1 Introduction

Monopoly is defined by the phrase "single seller", but that seems to characterize an auctioneer too. The two main differences distinguishing models of monopoly from auction models are related to the quantity of the good sold. First, monopolists typically sell multiple units, but most auction models analyze the sale of a single unit. In practice, though, auctioneers often sell multiple units of the same item. Second, we typically model monopolists choosing the quantity to supply, whereas most models of auctions focus on the sale of a fixed number of units. Again this is an artificial distinction. In reality the use of reservation prices in auctions endogenously determines the number the units sold as well.

There are several other differences between most auction and monopoly models. Monopolists price discriminate through market segmentation, but auction rules do not make the winner's payment depend on his type. A firm with a monopoly in two or more markets can sometimes increase its value by bundling goods together rather than selling each one individually. While auction models do not typically explore these effects, auctioneers also bundle goods together into lots to be sold as indivisible units, presumably to increase the net revenue from the auction.

The next two sections in this chapter review multiple unit auctions. We distinguish between two basic cases, when each bidder wants at most one unit, and when individual bidders are willing to pay extra for multiple units. The reason for distinguishing between these two cases is that many of the results on strategic and revenue equivalence derived for single unit auctions extend to multiunit auctions providing buyers only bid on one unit each. One auction we consider in Section 2 is a sequential auction where bidders have private values. As successive units are sold, there are less left to auction, but only the bidders with lower valuations remain. We show that these factors cancel each other out, and that the sequence of winning bids follows a random walk.

New complications are introduced when a bidder is willing to pay extra for a second unit, because the bid she makes on her second unit may affect the price she pays for the first, depending on the auction mechanism. We describe several multiunit auctions in Section 3 and analyze the Vickery auction, which is the natural generalization of a second price auction. In a Vickery auction, prices are depend on the number of units purchased and the identity of the bidder, and in equilibrium the auctioned units are allocated to their highest value use.

Section 4 relaxes the assumption that the number of units sold is fixed. We begin by analyzing the case in which demand at any given price is known. If in addition the valuations of each demander is known, then the monopolist may be able to extract all the surplus from the bidders by playing an ultimatum game with each consumer segment. In this case all the gains from trade are captured by the monopolist. There may be technical or legal reasons why price discrimination is impossible, and in such situations we might expect a uniform price to emerge. After showing how these principles are applied in a standard setting, we analyze how much more an entrepreneur seeking financial partners can gain from engaging in price discrimination.

Between perfect price discrimination and uniform pricing are many possibilities that allow monopolists to take advantage of consumer heterogeneity. Section 5 studies two, applying quantity discounts and segmenting the market by product bundling. Our analysis draws from the chapter on contracts. When segmenting the market by discounting for quantity or bundling products, consumers select from choice sets the product line that maximize their own utility subject to eligibility rules laid out by the monopolist. Their selection constrains the ability of the monopolist to extract rents, and in this way helps characterize the constraints that shape the monopoly optimization problem.

An interesting form of market segmentation arises in durable goods markets, the subject of Section 6. Here the monopolist may be able to introduce the product at a high price to the consumers who value it the most, and then reduce the price over time to attract residual consumers. Another aspect of supplying durable goods is how durable the monopolist makes a product. A natural benchmark is that as a value maximizer the monopolist is a cost minimizer. There are however mitigating circumstances that might lead a monopolist to produce something more or less durable than what is optimal, for example when consumers are partly responsible for how durable the good is by their own actions.

Finally, in Section 7, we turn to a situation where the monopolist does not know the demand for his product, but only the probability distribution from which his demand arises. Perhaps the simplest problem to solve within this class is a single unit auction, where the bidders values are independently and identically distributed, and where the seller can commit to destroying the unit if no bidder is willing to pay a reservation price that he sets before the bidding begins. In this case the monopolist chooses between selling zero or one units of the good. In a second application we study optimal pricing by a monopolist who is learning about his demand curve.

2 Single Unit Demand

Let us suppose there are Q identical units of a good up for auction, all of which must be sold to N bidders or potential demanders of the product. Each bidder values having one unit of the good, but no bidder places any value on owning more than one unit, and $Q < N$. Following previous notation, we denote their valuations for the first unit by v_1 through v_N. There are more ways to auction multiple units of the same good than there are ways to auction a single unit. The seller must decide whether to sell the objects separately in multiple auctions or jointly in a single auction. Moreover within each auction there is a range of formats for sellers to choose between. Following our discussion of single unit auctions, we divide the types of auctions up into various types of sealed bid, descending price, and ascending price auctions.

