are examples of relationships that lead to inside contacts. There are three limitations from turning this direction. Insiders are acquaintances, so few in number. Consequently they charge a premium for taking a major share in the project if they are risk averse. They might be tempted to steal the innovation, and betray the trust placed in them. Finally they are less qualified than outsiders specially picked because of their expertise to evaluate or contribute to the project, since their position as insiders are often unrelated to the project's challenges.

How would an entrepreneur write contracts with N insiders to partially insure himself about a project that outsider investors avoid? Setting $n = 0$ to indicate the entrepreneur and $n \in \{1, \ldots, N\}$ for the insider partners we suppose the utility function of the n^{th} player is given by:

$$-\exp(-\gamma_n a_n)$$

where a_n denotes his assets if he does not participate in the project, and γ_n is the

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coefficient of risk aversion. We assume the payoff from the project is a random variable which denote by $x,$ drawn from a normally distributed random variable with mean μ and variance σ^2. The n^{th} insider partner pays f_n in return for a share of s_n from the proceeds from the project, and thus receives a final payoff of $(f_n - a_n - s_n x).$ The formula for a moment generating function of the normal distribution implies that the expected utility of the partner is:

$$-\exp\left[\gamma_n(f_n - a_n - s_n\mu) + \frac{(\sigma\gamma_n s_n)^2}{2}\right] = -\exp\left[\gamma_n\left(f_n - a_n - s_n\mu + \frac{\gamma_n(\sigma s_n)^2}{2}\right)\right]$$

By inspection, the certainty equivalent to the n^{th} partner from the random payoff $s_n x$ bought with a fee of f_n is:

$$s_n\mu - \frac{\gamma_n(\sigma s_n)^2}{2} - f_n$$

The greater the variance of the random variable, and the greater the coefficient of risk aversion, the more the partner must be paid to accept a given share in the enterprise for a given certainty equivalent. With regards the entrepreneur, his expected utility from the partnership with the insiders, who pay f_1 through $f_N,$ for shares $s_1$through $s_N,$ is:

$$-\exp\left[-\gamma_0\left(a_0 + \mu + \sum_{n=1}^{N}(f_n - \mu s_n)\right) + \frac{1}{2}\left(\sigma\gamma_0 - \sigma\gamma_0\sum_{n=1}^{N} s_n\right)^2\right]$$

His certainty equivalent for undertaking the project in partnership is therefore:

$$\mu + \sum_{n=1}^{N}(f_n - \mu s_n) + \frac{\gamma_0\sigma^2}{2}\left(1 - \sum_{n=1}^{N} s_n\right)^2$$

One way of solving the entrepreneur’s problem is to maximize his expected utility subject to the constraints imposed upon him by his financial partners. An easier method is to maximize the entrepreneurs’s certainty equivalent, subject to the same set of constraints imposed upon him by his financial partners, but again expressed in terms of certainty equivalents. We adopt the second approach.

If the entrepreneur is fully informed about the opportunities and preferences of hias partners, then he would offer contracts that were individually tailored them. Determining the fee schedule for each insider is the entrepreneur’s first step towards solving the partnership offer. To construct the fee schedule that exhibits an partner’s willingness to buy shares in the venture, we compare the partner’s expected utility from participating in the venture with what he would obtain otherwise. The n^{th} partner accepts a nonnegotiable partnership offer (f_n, s_n) if it has a positive certainty equivalent. Since the entrepreneur’s certainty equivalent is strictly increasing in the fee, and each partner’s certainty equivalent is strictly decreasing in the fee, the entrepreneur can extract all the rent by making an all or nothing offer, equating the fee with the certainty equivalent value of the shares offered to each partner:

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$$f_n^o = s_n\mu - \frac{\gamma_n(\sigma s_n)^2}{2}$$

In the next step the entrepreneur determines how many shares to offer each partner. Substituting the optimal fee f_n^o for each partner $n \in \{1,\dots,N\}$ into the entrepreneur's certainty equivalent value for the contract, he chooses $(s_1,s_2,\dots,s_N)$ to maximize:

$$\mu - \sum_{n=1}^{N} \frac{\gamma_n(\sigma s_n)^2}{2} + \frac{\gamma_0\sigma^2}{2}\left(1 - \sum_{n=1}^{N} s_n\right)^2$$

Since the objective function is quadratic in each choice variable $s_n,$ there is a unique interior stationary point, which we denote now by $(s_1^o,s_2^o,\dots,s_N^o)$. Differentiating with respect to s_n, the first order condition reduces to

$$s_n^o = \left(1 - \sum_{n=1}^{N} s_n^o\right)\frac{\gamma_0}{\gamma_n} \equiv s_0^o\frac{\gamma_0}{\gamma_n}$$

