1 Introduction

Studying auctions provides a second look at price formation. In a typical bilateral bargaining situation of the type studied in the previous chapter, there is one seller, one buyer, and a single unit to trade. Auctions serve the dual purpose of eliciting preferences and allocating resources between competing uses. In the type of auctions studied in this chapter, there is still only one seller with a single unit to trade, but many bidders (or alternatively one buyer and many sellers). This simplifies the problem of solving the trading game, yet yields some insights that carry over to other trading mechanisms where there are multiple units for sale, and more than one player on both sides of the market.

Another reason for studying auctions is that the value of goods exchanged each year by auction is huge. What kinds of goods are sold by auction? Real estate, both commercial and residential, is auctioned in many countries. The U. S. government uses auctions to sell Treasury bills. Mineral, oil, and timber rights; emissions permits and property rights over the frequency spectrum are often disposed of through an auction. There are auctions for agricultural produce. Intangible properties (such as patents, trademarks) and art, and uniquely defined are auctioned. Firms procure inputs from other firms by seeking competitive contracts which have features of an auction. Contracts to build highways and undertake other public works projects are sold through auctions. Auctions have been proposed as an alternative to the existing quota system for allocating airport time slots to competing airlines. Privatization, takeover, mergers, and acquisitions sometimes follow an auction format. All manner of commodities are auctioned over the internet.

The outcome of an auction depends on the bidding rules, how bidders value the item up for auction, as well as the information each bidder has about her own valuation and the valuations of the other bidders. The next section provides a framework for characterizing the preferences and the information of bidders. With regards information, we focus on three special cases. We have already analyzed what happens when each bidder has perfect information about the valuations of everyone. In a private value auction valuations are independently drawn from probability distributions that everyone knows, each bidder knows her own valuation, but does not observe the other valuations. The third case is common value. Here everyone has the same valuation. Before bidding begins each player receives a signal that is correlated with the unknown common value, which is only revealed after the bidding is completed. The last case we investigate are auctions where one bidder knows the common valuation before making a bid, but the other bidders do not.

There are many different auction formats. Sections 3 and 4 describe the main ones. Our discussion is divided into two, sealed bid auctions, and open auction formats. In a sealed bid auction bidders simultaneously make their respective final bids, the auctioneer selects the highest bidder, and all the bidders pay according to the rules of the auction. For example in a first price sealed bid auction the winner pays his bid, and the other bidders do not pay anything. In an open auction bidders indicate whether their willingness to pay as the auctioneer changes the potential terms of trade to discover which bidder is willing to pay the most. In descending auctions the auctioneer lowers the price until a bidder snaps up the item; in ascending auctions the price increases until only one bidder indicates his willingness to pay.

Several auction formats we analyze share the same direct revelation game, at least for certain information sets. Appealing to the revelation principle discussed in the previous chapter, this implies they have the same solution payoffs to each bidder and the auctioneer. We discuss this form of equivalence in Section 5, before introducing revenue equivalence, a weaker form, in Section 6. When bidders are risk neutral, and their valuations are distributed as independently and identically across bidders, then a wide range of auction mechanisms yield the same expected payoffs to the auctioneer and each bidder. This is called the revenue equivalence theorem. Under these conditions, both the auctioneer and the bidders are indifferent between what auction mechanism is used. Indeed it is straightforward to calculate the solution bid functions for any auction satisfying the conditions of the revenue equivalence theorem from the solution to the second price auction (which itself satisfies the conditions).

2 Preferences and Information

Because auctions differ in their particulars, some of the notation we use to characterize them is necessarily unique to a given type of auction, and therefore best introduced in the subsection where it is actually used. There are, however, common elements to all auctions, so by way of introducing auction games, we now define and explain them. Throughout this chapter we shall consider auction games where N bidders and one auctioneer play. the bidders are labeled 1 through $N,$ and the auctioneer is Player 0. To simplify the exposition, but without loss of generality we shall also interpret the auctioneer as selling an object, and that the bidders are potential buyers. In the case of an auctioneer seeking bids from firms to supply a service, or undertake construction, in other words a procurement auction, the item up for auction is the contract which supplies the service. In a procurement auction the winning bid is negative, since none of the bidders would pay to accept the contract, but apart from that minor detail, nothing else in our analysis would change. The auctioneer, for example, would still prefer high bids, that is low negative prices, to low bids, which are large costs in this case.

2.1 Preferences

Let v_n denote the value the n^{th} player places on the auctioned item in the event of winning the auction. Depending on the particulars of the auction, this parameter might measure the expected value of discounted profit stream from a commercial venture, utility directly consumed by the bidder, or some combination of both. For example when a nation state dispose of government owned and operated airline, commercial airlines bidding for the value of the aircraft fleet, the route slots, and grand-fathered contracts with the employees. They differ in their valuations mainly because of the networking synergies the plant and workforce of the national airline creates with their own, and also the threat that it would pose as an addition to their rivals' network. In any auction, the winning bidder derives pleasure, utility and/or profit from the auctioned item. An example of the second kind is a body massage administered by a film star to the winning bidder in a auction whose proceeds benefit a worthy cause. Bidders at the auction differ in their taste for this form of relaxation and also place different values on the opportunity to meet the film star. Artwork purchased at auctions by private collectors are typically an example of the third kind. The collector may benefit personally from displaying the masterpiece in his private residence, but also recognize that part of its utility lies in its resale value, when he decides to replace it with another or use the resources it generates in some other way. Of course not all bidders necessarily have the same kinds of motives for winning the auction. A waterfront property up for auction might have consumption and investment value to a bidder seeking to live there, only commercial value to a real estate developer, and only utility to a conservation group seeking to preserve the area in its pristine state.

Weighed against the potential benefit of winning the auction is the utility and resources forgone from bidding in the auction. In this chapter we shall assume that all of those costs can be monetized, and p_n denote the total payment of the n^{th} player to the auctioneer. Consequently the utility from paying p_n for the object as $u_n(v_n, p_n)$, which is increasing in its first argument and decreasing in its second. In most of the examples we analyze, the nonlinearites in the utility function are ignored, so as a first approximation we model the net benefit from participating in the auction as:

$$u_n(v_n, p_n) = v_n - p_n$$

Depending on the type of auction, the auctioneer might receive payments from all the bidders, and also might value the item herself. Accordingly, let $a(v_0, p_0)$ denote the value of the auction to the seller, when he values the auctioned item at v_0 and receives a total payment of p_0 for all the bidders, defined as:

$$p_0 = \sum_{n=1}^{N} p_n$$

Having defined prefences over the object on the auction block and the cost of bidding, we are now in a position to define and investigate many trading games of perfect foresight in which several buyers, or bidders, compete with each other for an object for sale by a single seller, the auctioneer.

2.2 Information

The exercise above establish that just by varying the rules about which players are cerified to make limit and market orders, there are many formats for allocating an object, and illustrate the crucial role that information about actions can play, as well as teh many times in which the format does not affect the outcome, something we shall analyze in more detail in the last part of the chapter.

Apart from the bidding rules and the rules about payment, auctions may differ because of what bidders know about their valuations. The case of full information has been analyzed in Chapter 7. Thsi chapter concentrates on two information structures, called private values and common value. When there are private values, each bidder knows her own valuation, but is not privy to any confidential information about the valuations of the others. In private value information structures each valuation is an independently distributed random variable, and each bidder observes only her draw.