Revenue equivalence revisitedAn important question is whether the revenue equivalence theorem extends to multiunit auctions. Initially we suppose that each bidder knows her own valuation, is risk neutral, and wishes to buy at most one of the items. As in the single unit case we assume that each valuation is independently and identically drawn from the same distribution, and that the two auction mechanisms under comparison support equilibrium in which the q highest valuation bidders win. In this case the revenue equivalence theorem applies, implying both mechanisms yield the same expected revenue and that conditional upon drawing a valuation, all bidders are indifferent between the form of the auction.

Exercise *Comparing the restuls from the previous auction experiments. Consider a private value auction where the number of bidders is* $N = 10$, *valuations* v_n *are independently, identically and uniformly distributed with support* $[0, 100]$ *for* $n \in \{1, \ldots, 10\}$, *and the number of units on the auction block is* $Q = 4$.*Conduct five experiments in which winning bidders pay the prices they submitted, in other words a discriminatory auction. Then conduct five experiments in which winning bidders pay the sixth best bid. Finally conduct five experiments in which each player pays what they bid. Plot the bids as a mapping of the valuations, taking care to distinguish between the three auctions. Find an estimate of the unconditional expected surplus to bidders in each case. For each type of auction, compute an estimate of the expected surplus for bidders conditional on having valuations in the range of* $[0, 20)$, $[20, 40)$, $[40, 60)$, $[60, 80)$ *and* $[80, 100]$. *Are there significant differences? Test whether the differences between the average revenues obtained from three auction types are significant.*

2.1 Auction formats

Sealed bid auctions for multiple units are conducted by inviting each bidder to submit a single bids, and allocating the available units to the highest bidders. Following our notational convention for single unit auctions, we denote by b_n the price submitted by the n^{th} bidder. Typically those bidders submitting bids ranked lower than the Q^{th} price pay and receive nothing, a notable exception being all-pay sealed bid multiunit auctions. Thus sealed bid auctions are usually distinguished from each other by the rules which determine the prices that successful bidders pay. One convention is for winning bidders to pay the respective prices they posted. This is called a discriminatory auction. As before, let $b^{(q)}$ denote the q^{th} highest bidder (where ties are decided randomly). The revenue generated from a discriminatory auction is thus

$$\sum_{q=1}^{Q} b^{(q)}$$

Denoting by u_n the net payoff to the n^{th} bidder, we see that in a private value auction:

$$u_n = \begin{cases} v_n - b^{(q)} \text{ if } b_n = b^{(q)} \text{ for some } q \leq Q \\ 0 \text{ if } b_n < b^{(Q)} \end{cases}$$

Another convention is to charge winning bidders a uniform price, such as the j^{th} highest bid, for some $j \in \{1, \ldots, N\}$. Then the revenue generated from the j^{th} price uniform auction is $Qb^{(j)}$ and the net payoffs the n^{th} bidder is:

$$u_n = \begin{cases} v_n - b^{(j)} \text{ if } b_n \geq b^{(Q)} \\ 0 \text{ if } b_n < b^{(Q)} \end{cases}$$

Discriminatory price private value auctions

Sealed bid

Descending price multiunit (Dutch) auctions

Similar to a single unit descending auction, the auctioneer gradually reduces the price, and records the points at which bidders indicate their willingness to buy. Descending price multiunit auctions provide a fast way of disposing of several objects which might differ in many ways. The bidder willing to pay the highest price chooses the object he ranks most highly, the price continues to fall until a bidder indicates his willingness to buy a second object, which is then removed. The process continues until all the objects are sold. In a Dutch descending auction, the first Q bidders to signal pay the price they indicated in return for a unit, while the remaining bidders pay and receive nothing. Since the auction does not end with the first bidder's signal, the strategy space for bidder's is more complicated than in the single unit case. Each bidder must decide when they will signal conditional upon observing q of the Q units have already been assigned to other bidders.