Summing over n yields

$$\sum_{n=1}^{N} s_n^o = 1 - s_0^o = s_0^o \sum_{n=1}^{N} \frac{\gamma_0}{\gamma_n}$$

and solving for the optimal residual stock holdings of the entrepreneur, $s_0^o,$and hence for his partners', we obtain for all $n \in \{0,\dots,N\}$ the solution shares:

$$s_n^o = \left[\sum_{m=0}^{N} \frac{\gamma_n}{\gamma_m}\right]^{-1}$$

Although not imposed, this interior solution satisfies the constraints that each player is allocated a strictly positive share in the firm, that is $0 \le s_n^o \le 1$ for all $n \in \{0,\dots,N\},$ and is therefore the global optimum.The fees to insiders are increasing in μ and declining in $\sigma,$ projects with higher means and lower variances commanding greater fees. However the solution shares do not depend on mean or variance of the project, but only on the attitudes of each member towards risk. Furthermore the ownership share of the entrepreneur is determined exactly the same way as his partners, the inverse of a weight reflecting his risk aversion compared to everyone else. This allocation of shares maximizes the sum of the certainty equivalents, a necessary condition for the entrepreneur to extract all the potential gains from the partnership.

Discriminatory pricing is more lucrative than uniform pricing, since it encompasses it as a special case, and offers many other options besides. However discriminatory pricing might not be feasible if, for example, insiders can trade their shares or contract dividends from their shares with each other. In this case any attempt to price discriminate would unravel. Rather than pay a higher price to join the partnership than some other insiders, an insider offered relatively unfavorable terms would simply approach a partner who was being offered a better deal, and agree on terms of trade that benefited both partners. The potential for retrading effectively constrains the entrepreneur to charge a uniform price for each share. For similar reasons the entrepreneur cannot, in this case, control how many units each shareholder should

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buy.

Accordingly we now suppose that the entrepreneur sets a price p for the whole venture, and each insider decides s_n, what share to purchase, costing ps_n. The solution to this problem can be solved in three steps. First we derive an insider's demand for shares in the partnership for any given price p. This yields a demand schedule for the n^{th} insider. Then we aggregate demand across insiders to find the total number of shares demanded at any given price. The last step is to choose the price which maximizes the certainty equivalent of the entrepreneur anticipating the demand response derived in the first two stages.

Deriving insider demand for shares in the venture is essentially a simple portfolio problem. Given p insider n picks to s_n to maximize the certainty equivalent

$$ps_n - a_n - s_n\mu + \frac{\gamma_n(\sigma s_n)^2}{2}$$

Differentiating with respect to the share s_n yields the first order condition:

$$(p-\mu) + \sigma^2\gamma_n s_n = 0$$

or the linear demand schedule:

$$s_n(p) = \frac{\mu - p}{\gamma_n\sigma^2}$$

Summing the individual demands we obtain the aggregate demand for shares by the insider:

$$s(p) = \left(\frac{\mu - p}{\sigma^2}\right)\left(\sum_{n=1}^{N}\frac{1}{\gamma_n}\right)$$

We are now ready to take the last step. The entrepreneur chooses p to maximize his certainty equivalent:

$$\mu + (p-\mu)s(p) - \frac{\gamma_0\sigma^2[1-s(p)]^2}{2}$$

The first order condition is

$$s(p^o) + (p^o - \mu)s'(p^o) + \gamma_0\sigma^2[1-s(p^o)]s'(p^o) = 0$$

The three expressions determining the optimal price have intuitive interpretations. As a first approximation, raising the price of a share increases revenue by the number of shares sold, namely $s(p^o)$. Also as the price rises, demand contracts at the rate of $s'(p^o)$ eliminating the expected losses from selling the marginal shares, which are $(\mu - p^o)$ per unit. (Recall from above that $s(p^o) > 0$ implies $\mu > p^o$.) Both expressions are positive. But retaining greater ownership in the project exposes the entrepreneur to more risk. Noting his share is $s_0 = 1 - s(p^o)$, he discounts his whole portfolio in the project by an extra $\gamma_0\sigma^2 s_0 s'(p^o)$ for the marginal price increase due to greater exposure.

An expression for the optimal price can be derived from the first order condition. Differentiating aggregate demand equation for $s(p)$ with respect to price yields:

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$$s'(p) = \frac{-1}{\sigma^2}\left(\sum_{n=1}^{N} \frac{1}{\gamma_n}\right) = \left(\frac{-1}{\mu - p}\right)s(p)$$

Substituting the expression for $s'(p)$ into the first order condition, some algebraic manipulations yield:

$$2(\mu - p^o) + s(p^o)\sigma^2\gamma_0 = \sigma^2\gamma_0$$

Now substituting the equation for $s(p)$ we solve for p^o to obtain:

$$p^o = \mu - \sigma^2\gamma_0\left(1 + \sum_{n=0}^{N} \frac{\gamma_0}{\gamma_n}\right)^{-1}$$

The optimal price varies one-for-one with the mean return from the project. The higher its variance, and the more risk averse is any player, the lower the price.