In common value information structures every bidder would place the same value on the object if they had the same information about it. Before the auction each bidder receives a signal about the value of the object, that is related but not identical to the common value. We denote the n^{th} bidder's signal by s_n. Moreover we assume that the signals bidders receive may differ, implying that their information sets differ, and that the union of their information is superior to either signal received alone. Accordingly let v denote the value of the object conditional on all the information available to the bidders after the auction, and suppose that each bidder receives a signal denoted denoted s_n. Without loss of generality we may assume:

$$v = v(s_1, \ldots, s_N)$$

For example suppose that an oil well if s_n is independently distributed with mean μ and standard deviation σ, where μ is itself the unknown value of teh object in use, then

$$v = \sum_{n=1}^{N} s_n$$

the third form of information structure we analyze is where some bidders have more information than others. We typically will look at situations in which

what about a private value setup in which one bidder knows the valutiaons of everybody and the other bidders don't know their own valuation?

3 Sealed Bid Auctions

In sealed bid auctions each bidder in a sealed bid auction simultaneously submits a single price to the auctioneer, and the highest bidder receives the auctioned item. For convenience, we assume ties are broken by a randomization, although in many of the models we examine in the next chapter this is inconsequential, since the probability of observing two equal bids is zero. Accordingly, let b_n denote the bid of the n^{th} player, and let $b^{(n)}$ denote the n^{th} highest bid. Thus

$$b^{(1)} = \max_{j \in \{1,\ldots,N\}} \{b_j\}$$

$$b^{(N)} = \min_{j \in \{1,\ldots,N\}} \{b_j\}$$

and so on.

Sealed bid auctions only differ in how much bidders pay. We investigate three variations, first price, second price, and all pay. In a first price and second price sealed bid auction, only the highest bidder anything at all. In a first price sealed bid auction the winner pays the price he or she bid, and in a second price sealed bid auction, the highest bidder pays the second highest bid. In an all pay auction, every bidder pays what he or she bid. We proceed to examine each one in turn.

3.1 First price sealed bid auctions

A first price sealed bid auction can be defined in terms of limit and market orders. Bidders simultaneously submit a single limit order to buy a unit without knowing any of the other bids, and then the auctioneer places a market order to sell one unit. In a first price sealed bid auction, the winning bidder pays the amount she bid in exchange for the object up for auction. Therefore, u_n, the net payoff to the n^{th} player is defined as:

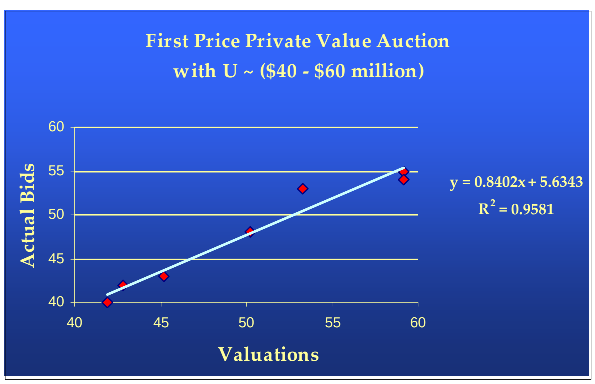
$$u_n = \begin{cases} u_0(0, b^{(1)}) \text{ if } n = 0 \\ 0 \text{ if } n \neq 0 \text{ and } b_n \neq b^{(1)} \\ u_n(v_n, b^{(1)}) \text{ if } n \neq 0 \text{ and } b_n = b^{(1)} \end{cases}$$

Each experimental session consisted of a series of first price sealed bid auctions in which a single unit of a commodity was awarded to the high bidder. The first two auctions were private value auction and the third auction was a common value auction. All the auctions were motivated with the following statement: "Due to recent accounting scandals the Houston Astros (A Major League Baseball Team) are looking to change the name of their staduim from Enron field. Since this is an important piece of revenue for the Astros and whomever eventually puts there name on the stadium, future naming rights of the stadium will be auctioned." In the private value auctions the valuations of the bidders are drawn from a uniform distribution with minimum 40 and maximum 60 and with minimum 30 and maximum 70. Each bidder submits a bid for the auctioned item by submitting a "limit order" for one unit. The auctioneer closes the bidding by making a "market sell". The auction was motivated with the following statement: "Due to recent accounting scandals the Houston Astros (A Major League Baseball Team) are looking to change the name of their staduim from Enron field. Since this is an important piece of revenue for the Astros and whomever eventually puts there name on the stadium, future naming rights of the stadium will be auctioned."

In the common value auction, the value of the item, v_0, was unknown at the time bids were submitted. V_0 was drawn randomly from a uniform distribution with minimum 40 and maximum 60. Each bidder received his own private information signal, x, randomly drawn from a uniform distribution on [v_0 - e, $v_0 +$ e]. The number of bidders (8), the value of e, the distribution undelying both the value of the item, v_0 and the private estimate x were common knlowledge.

More specifically the instructions on the common value auction were read as follows: "If you submit the highest bid in your auction you will win the auction, and your profit is: the value of the of the naming rights minus the HIGHEST bid. This can be gain or a loss. The naming rights are worth between $40 and 60. Operationally, a computer will generate a random number between $40 and $60, so that the range is equally likely. You will not know the value of the naming rights during the auction, but you will be given a private estimate. This estimate will help to narrow down the range of possile values for determining your bid. Each bidder will get a different private estimate. The value of the of each private estimate will be generated from the value of the naming rights, and a random variable, epsilon (e). This means that the values each bidder recieves will be somewhere in the range of (value of the naming rights plus or minus epsilon). Any number in the interval has an equal chance of being drawn. The value of epsilon is $10. The auctioneer in each auction closes the bidding by making a "market sell".

Figure 1 and 2 show the relationship between the private valuation that a subject was assigned and actual bids. The red dimonds represent the actual bids and the solid line is the predicted value obtained from the regression with the bids as dependant variable and valuations as the independant values. As it can be observed from these graphs the higher the private valuation or in the common value auction, private estimate (see Figure 2) the higher the bids are. So bids are increasing in private values and in private estimates. The coefficient on valuations was 0.840 with standard errors 0.231 for the first price private sealed bid auction with private valuation drawn from the uniform distribution with minimum 40 and maximum 60 (Figure 1) and 0.773 (0.382) in the first price common value auction (Figure 2). The value less than 1 indicates that subjects were bidding less than their valuations. In the private value auction valuations drawn from uniform distribution with minimum 30 and maximum 70 the regression coefficient 1.07 on valuation was not significantly different from zero (standard errors were 0.859).



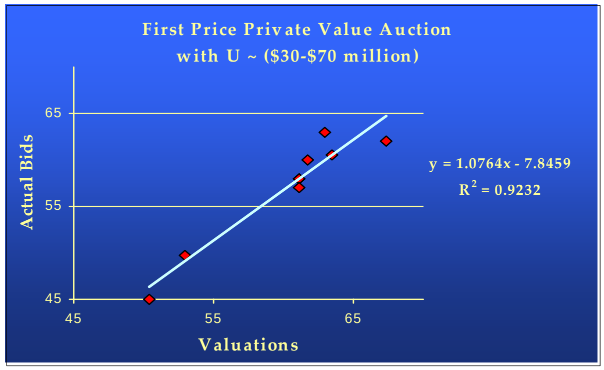


Figure 1:First price private value auction with private value drawn from the uniform distribution with minimum 40 and maximum 60 and with minimum 30 and maximum 70 .