In a private value auction, there is no reason to bid more for the second item if the first has already been taken, since this would imply that it would have been optimal to bid higher for the first. It follows therefore that each bidder picks a declining sequence of bids, and we denote a strategy by the n^{th} bidder as $b_n \equiv (b_{n1}, \ldots, b_{nQ})'$ where b_{nq} is the price which the bidder indicates he will pay once $(q-1)$ units have already assigned, and $b_{n,q} \geq b_{n,q+1}$. The strategic form of this auction is completed by defining the payoffs as a function of the strategies. Labeling as $b^{(1)}$ the first bid, we can recursively define who wins units at this auction. At the q^{th} stage, we choose amongst those remaining the bidder with the highest $\{b_{1q}, \ldots, b_{Nq}\}$ after eliminating from consideration those bidders who have already won a unit, which we label $N(q-1)$:

$$b^{(q)} \in \max_{n \in N(q)} \{b_{nq}\}$$

$$N(q) + \sum_{n=1}^{N} n 1\{b_{nq-1} = b^{(q-1)}\} = N(q-1)$$

The resulting declining sequence $\{b^{(q)}\}_{q=1}^{Q}$ is summed to form the revenue generated by the auction:

$$\sum_{q=1}^{Q} b^{(q)}$$

The payoffs to each bidder may be written as

$$u_n = \begin{cases} v_n - b^{(q)} \text{ if } b_n = b^{(q)} \text{ for some } q \leq Q \\ 0 \text{ if } b_n < b^{(Q)}. \end{cases}$$

In a uniform price descending auction bidders pay the same price, for example the price at which the l^{th} bidder tendered for some $l \in \{1, \ldots, N\}$, denoted $b^{(l)}$. In this case the payoffs are written as

$$u_n = \begin{cases} v_n - b^{(l)} \text{ if } b_n = b^{(q)} \text{ for some } q \leq Q \\ 0 \text{ if } b_n < b^{(Q)}. \end{cases}$$

Experimental design

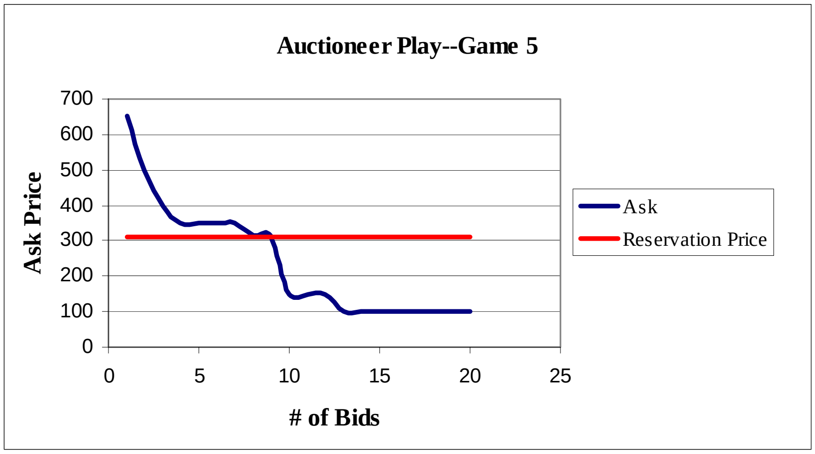
Game Background: Four bottles of wine are auctioned. One player is designated the auctioneer, the first one to log in. The other players are consumers. The valuation of the consumers are drawn from uniform distributed with a minimum value of $100 and a maximum of $700. Each bidder is endowed with $1000.

The auctioneer in each auction successively submits limit orders to sell one unit, gradually decreasing the price at which the auctioned item is available. The auction concludes when all four bottles of wines are sold, by submitting a market order to buy the auctioned good. If you win the auction, your profit is: the value of the company minus the price. This can be a gain or a loss.

Name	**Time of Bid**	**Subjective Value**	**Actual Bid**	**Profit**
Supastar	18:54:20	594.16	590	4.15
XXXX	18:54:21	627.12	590	37.12
YYYY	18:54:23	690.08	590	100.08
ZZZZ	18:55:26	549.87	545	4.87

The names are listed in order of how bidders won bottles. Supastar retrieved the first bottle of the auction. XXXX instead waited to receive the best price on the bottle based on his valuation. This is shown by his high valuation and by his bid time being right after Supastar's bid. He obviously waited for someone to buy the first bottle and then immediately bought the second to ensure purchase, but at a price significantly less than his valuation. YYYY likely played a similar strategy to XXXX: waited for someone to take the first bottle and then purchased. This is further shown by his bid time being 2 seconds different than XXXX's.

The auctioneer simply played the game as they were supposed to, not attempting to gain any special advantage with their price offerings or tick time.