Under the perfect price discrimination, the entrepreneur achieves a higher expected utility and sells more shares than under uniform pricing arrangement. The first point is a direct consequence of the fact that it is not optimal to present the same terms to partners if they have different attitudes towards risk. As to the second claim, we now prove that the entrepreneur retains more shares when there is uniform pricing. Substituting the solution for p^o into the aggregate demand equation proves that

$$s(p^o) = \left[\sum_{n=0}^{N} \frac{\gamma_0}{\gamma_n} - 1\right]\left[1 + \sum_{n=0}^{N} \frac{\gamma_0}{\gamma_n}\right]^{-1}$$

Now note that $1 - s(p^o)$, shares held by the entrepreneur under the optimal uniform pricing scheme, exceeds s_0 because:

$$\frac{1 - s(p^o)}{s_0} = 2\left[\sum_{n=0}^{N} \frac{\gamma_0}{\gamma_n}\right]\left[1 + \sum_{n=0}^{N} \frac{\gamma_0}{\gamma_n}\right]^{-1} > 1$$

7 Summary

The example at the beginning of this chapter sets its tone. In order to procure a component from a supplier, a firm must offer a contract that the supplier accepts, and a contract that ensures the supplier will ifnd that it is in its own to produce according to the specifications that the procurement firm wants. These two conditions are called the participation constraint and the incentive compatitibility constraint. The procurement firm minimizes its costs subject to these two constraints.

The application which follow build upon this theme.

When all the information sets at the terminal nodes of a contract are singletons, the contract has a full set of contingencies. The most straightforward mechanism design problems are those where there are fully contingent contracts. We analyzed three applications of this problem, contracts with unskilled laborers and itinerant workers, procurement procedures from component suppliers and service providers. In these examples the principal optimally designs a contract that maximizes the social surplus, that is the aggregate value of the game to all the players. In these examples the agent has the option of rejecting the contract offer, and seeking employment or business elsewhere. This alternative constrains the contracts that the principal will offer to avoid

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rejection. Thus the participation constraints restrict the principal to those contracts where the best response of agents is to accept the contract the principal offers. These two principles, maximizing social surplus and obeying participation constraints, lead us to a general characterization of the solution. When contracts are fully contingent, the principal should design a contract that maximizes the social surplus and expropriates all the gain to himself subject to the participation constraints applying to the agents.

For example when a firm contracts with a builder to undertake to undertake some construction, the quality of the work is sometimes hard to judge, because if there is structural failure the reasons might not be clear. Unfortunately errors of both types (punishing innocent parties and failing to convict guilty parties) in courts determining liability. In cases like the builder might be the only party to know whether his construction methods are sound or not. Recognizing the agent makes choices that the principal does not observe, the principal offers contracts that provide incentives for the agent to adopt construction methods that are compatible with the principal's objectives. This is an example of an incentive compatibility constraint, which are relevant whenever the agent makes choices affecting the principal's expected payoffs that the principal cannot observe or infer.

Another type of example is when agents know more about the problem, observing finer information partition at the terminal nodes than the principal. A question arises does. This chapter has analyzed games with first movers as problems in contract design. The first half of the chapter concentrated on games where there is a single principal designing a contract and one or more agents who have the opportunity to implement it. In a full information game with a single first mover, only the participation constraint limits the principal's power to extract rent from the agents. If the principal attempted to extract more than is dictated by the participation constraint, the agents would reject the contract. In first mover games with perfect information

One area where incomplete information about a valuations is in employment. The last topic of this chapter was on search games, in which an employer seeks to fill a position by sequentially discovering information about candidates and their abilities until one is hired, thus stopping the process. This topic also extends the earlier work on multilateral bargaining, because it involves selecting a recruit from a set of job candidates. We analyzed games where there are only a finite number of candidates who seek the job and look qualified enough to justify serious consideration, and showed how the reservation quality the firm demands increases, as the field of qualified candidates increases, and falls as the number of rejected candidates increases. We also discussed how recall affects the list of offers; if candidates who reject offers that the firm makes have the opportunity to reconsider better offers as they are made to subsequent candidates, then offers a firm makes do not increase as steeply as when candidates have no right of recall. Increased competition between candidates reduces the firm's wage bill. Finally we discussed the trade-off between two partners as they search for ways of achieving a better match as they

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simultaneously bargain over the share of their resulting partnership, and compare this with the results from an dynamic optimizing model of the type analyzed in Chapter 11. Does bargaining lead to too much investment, or too little?

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