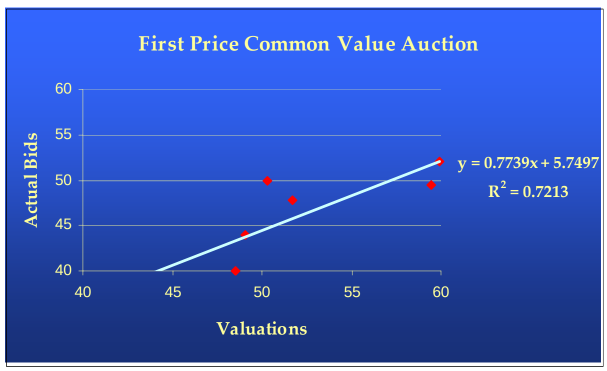
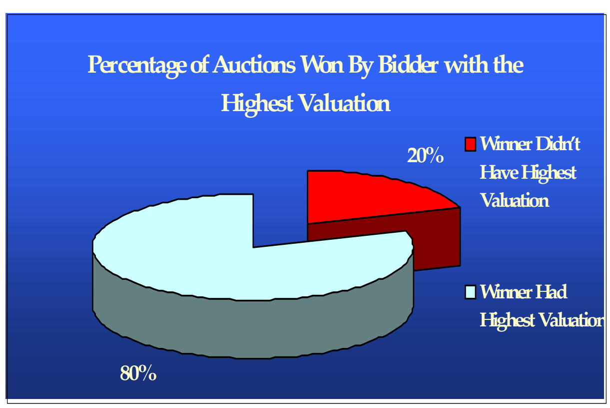


Figure 2: First price common value auction with minimum 40 and maximum 60 and espilon is 10.

Further determination about overall player's strategy could be made by seeing if the winner of each auction correspond to the bidder with the highest valuation. Graph 3 shows that this was the case in 80 percent of the cases (i.e. in four out of the five auctions).



3.2 Second price sealed bid

The second price sealed bid auction although not as common as a first price auction, is also closely related to oral auctions we discuss below. The bidder submitting the highest price pays the second highest price submitted. In terms of the notation we have devloped, the net payoff to the n^{th} player is defined as:

$$u_n = \begin{cases} u_0(0, b^{(2)}) \text{ if } n = 0 \\ 0 \text{ if } n \neq 0 \text{ and } b_n \neq b^{(1)} \\ u_n(v_n, b^{(2)}) \text{ if } n \neq 0 \text{ and } b_n = b^{(1)} \end{cases}$$

Notice that $b^{(2)} \leq b^{(1)}$ so that if all bidders adopted the same bidding strategy for both auctions, then the second price sealed bid auction would yield less revenue than the first price sealed bid auction, and the winners would pay more.

3.3 All pay sealed bid auctions

In an all-pay sealed bid auction, each bidder pays what she bids, and the highest bidder wins the auction. The net payoff to the n^{th} player is defined as:

$$u_n = \begin{cases} u_0\left(0, \sum_{n=1}^{N} b_n\right) \text{ if } n = 0 \\ u_n(0, b_n) \text{ if } n \neq 0 \text{ and } b_n \neq b^{(1)} \\ u_n(v_n, b_n) \text{ if } n \neq 0 \text{ and } b_n = b^{(1)} \end{cases}$$

All pay auctions include raffles and lotteries, but also approximate models of research patent races and political contests. A lottery or a raffle In an all-pay sealed bid auction, each bidder simultaneously pays their bid, and person making the highest payment wins the auctioned item. An all-pay sealed bid auction can also be represented in market as a two stage process. In the first stage bidders purchase as many tokens for a common low price per unit. The auctioneer offers an unlimited quantity of these tokens at a constant low price per unit, and bidders submit market buy orders to obtain them. Like raffle tickets these tokens only have value in the bidding competition which follows. They are used to bid for the item, the owner of the most tokens winning the auction. All-pay auctions are also a paradigm for modeling competitions of various kinds, rather than an institution for literally conducting auctions. For example supply contracts are like all-pay auctions. Bidders expend considerable resources preparing a proposal, but only one bidder is awarded the contract. Similarly research teams in the same field use resources competing with each other, but the first team to make a discovery benefits disproportionately in the rewards from their discovery through patenting, first mover advantages, and so on.

For example in the America’s cup yacht race, held every four years, several syndicates identify long before the elimination rounds, which determine the single competitor who will have the right to challenge the defender, and spend millions of dollars on researching better sailing designs and contracting with some of the world’s leading sailors. However these race preparations are cloaked in secrecy, so as a first approximation it is reasonable to suppose that each syndicate chooses their expenditure without knowing what the other syndicates have chosen. Once the races start, luck, sailing strategy and the boats themselves all contribute to determining the outcome. The pre-race preparations can be modelled as an all-pay simultaneous move auction. Each syndicate player n raises its bid, or holds. the bidding stops when . At that point a lottery is held to determine the winner of the auction.

4 Open Auctions

Rather than attempting to preserve the confidnetiality of secret bids, an alterntive is to conduct the auction

4.1 Descending Auctions

Descending auctions are also known as Dutch auctions, in deference to the most famous of descending auctions, the daily flower auctions in the Netherlands, which now attract both suppliers and demanders from all over the world, much of the produce being airfreighted into and out of that country. The auctioneer begins by offering the item at a very high price which he confidently believes exceeds the willingness to pay of any bidder, and then continuously lowers it until one bidder announces that he is willing to pays the current price. At that point the auction ends, the bidder buying the item at the lowest price offered. To recapitulate, the auctioneer sequentially submits a sequence of declining limit orders to sell a unit, and the auction ends when any bidder submits a market order.

To formally describe the payoffs from this game, let a_t denote the limit sell order in period t, or the auctioneer's ask price at that time. (It is straightforward to show that even if he is not required to do so, the auctioneer will choose $a_t < a_s$ if $s < t$.) Also denote by t_n the time at which the n^{th} bidder submits a market order, conditional on the contingency that no other bidder has submitted an order by that time, meaning the auction has not ended yet. Analogous to our ranking of the bids, let $t^{(n)}$ denote the n^{th} earliest, which implies $t^{(1)} \leq t^{(2)} \leq \ldots \leq t^{(N)}$. The player's payoffs can be then defined as:

$$u_n = \begin{cases} u_0(0, a_t) \text{ if } n = 0 \text{ and } t = t^{(1)} \\ 0 \text{ if } n \neq 0 \text{ and } t_n \neq t^{(1)} \\ u_n(v_n, a_t) \text{ if } n \neq 0 \text{ and } t_n = t^{(1)} \end{cases}$$

4.2 English auction

In ascending auctions, the auctioneer raises the price as long as more than one person is willing to pay the current price. Thus we define an ascending auction in which the price rises until only one bidder is willing to pay it. Consequently the feature differentiating ascending auctions is how much the bidders observe as the auction proceeds. We focus on two special cases, an English auction, and a Japanese auction.

In an English auction bidders compete against each other by successively raising the price at which they are willing to pay for the auctioned object. In some auctions the bidders themselves raise the prices, in other auctions the auctioneer raises the prices and seeks confirming signals from those bidders who are willing to pay that price. The bidding stops when nobody is willing to raise the price any further, and the item is sold to the person who has bid the highest price, at that price. In practice the end of the bidding phase is indicated by an auctioneer announcing "going once, going twice, sold to the lady (or gentleman) . . .", or in eletronic auctions by a set time by which all bids must have been submitted. English auctions can therefore be described in terms of limit and market orders. Bidders submit an increasing sequence of limit buy orders, and once it becomes apparent that no one is willing to raise the highest limit order any further, the auctioneer submits a market sell order.