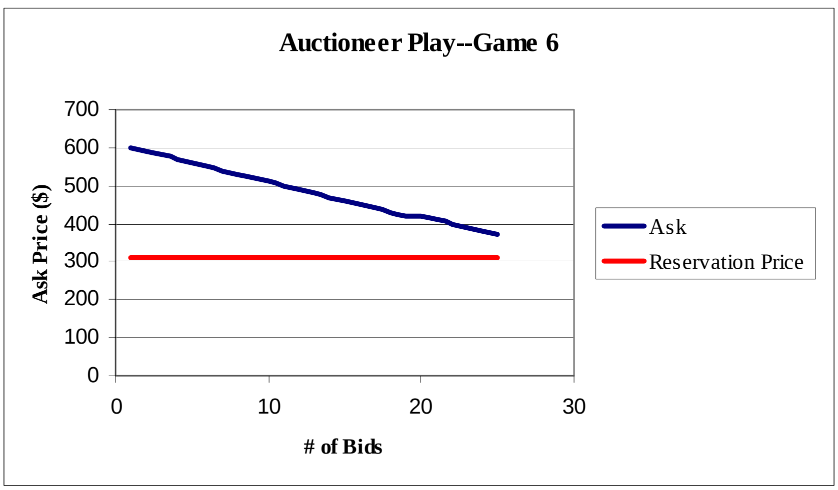
Name	Time of Bid	Subjective Value	Actual Bid
Saida	19:13:25	676.498	350
gb	19:13:25	389.027	350
Supastar	19:13:27	316.783	315
charness	19:13:31	187.788	150
kia	19:13:33	540.813	350
fernando	19:13:34	566.641	100
restifo	19:13:34	129.004	100
restifo	19:13:37	129.004	100
thay	19:13:42	309.842	100
thay	19:14:01	309.842	100
Start Time: 19:04:14			

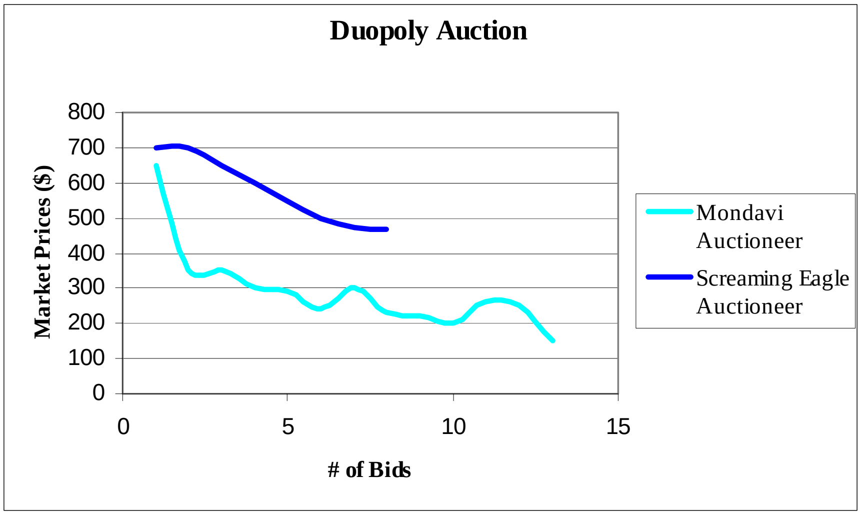
Game 6: Monopoly Wine Game With Variable Cost Per Unit
Activity in this auction mirrored that of Game 5, the other monopoly with 5 bottles. The results are shown in the table.

Game 6--Bidding Strategy and Final Profits

Player	# of bids	% bids removed unnecessarily	Winning Bid*	Subjective Valuation	Sub. Val - Final Bid
G Martin	1	0%	$ 100.00	$ 102.95	$ 2.95
restifo	2	50%	**$ 420.00**	$ 431.59	$ 11.59
kia	1	0%	$ 120.00	$ 621.43	$ 501.43
saida	2	50%	**$ 390.00**	$ 510.83	$ 120.83
gb	4	0%	$ 180.00	$ 347.97	$ 167.97
thay	4	0%	$ 182.00	$ 183.57	$ 1.57
supastar	2	50%	$ 300.00	$ 307.96	$ 7.96
fernando	4	75%	**$ 380.00**	$ 491.93	$ 111.93