How much the bidder observes about the other bidders during the auction depends on the setting. During the auction phase a bidder might be able to observe a sample of bidders who make bids, and thus update his beliefs about the value of the item as the auction progresses. The most restrictive assumption is that the bidders do not observe the identity of the other people making bids, and that to win the auction, a bidder must continuously indicate his willingness to pay successively higher prices. This simplification implies that as the auction progresses, a bidder willing to pay for the auctioned item at the current quote knows only that at least one other bidder has also signalled. It corresponds to the notion that bidders must bid frequently in order to attract the attention of the auctioneer. In this text we shall refer to this game as an English auction. Reverting to the terminology for limit order markets, the players observe the highest bid at all times, but not the rest of the book.

Defined in this way, the bidder n^{th} bidder sets a limit denoted b_n and . Denote by b_{nt} the price of a limit buy order previously submitted by the n^{th} bidder and still active at time t, and let ϕ_t denote the highest limit active limit buy order at time t.

$$\phi_t = \max_{n \in \{1 \ldots, N\} s \in [0,t]} \{b_{nt}\}$$

An English auction is a game of attrition. At time t the n^{th} bidder can prevent the item on the auction block from being sold to another bidder for ϕ_t by submitting a limit order exceeding ϕ_t for the item. At the beginning of the auction the bidder n^{th} bidder sets a bound denoted b_n, the maximal amount she is prepared to bid for the object conditional upon at least one other bidder is raising ϕ_t over time. Then she submits an increasing sequence of limit orders, until she reaches her own bound, meaning $\phi_t \geq b_n$, or all rival bidders stop bidding because they have reached their respective bounds, whichever event comes first. We rank the bounds of the bidders from the highest to the lowest as $b^{(1)}$ through to $b^{(N)}$. If the n^{th} bidder drops out of the auction, then she can infer that at least one other bidder has a higher bound than her, or that $b_n < b^{(1)}$. Upon winning the auction bidder n could deduce that her bound exceeds all the others, that is $b_n = b^{(1)}$, and that the bidding has stopped at the next highest bound, meaning $\phi_t \geq b^{(2)}$. For this reason bidders only increases their bids gradually, so that the auction ends when $\phi_t = b^{(2)}$. Thus a player's payoffs in an English auction may be defined as:

$$u_n = \begin{cases} u_0(0, b^{(2)}) \text{ if } n = 0 \\ 0 \text{ if } n \neq 0 \text{ and } b_n \neq b^{(1)} \\ u_n(v_n, b^{(2)}) \text{ if } n \neq 0 \text{ and } b_n = b^{(1)} \end{cases}$$

The bidders private values for the license are drawn from a uniform distribution with minimum $40 and maximum $60. Bidders succesively submit a bid for the auctioned item by submitting a "limit order" for one unit. (Previous limit orders should be deleted as bidders replace them with more attractive ones.) The auctioneer closes the bidding by making a "market sell".

The English Auction was included to observe ascending-value auction play. What was first noted about this auction was that only 6 of the 7 bidders participated. The starting bid of $45 exceeded the initial valuation of a player who did not submit a bid. Other players did not necessarily bid up to their valuations, but they did bow from bidding once the bid price exceeded their valuation price which can be seen in the Table 1 below.

English Auction Results				
Player ID	Final Bid	Valuation	Next Highest Bid	# Bids Above Final and Below Valuation
5	47	47.2808	48	0
6	48.5	54.3755	49	10
2	51	52.4603	55	0
1	52	52.9776	55	0
3	54	54.1201	55	0
12	55	59.512	Winner!	NA

Table 1: Results for the english private value auction.

Player 12 won the auction with the bid of 55 which was less than his valuation of 59.51 and just above the second highest valuation player 3. It is interesting to note that player 12 jumped to the bid 55 when the highest bid was 52. Player 12 could have increased the bid increments more slowely to better determine the valuations of the remainining bidder(s). With this strategy, the final winning bid could have been 54.13.

4.3 Japanese

Japanese auctions are also ascending auctions. Everyone willing to pay the current price for the auctioned indicates this to the auctioneer. The bid price increases until the second last bidder withdraws from contention, and the winner is assigned the item at that price. When a bidder withdraws the auction, every remaining bidder observes and may revise the value of the auctioned item. In this important respect this auction differs from the English auction described above.

When there are N bidders at the outset, the n^{th} bidder decides at what value he will drop out conditional on surviving up until there are only $N-K$ bidders left, having observed the prices at which the remaining K have withdrawn. We denote this by $b(\quad)$This is a increasing sequence of numbers $\{b_{n1}, b_{n2}, \ldots, b_{n,N-1}\}$. the algorithm for eliminating the first bidder is to find the player labelled k_1, who has the minimum threshold for dropping out of the auction:

$$k(1) \equiv \arg\min_{n \in \{1,\ldots,N\}} \{b_{n1}\}$$

Proceeding recursively, we define k_j as the j^{th} bidder to drop out, by comparing the j^{th} element in the sequence of $(N-j+1)$ bidders who remain in the auction:

$$k(j) \equiv \arg\min_{n \in \{k(1),\ldots,k(j-1)\}^c} \{b_{nj}\}$$

where $\{k_1, \ldots, k_{j-1}\}^c$ is just the set of players in $\{1, \ldots, N\}$ who remain after eliminating $\{k_1, \ldots, k_{j-1}\}$, the bidders who have already dropped out. The winning bidder pays the reservation price of the last bidder to drop out, namely $b_{k(N-1),N-1}$

$$u_n = \begin{cases} 0 \text{ if } n \in \{k_1, \ldots, k_{N-1}\} \\ v_n - b_{k(N-1),N-1} \text{ if } n \notin \{k_1, \ldots, k_{N-1}\} \end{cases}$$

The player's payoffs can be then defined as:

$$u_n = \begin{cases} u_0(0, c + b^{(2)}) \text{ if } n = 0 \\ 0 \text{ if } n \neq 0 \text{ and } b_n \neq b^{(1)} \\ u_n(v_n, b^{(2)}) \text{ if } n \neq 0 \text{ and } b_n = b^{(1)} \end{cases}$$

5 An Application of the Revelation Principle

The revelation principle defined and discussed in the previous chapter can be directly applied in auction theory too. recall that two games are congruent they share the same direct revelation game. In congurent auctions, bidders receive the same payoffs as a function of the signals and information they have reveived through the auction. As we explained in teh preious chapter, congruence has a useful implication for deriving the equilibrium bid function, because you should play the same strategy in two

5.1 First price sealed bid and descending auctions

In a descending auction, the auctioneer successively reduces his offer price until stopped by one of the bidders. Each bidder can stop the auctioneer from lowering the price, by buying it at the current offer price. In this way the player who makes the first and highest bid wins, paying the price at which he stops the auctioneer. Therefore nobody receives any information about the value of the auctioned object or about the valuations of other bidders during the auction. It follows that a descending auction is congruent to a first-price sealed bid auction. In a descending auction the outcome would be the same if one or more bidders secretly submitted a price to the auctioneer before it started that announced at what price they would stop the auctioneer from further reducing the price if the ask price fell to the price they submitted.

During the course of a descending auction no information is received by bidders. Therefore each bidder sets his reservation price before the auction, and submits a market order to buy if and when the limit auctioneer's limit order to sell falls to that point. Consequently Dutch auctions and first price sealed bid auctions share the same direct revelation mechanism for implementing the auction, and hence yield the same realized payoffs if the initial valuation draws are the same. Denoting the reservation price of the n^{th} bidder by b_n, the payoff equation is then identical to that of a first price sealed bid auction.

Rule 1 Pick the same reservation price in Dutch auction that you would submit in a first price auction

We remark that this rule does not depend on the information structure associated with the valuations. Congruence holds regardless of whether the information strucure is private values, common value, or more more complicated than either of those two forms.

5.2 Second price sealed bid and English auctions

Information received during the auction In the second price sealed bid auction, bidders receive no information about the bids of other players during the auction itself. Similarly in an English auctions, each player cannot react to information revealed by the other player during the auction. this is evident from the fact taht the reservation price for the English auction is chosen before teh auction begins.

Rule 2 Choose the bid you would submit in a second price sealed bid auction as your reservation price in an English auction.

Observe that the payoffs for teh English and second price sealed bid auctions are the same if players adhere ot the second rule.

5.3 Second price sealed bid and Japanese auctions

English and Japanese auctions are both ascending auctions, but are conducted in slightly different ways. In both mechanisms the highest bidder wins the auction. In a Japanese auction all those willing to pay at least as much as the cu

Strategic equivalence in private value ascending auctions.Similarly in a Japanes auction, each player cannot react to information revealed by the other player during the auction when there are only 2 bidders. (Each bidder can set his optimal reservation price before the auction begins.) When there more than 2 bidders, each bidder receives more information during the bidding process in a Japanese than an English auction, but even in the English auction bidders receive some information during the auction.

The additional information is worthless in private value auctions, because the optimal strategy is to bid (up to) your true valuation. In this case the extra information is not payoff relevant. Therefore the two ascending auction mechanisms are strategically equivalent to the second price sealed bid auction if all the players have independently distributed valuations.When there are only 2 bidders, all three auctions are strategically equivalent (because no information is received during the auction). In common value auctions the 3 mechanisms are not strategically equivalent if there are more than 2 players. For example in a Japanese auction a bidder would lower her reservation price as other players drop out of the bidding. In this case players learn about the common value when other players quit an ascending auction and condition on this information.

Rule 2 In private value auctions, or if there are only two bidders, choose the reservation price for an English or a Japanese auction that you would submit in a second price sealed bid auction.

Consequently the extensive form does not convey any information not included in the strategic form, and the payoffs are identical to those in a second price auction. Therefore the most restrictive form of an English auction corresponds to both games have the same extensive form representation, and hence the same solutions.

6 Revenue Equivalence

Auctions with the same direct revelation game yield the same auction revenue when the valuations of the players are the same. Two auction mechanisms are revenue equivalent if, given a set of players their valuations, and their information sets, the expected surplus to each bidder and the expected revenue to the auctioneer is the same. Thus revenue equivalence is a weaker condition. Revenue equivalence is interesting for two reasons. It tells the auctioneer the conditions under which auctions are not revenue It also provides bidders a straightforward way of deriving the optimal bid strategy We now go on to explore weaker conditions under which revenue equivalence holds.

6.1 Revenue Equivalence theorem

We suppose preferences are additive, symmetric and private, with

$$u(v_1, v_2, \ldots, v_N, p_n) = v_n - p_n$$

Let $\Pr(v_n)$ denote the probability the n^{th} bidder will win the auction when all players bid according to their equilibrium strategy, and let $U(v_n)$ denote the expected value of the n^{th} bidder from participating. Then

$$U(v_n) = E[v_n - p_n]$$

We define the expected payments form participating in the auction (including any fees to enter the auction, payments in the case of submitting a winning bid) as

$$C(v_n) \equiv \Pr(v_n)v_n - U(v_n)$$

Suppose the n^{th} bidder acted as if his valuation was v instead of v_n. Then his net payoff would be

$$\Pr(v)v_n - C(v) = U(v) + v_n \Pr(v) - v \Pr(v)$$

Assuming $\Pr(v)v_n - C(v)$ is differentiable at v we obtain

$$\frac{\partial U(v)}{\partial v} + (v_n - v)\frac{\partial \Pr(v)}{\partial v} - \Pr(v)$$

since there is a stationary point at $v_n = v$ we obtain

$$\frac{\partial U(v_n)}{\partial v} = \Pr(v_n)$$

Appealing to the fundamental theorem of calculus we integrate from the lowest possible valuation up to v_n and thus obtain:

$$U(v_n) = U(\underline{v}) + \int_{v=\underline{v}}^{v_n} P(v)dv$$

This equality shows that in private value auctions, the expected surplus to each bidder does not depend on the auction mechanism itself providing two conditions are satisfied:

1. In equilibrium the auction rules award the bid to the bidder with highest valuation.
2. The expected value to the lowest possible valuation is the same (for example zero).

Note that if all the bidders obtain the same expected surplus, the auctioneer must obtain the same expected revenue.

Theorem *Assume each bidder is a risk-neutral demander for the auctioned object; draws a signal independently from a common, strictly increasing, cumulative continuous distribution function. Consider auction mechanisms where the buyer with the highest signal always wins the bidder with the lowest feasible signal expects zero surplus. Then the same expected revenue will be generated by the auctions, and each bidder will make the same expected payment as a function of her signal.*

The exercise below investigates whether revenue equivalence holds in several auctions that meet the conditions of the theorem, that is assuming that the subjects are risk averse.

6.2 Revenue from private value auctions

Having determined that a wide range of auctions yield the same revenue, we now derive what that revenue is. Since any auction satisfying the conditions for the theorem can be used to calculate the expected revenue, we select the second price, or English auction, to accomplish this task. Recall that from the last chapter each bidder in a second price sealed bid auction bids her valuation. The expected revenue from this auction, and hence the expected revenue from any auction satisfying the conditions of the theorem, is the expected value of the second highest bidder. To obtain this quantity, we proceed in two steps, first deriving and analyzing the probability distribution of the highest valuation, and then deriving the probability distribution of the second highest bidder.

From the perspective of the auctioneer, or an outsider who does not know the valuation of any player, each player has an equal chance of winning the auction. Given N bidders, the probability of bidder n winning the auction is $\frac{1}{N}$. Each player knows his own valuation v_n and consequently has more information than the auctioneer. In the solution to a second price sealed bid auction, each bidder submits her own valuation to the auctioneer, and therefore the winning bidder is the player with the highest valuation. So far as the n^{th} player is concerned, the probability that she has a higher valuation than another player selected at random is $F(v_n)$, the probability that she has a higher valuation than two players selected at random is $F(v_n).F(v_n) = F(v_n)^2$, and appealing to the principle of induction, the probability that she has a higher valuation than the $(N-1)$ rival bidders is $F(v_n)^{N-1}$. That is the probability of winning an auction ion equilibrium is just the probability of being endowed with the highest valuation. If valuations are identically and independently distributed with cumulative probability distribution function $F(v)$, the probability that v_1, for example, is the highest of the N valuations equals:

$$\begin{aligned}
&\Pr\{(v_2 < v_1) \cap (v_3 < v_1) \cap \ldots \cap (v_N < v_1)\} \\
&= \Pr\{v_2 < v_1\} \times \Pr\{v_3 < v_1\} \times \ldots \times \Pr\{v_N < v_1\} \\
&= F(v_1) \times F(v_1) \times \ldots \times F(v_1) \\
&= F(v_1)^{N-1}
\end{aligned}$$

The first line in the equation sequence is simply a statement that the probability of winning is simply that the valuation is the highest, the second line follows from the fact that since the valuations are distributed independently the joint probability is the product of the marginal probabilities. The third statement is based on the fact that all the marginal distributions are identical, while the final line simply counts the number of times $F(v_1)$ is multiplied together.