*****actual winning bids are in boldface type. All other bids here are simply non-winning, final bids





Name	Time of Bid	Subjective Value	Actual Bid
charness	19:21:23	458.928	350
charness	19:21:26	458.928	350
fernando	19:21:31	617.073	300
fernando	19:22:04	617.073	290
Supastar	19:22:15	255.268	240
Thay	19:22:41	248.419	230
Kia	19:22:41	433.943	230
Gb	19:22:51	246.372	220
Start Time: 19:04:14			

Experiment 16.2

Consider a private value auction where the number of bidders is $N = 10$, valuations v_n are independently, identically and uniformly distributed with support $[0, 100]$ for $n \in \{1, \ldots, 10\}$, and the number of units on the auction block is $q = 4$. Conduct five experiments in extensive form using a descending auction format with discriminatory pricing, and another five with a uniform price of the fourth highest bidder, that is $4\{b_1, \ldots, b_{10}\}$. Then repeat the experiments using the strategic instead of extensive form.

Exercise **1.** *Plot the extensive and strategic form (valuation, bid) coordinates for the discriminatory and uniform price auctions.*

2. *Compare the sequence of prices $k\{b_1, \ldots, b_{10}\}$ for the strategic and extensive forms.*

3. *Compare the surplus from all four auctions, and test whether there is* *any statistical difference between them.*

2.2 First price sequential auction

The Random Walk Hypothesis

In this section we examine dynamic markets by analyzing sequential auctions. First we look at supply be a fixed number of firms each supply a fixed quantity, then we see how the industry determines supply, and finally we ask how a monopolist would conduct this industry.

Throughout we suppose there are N bidders, each with a private valuation drawn from the probability distribution function $F(v)$ and K items.

Much of the intuition comes out of a two auction model. Note that in the final period there are only $N-1$ bidders, each of whom has a valuation of $v_n < v^{(1)}$. Following the rule derived in the previous chapter, the optimal bidding strategy is

$$b_2(v_n) = E\left[V_n^{(2)} \middle| V_n^{(2)} < v_n\right]$$

where as before $V_n^{(2)}$ is the second highest value excluding V_n, or the highest of the remaining values excluding the value of the winner in the first auction and V_n itself:

$$V_n^{(2)} \equiv \sec\max\{V_1, V_2, \ldots, V_{n-1}, V_{n+1}, \ldots, V_N,\}$$

We remark that this value does not depend on the price or the value of the first auction winner. Now consider the first period. As before we focus on a necessary condition that comes from considering a bidder with valuation v_n submitting $b_1(v)$ instead of $b_1(v_n)$, where $b_1(v) > b_1(v_n)$. In this case the probability that she wins the first auction is $F^{N-1}(v)$ in which case her net payoff is $[v_n - b_1(v)]$, the probability that she loses the auction to the first bidder by bidding $b_1(v)$, but wins the second auction by bidding $b_2(v_n)$ is $[1-F(v)]F^{N-2}(v_n)$, in which case she earns $[v_n - b_2(v_n)]$. She could also lose to the second bidder in the first auction but win the second auction, and so it goes on with respect to the other $N-1$ bidders. Therefore her expected payoff from bidding $b_1(v) > b_1(v_n)$ in the first auction, and in the case of losing, bidding $b_2(v_n)$ in the second auction is

$$F^{N-1}(v)[v_n - b_1(v)] + (N-1)[1-F(v)]F^{N-2}(v_n)[v_n - b_2(v_n)]$$

Differentiating with respect to v, we obtain:

$$0 = (N-1)F^{N-2}(v)[v_n - b_1(v)] - F^{N-1}(v)b_1'(v) - (N-1)F'(v)F^{N-2}(v_n)[v_n - b_2(v_n)]$$

which is must be an inflexion or a stationary point for the optimal solution.

Since the optimum occurs where the one sided derivative has zero $v = v_n$ and

$$b_1'(v_n) = \frac{(N-1)F^{N-2}(v_n)}{F^{N-1}(v_n)}[b_2(v_n) - b_1(v_n)]$$

Second Price Sequential Auction

Note that the revenue equivalence result implies

A noteworthy feature of this solution is that the successive winning prices follow a random walk. This is an example of the efficient markets hypothesis, a proposition that assets this rule holds more generally. It is based on the arbitrage pricing rule. If a bidder expected the winning price of the second item to be lower than the first, he would bid less aggressively in the first auction and more aggressively in the second.