Notice that as N increases the probability of winning the auction declines. for example if v_1 is the median of the distribution, the probability of winning the auction when $N = 2$ is 0.5, but the probability of winning the auction when $N = 10$ is, which is orders of magnitude less than $\frac{1}{10}$. More specifically the formula for computing the unconditional probability of winning the auction for a value in the q^{th} quantile is:

$$\Pr[q > \max\{v_2, \ldots v_{N1}\}] = q^{-N}$$

Figure 16.1 illustrates how the probability of winning an auction declines as the numbers of bidders increases for valuations at different quantiles in the distribution, when the bidder with the highest valuation wins the auction.

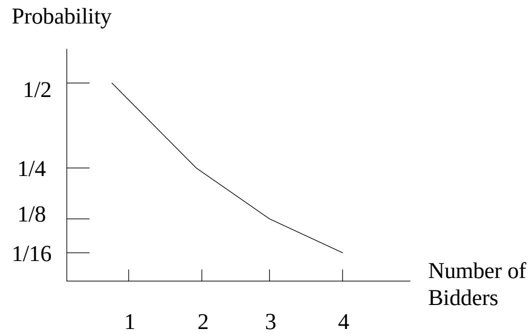


Figure 16.1
The unconditional probability of winning an auction
with a median valuation as a function of the number of bidders

The expected revenue to the auctioneer is the expected value of the second highest valuation,. The event that v is the second highest value is the union of $N = 1$ disjoint events. First all valuations could be less than $v;$ also all but any of the valuations, say $v_n,$ could be smaller than v for all $n \in \{1, \ldots, N\}.$ From above we have that:

$$\Pr[v^{(1)} \leq v] = F(v)^N$$

Also the probability that $v_m \leq v \leq v_n$for some $n \in \{1, \ldots, N\}$ and all $m \neq n$ is:

$$\Pr[v_m \leq v \leq v_n] = F(v)^{N-1}[1 - F(v)]$$

Therefore the probability that all but one of the valuations is less than v is

$$\Pr[v_m \leq v \leq v_n] = NF(v)^{N-1}[1 - F(v)]$$

Combining these results, the probability that is the second highest value is less than or equal to v is the sum of the probabilities that we have derived, namely

$$\begin{aligned} G_2(v) &\equiv \Pr[v^{(2)} \leq v] \\ &= F(v)^N + NF(v)^{N-1}[1 - F(v)] \\ &= NF(v)^{N-1} - (N-1)F(v)^N \end{aligned}$$

Differentiating $G_2(v),$ the cumulative distribution for $v^{(2)}$, we obtain the probability density function for the second highest value

$$\begin{aligned} G_2'(v) &= N(N-1)F(v)^{N-2}F'(v) + (N-1)NF(v)^{N-1}F'(v) \\ &= N(N-1)F(v)^{N-2}[1 - F(v)]F'(v) \end{aligned}$$

We now compute the expected revenue for a private values auction satisfying the conditions of the theorem as:

$$E[v^{(2)}] = N(N-1)\int_{v=\underline{v}}^{\bar{v}} vF(v)^{N-2}[1 - F(v)]F'(v)dv$$

Exercise *Compute the expected revenue from an auction in which valuations* *are distributed on the support* $[\alpha, \infty]$ *as an exponential random variable with minimal value* α *and decay parameter* β. *That is each bidder* $n \in \{1, \ldots, N\}$ *independently draws his valuation* v_n *from the probability distribution*

$$Pr[v_n \leq v] = \beta[1 - exp(\alpha - v)]$$

1. *Compute the expected revenue for different values of the parameters* (α, β) *and the the number of bidders* N.

2. *Conduct auctions of different types and record the auction payments.*

3. *Test the hypothesis that the revenue equivalence theorem holds by comparing the revenues from the different auctions.*

4. *Test the hypothesis the revenue of each auction is distributed about the mean*

5. *Plot the empirical distribution of the bids against their theoretical counterparts. Which auctions have more variance, both in theory and in practice?*

6.3 Bidding rules for private value auctions

Since the equilibrium solution to many private value auctions awards the object to the bidder with the highest valuation, the revenue equivalence theorem can be used to derive the bidding rules by calculating the revenue. We now turn to a derivation of the optimal bidding rules. the revenue equivalence theorem can be exploited to

The revenue equivalence theorem is also a useful tool for deriving the equilibrium bidding rules. a generalization of a principle we developed for second price sealed bid auctions where there is perfect foresight. We have already proved that bidding your valuation in second price auctions is a weakly dominant strategy. In the early parts of this chapter we show that the second price auction is strategically equivalent to several other auction mechanisms, including oral auctions with limited feedback, ascending private value auctions, and all ascending auctions in only two bidders participate. We use the optimal bidding rule of the second price auction and the revenue equivalence theorem to derive the optimal bidding rule for the first price auction and the all pay auction.

First price sealed bid auctions with private valuations

Consider, for example the first price sealed bid auctions with independent valuations.

The probability of the remaining valuations being less than w when the highest valuation is $v^{(1)}$ is

$$\Pr\{v^{(2)} < w | v^{(1)}\} = F(v^{(1)})^{1-N} F(w)^{N-1}$$

so the probability density of the second highest valuation when $v_n = v^{(1)}$ is

$$(N-1)F(v_n)^{1-N} F(w)^{N-2} F'(w)$$

The expected value of the second highest valuation, conditional on $v_n = v^{(1)}$ is therefore

$$(N-1)F(v_n)^{1-N}\int_{\underline{v}}^{v_n} vF(v)^{N-2}F'(w)dv$$

Integrating by parts we obtain the bidding function

$$b(v) = F(v_n)^{1-N}\left[vF(v)^{N-1}\right]_{v=\underline{v}}^{v=v_n} - F(v_n)^{1-N}\int_{\underline{v}}^{v_n} F(v)^{N-1}dv$$
$$= v_n - F(v_n)^{1-N}\int_{\underline{v}}^{v_n} F(v)^{N-1}dv$$

Let $G(v|v^{(1)})$ denote the cumulative probability distribution function for the second highest valuation, given the highest valuation is $v^{(1)}$. That is $G(v|v^{(1)})$ is the distribution function for the highest valuation of the $(N-1)$ losing bidders

$$G(v|v^{(1)}) = \prod_{n=1}^{N-1}\Pr\{v_n < v|v_n < v^{(1)}\}$$

To simplify the expression on the right side of the equation, we first note that the definition of a conditional probability implies

$$\Pr\{v_n < v|v_n < v^{(1)}\}\Pr\{v_n < v^{(1)}\} \equiv \Pr\{v_n < v, v_n < v^{(1)}\} = \Pr\{v_n < v\}$$

the second equality following from the fact that $v_n < v^{(1)}$. But since $\Pr\{v_n < v\} \equiv F(v)$, we have

$$\Pr\{v_n < v|v_n < v^{(1)}\} = \Pr\{v_n < v\}/\Pr\{v_n < v^{(1)}\} = F(v)/F(v^{(1)})$$

Substituting this expression into the top equation thus proves

$$G(v|v^{(1)}) = [F(v)/F(v^{(1)})]^{N-1}$$

Figure 16.1 above also depicts the values of $G(v|v^{(1)})$ for different values of N. For example if $F(v)/F(v^{(1)}) = 0.75$ and $N = 2$, then $G(v|v^{(1)}) = (0.75)^{-4}$ as shown. This implies that if there are three bidders whose values are lower than the winning one, the probability that the highest of the three lies in the bottom three fourths of the truncated distribution is negligible.

Differentiating $G(v|v^{(1)})$ with respect to v we obtain $G'(v|v^{(1)})$, the probability density function for the second highest valuation:

$$G'(v|v^{(1)}) = (N-1)F'(v)F(v)^{N-2}/F(v^{(1)})^{N-1}$$

Therefore the expected value of the second highest valuation, conditional upon the first highest value of $v^{(1)}$ is found by taking the product of v and $G'(v|v^{(1)})$, and then integrating over the range of the losing valuations, which is the interval $[\underline{v}, v^{(1)}]$:

$$E[v^{(2)}|v^{(1)}] = \int_{v=\underline{v}}^{v^{(1)}} vG'(v|v^{(1)})dv$$
$$= v^{(1)} - \frac{\int_{v=\underline{v}}^{v^{(1)}} F(v)^{N-1}dv}{F(v^{(1)})^{N-1}}$$

The second line of this formula is derived by substituting the formula for $G'(v|v^{(1)})$ into the integral, and then integrating by parts:

$$F(v^{(1)})^{N-1}\int_{v=\underline{v}}^{v^{(1)}} vG'(v|v^{(1)})dv = \int_{v=\underline{v}}^{v^{(1)}} v(N-1)F'(v)F(v)^{N-2}dv$$

$$= \left[vF(v)^{N-1}\right]_{v=\underline{v}}^{v^{(1)}} - \int_{v=\underline{v}}^{v^{(1)}} F(v)^{N-1}dv$$

To summarize, in a symmetric equilibrium to first price sealed bid auction a bidder with valuation v_n bids:

$$b(v_n) = v_n - \frac{\int_{v=\underline{v}}^{v_n} F(v)^{N-1}dv}{F(v_n)^{N-1}}$$

For example, suppose valuations are uniformly distributed between $\underline{v}$ and $\bar{v}$. Then:

$$F(v) = \frac{v-\underline{v}}{\bar{v}-\underline{v}}$$

and thus

$$b(v_n) = v_n - F(v_n)^{1-N}\int_{\underline{v}}^{v_n} F(v)^{N-1}dv$$

$$= v_n - (v_n-\underline{v})^{1-N}\int_{\underline{v}}^{v_n}(v-\underline{v})^{N-1}dv$$

$$= \underline{v}/N + v(N-1)/N$$

To interpret the optimal bidding rule for a first price sealed bid auction, note for the formula that your optimal bid is a weighted average of the lowest possible valuation and your own valuation. The weight you put on the lowest valuation is the inverse of the number of bidders. The larger the number of bidders, the closer your bid is to your valuation. Finally, since F (v1)I-1 is the probability a bidder with valuation v1 wins the auction, this expression shows that in equilibrium, the probability of winning the auction declines with the number of bidders.

Comparing bidding strategies in first and second price sealed bid auctions, we see that the bidding strategies in the first and second price auctions markedly differ. In a second price auction bidders should submit their valuation regardless of the number of players bidding on the object. In the first price auction bidders should shave their valuations, by an amount depending on the number of bidders.

Bidding in an all pay auction

In this case the bidders actually pay what they would expect to pay in a first price auction

$$
\begin{aligned}
b_{\text{all pay}}(v_n) &= \Pr\{n \text{ wins auction}\} b_{\text{first price}}(v_n) \\
&= F(v_n)^{N-1}\left[v_n - F(v_n)^{1-N} \int_{\underline{v}}^{v_n} F(v)^{N-1} dv \right] \\
&= v_n F(v_n)^{N-1} - \int_{\underline{v}}^{v_n} F(v)^{N-1} dv
\end{aligned}
$$

In the uniform distribution case we have

$$
b_{\text{all pay}}(v_n) = \left(\frac{v - \underline{v}}{\overline{v} - \underline{v}} \right)^{N-1} \left(\frac{\underline{v}}{N} + \frac{N-1}{N} v \right)
$$

Experiments

The experiments were run over three sessions. At each round a single private value item called wireless comunication channel was auctioned simultaneously in two parallel markets. The number of bidders per markets, and the total number of markets was held constant during each session. In each auction, all bidders in the room paticipated in one market or were randomly assigned to different markets. This random assignment prevented the auction from mimicking a repeated game when the number of bidders was small. Thus bidders did not know ex ante who they were bidding against. Table 1 lists sessions that were run.

Session	Type	Endowment	Private Value	# Playing
#1	First price sealed bid, private	$800K	V ~ [400, 700]	8 players
#2	First price sealed bid, private	$800K	V ~ [400, 700]	3 players
#3	First price sealed bid, common	$800K	V ~ [400, 700]	4 players

Table 1: Description of sessions

In all auctions, bidders values for the item are drawm from from a uniform distribution on with minimum 400 and maximum 700 and they were endowed with $800. Each auctioneer is endowed with one item to sell and has no value for the item. Each subject was informed of the number of bidders in the market. Each bidder submits a bid for the auctioned item by submitting a *limit order* for one unit. The auctioneer in each auction closes the bidding by making a *market sell*.

Table 2 reports the summary results for the first auction in which eight players participated. The winning bid is in bold. We have a closed form for the bid function over the entire range: $b(v) = \underline{v}/I + v(I-1)/I$. For each bid observation, we computed the theoretical bid $b(v_i)$, in a symmetric equilibrium, given the private value and number of potential bidders (I).

Bid	Private Valuation	Private value – Bid	Theoretical Bid	Δ= Bid - Theoretical Bid
400	470.64	70.64	461.81	-61.81
414.53	416.35	1.82	414.30	0.23
415	431.66	16.66	427.71	-12.71
433	435.20	2.20	430.80	2.20
450	546.13	96.13	527.86	-77.86
490	491.15	1.15	479.76	10.24
517.16	546.46	29.30	528.15	-10.99
570	**564.60**	**-5.40**	**544.03**	**25.97**

Table 2: Relationship between the theoretical prediction and the actual bidding behavior for n = 8.

Define $\Delta = b(v_i) - b_i$ as the difference between the actual bid by an agent, b_i, and the theoretical bid in symmetric equilibrium, $b(v_i)$. One prediction of our model is that Δ has mean zero. First, observe that on the basic means test, the model is accepted, the mean of Δ is negative -15.59 and standard error is 12.68. At the 95% confidence level, the t-test yielded a P value of 0.259 indicating that players appeared to behave in a manner consistent with theory. This also shows that players behaved as risk neutral bidders as seen in the chart below.

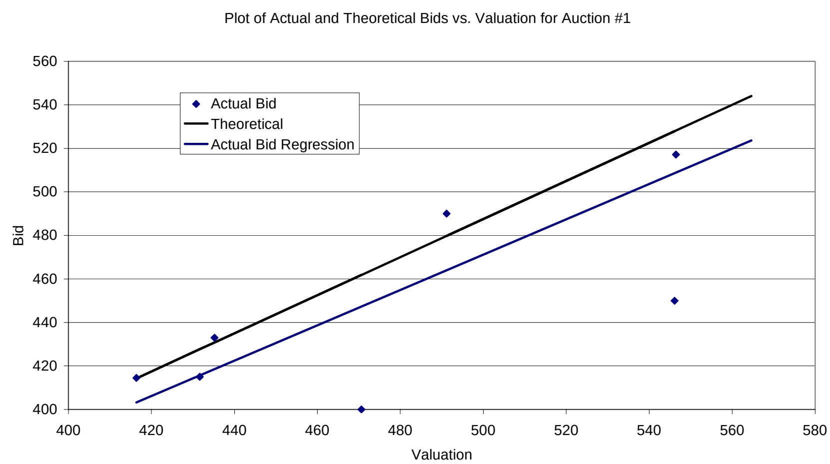


Figure 1: Plot of Actual and Theoretical Bids as a function of valuations for n= 8.

In Table 3 we present results from the three different runs with n=3 and n=4.

Run	N	Bid	Private Valuation	Private value – bid	Theoretical Bid	Δ=Bid - Theoretical Bid
a	3	470	574.56	104.56	516.37	-46.37
		478	493.90	15.90	462.60	15.40
		525	**587.09**	**62.09**	**524.72**	**0.28**
b	4	425	449.81	24.81	437.35	-12.35
		445	476.18	31.18	457.13	-12.13
		475	481.45	6.44	461.08	13.92
		490	**507.89**	**17.89**	**480.92**	**9.08**
c	3	450	524.66	74.66	483.11	-33.11
		500	543.87	43.87	495.91	4.09
		545	**599.07**	**54.07**	**532.71**	**12.29**

Table 3: Relationship between the theoretical prediction and the actual bidding behavior for n = 3 and n=4.

For n=3, the mean of Δ is negative -7.9 and standard error is 10.45. At the 95% confidence level, the t-test yielded a P value of 0.48 indicating that players appeared to behave in a manner consistent with theory. The result is similar for n=4 where Δ = -0.37 and standard deviaion is 6.92. At the 95% confidence level, the t-test yielded a P value of 0.96. This also shows that players behaved as risk neutral bidders as seen in the figures below.

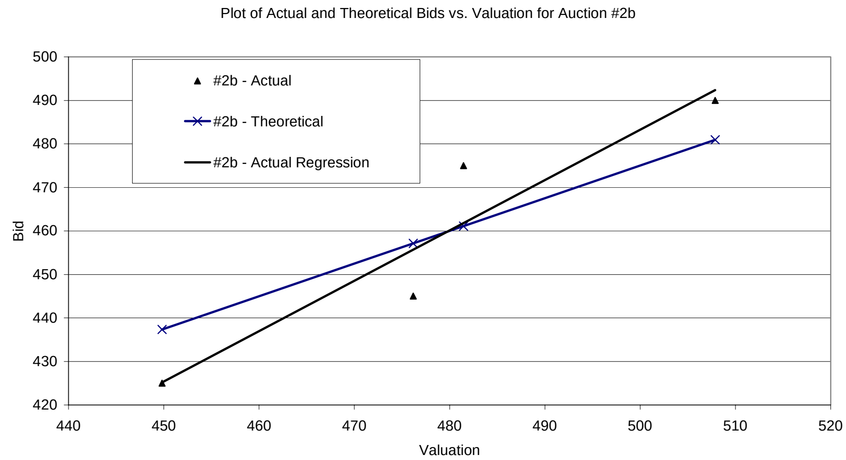


Figure 2: Plot of Actual and Theoretical Bids as a function of valuations for n= 3.

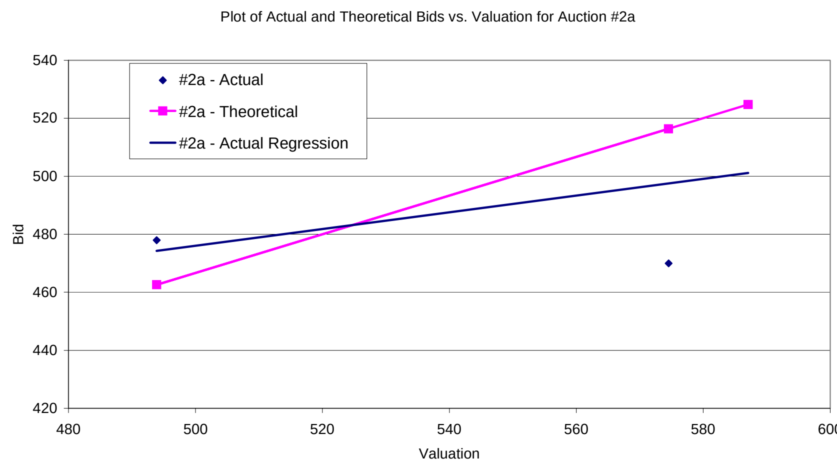


Figure 3: Plot of Actual and Theoretical Bids as a function of valuations for n= 4.

We also regressed bid on number of bidders to test if bidders bid more aggressively when we increase the number of bidder. The coefficient on number of bidders is -4.94 (4.83) and is not significantly different from zero.

All pay sealed bid auction with private valuations

The revenue equivalence theorem implies that the amount bidders expect to pay in an all-pay auction as in all other auctions satisfying the conditions of the theorem. In contrast to a first or second price sealed bid auctions where only the winner bidder pays his bid or the second highest bid in an all pay auction losers also pays their bids. In this case, the amount paid by the n^{th} bidder is certain, and not paid with the probability of winning the auction, that is $F(v_n)^{N-1}$. By the revenue equivalence theorem the amount each bidder expect to pay in the first two auctions, upon seeing their valuation, equals the amount the bidder actually does pay in all pay auction. Thus we see that in an all pay auction the expected revenue from a bidder with v_n bids $F(v_n)^{N-1}$ multiplied by the amount he would bid in a first price auction. From the equations above this is:

$$v_n F(v_n)^{N-1} - \int_{v=\underline{v}}^{v_n} F(v)^{N-1} dv$$

10 Summary

This chapter described several kinds of auction mechanisms and demonstrated their relationship to one side of the market when the other side is controlled by a single player with no discretion about how much to trade. Then we showed certain equivalences between different auction mechanisms. We argued that first price sealed bid and the descending auctions are congruent, and that the second price sealed bid auction is congruent to the English auction as defined here.

When a bidder in a second price auction knows his own valuation, then it is a weakly dominant strategy to bid it. Bidders should bid (up to) their valuations. This result holds independently of what others know either about the valuations of rivals or the their own valuations. A second result we derived applies to many kinds of auctions, but simplifies the information structure considerably, by assuming that each bidder knows not only how they themselves value the auctioned item but how every other bidder does too. In this case all the auction mechanisms we described yield the same revenue to the auctioneer. In the next chapter we relax the strong assumptions about information and ask how far the result on revenue equivalence can be extended.

We then derived and analyzed the solutions for several different types of symmetric auctions, starting first with private value auctions, before moving to symmetric auctions with a more general information structure. We began by demonstrating the revenue equivalence result for private value auctions.