Experiment 16.3

The uniform distribution

To be more concrete, we now consider the case in which private valuations are identically distributed on $[\underline{v}, \bar{v}]$ amongst N bidders. In this case the expected value of the Q^{th} highest valuation is

$$E[v^{(n)}] = \underline{v} + \frac{N+1-Q}{N+1}(\bar{v} - \underline{v})$$

Therefore if the firms had to make their production decision before the first bid is made then Q would be chosen as $N = 1$ if $C < \underline{v}$, and if $\bar{v} > C \geq \underline{v}$ to solve

$$C = \underline{v} + \frac{N+1-Q}{N+1}(\bar{v} - \underline{v})$$

which implies

$$Q = N + 1 - (C - \underline{v})(N+1)/(\bar{v} - \underline{v})$$
$$= (N+1)(\bar{v} - C)/(\bar{v} - \underline{v})$$

As N increases, the number of units produced as a fraction of total demand converges to the number of bidders with valuations above C.

Now considering a monopolist or cartel who pre-commits to a production level, his expected total profit is

$$\left[\underline{v} + \frac{N+1-Q}{N+1}(\bar{v} - \underline{v}) - C\right]Q$$

Differentiating to obtain the first order condition we see that

$$\frac{Q}{N+1}(\bar{v} - \underline{v}) - \left[\underline{v} + \frac{N+1-Q}{N+1}(\bar{v} - \underline{v}) = C\right]$$

3 Multiunit Demand

Many equivalences between different types of auctions break down. Intuitively, each bidder recognizes that his bid on one item may compete with a bid he submits for another unit. The Vickery auction is a mechanism that prices winning bids to avoid this problem

Auctions with multiunit demands are those where one or more bidders seek to purchase at least two of the units on the auction block. Without loss of generality we suppose each player submits bids on all Q units for sale.

3.1 Vickery auction

In sealed bid auctions where N bidders compete for a total of Q units by submitting demands for multiple units, we may assume without loss of generality that the auctioneer receives QN bids from the N bidders for the Q units. We denote by $b_n^{(q)}$ the bid player n makes on his q^{th} unit where $q \in \{1, \ldots, Q\}$. As before we rank the winning bids $b^{(1)}$ through $b^{(Q)}$. This ranking determines how the units are allocated, by pairing the Q highest bids with the players who submitted them. Allocating the units this way is one feature shared by practically all sealed bid auctions where there is multiunit demand.

The distinguishing feature of Vickery auctions are the prices that each bidder pays for the units he has won. Suppose the n^{th} bidder places wins Q_n objects, implying that Q_n of his Q bids were at least $b^{(Q)}$ and that

$$Q = \sum_{n=1}^{N} Q_n$$

To calculate what the n^{th} bidder must pay in a Vickery auction for the Q_n objects he has won, note that there are $(N-2)Q + Q_n$ losing bids by the other $(N-1)$ bidders, and rank the top Q_n of those losing bids from the other players by $b_{-n}^{(1)}$ through $b_{-n}^{(Q_n)}$. Clearly $b^{(Q)}$ exceeds $b_{-n}^{(1)}$ through $b_{-n}^{(Q_n)}$. Furthermore if the n^{th} player had suddenly withdrawn all his bids just before the auctioneer started announcing the winning bids, and the other $(N-1)$ players did not change their bids, then the players whose bids corresponded to $b_{-n}^{(1)}$ through $b_{-n}^{(Q_n)}$ would have won the extra units now released to them by bidder n withdrawing from the auction. In a Vickery auction, the n^{th} bidder pays the bids $b_{-n}^{(1)}$ through $b_{-n}^{(Q_n)}$ for the units he has won. That is

$$u_n = \begin{cases} \sum_{q=1}^{Q_n} \left(v_n^{(q)} - b_{-n}^{(q)} \right) \text{ if } Q_n > 0 \\ 0 \text{ if } Q_n = 0 \end{cases}$$

If each demand bids a strictly positive amount only only one item, then every winning bidder pays the same price $b^{(Q+1)}$, the highest losing bid. This situation corresponds to the sealed bid uniform price auction we discussed above, and specializes to the second price sealed bid auction, analyzed in Chapters 15 and 16, when $Q = 1$, meaning only one unit is auctioned. More generally, however, bidders might pay different amounts for their items, even if they win the same number. Thus Vickery auctions yield discriminatory prices.

There are two main properties that characterize Vickery auctions with private values. It is a weakly domininant strategy for each bidder to submit their valuations for the Q units they demand, and the auction allocates the items to the highest valuation bidders. The second claim may be restated as the following. In a Vickery auction the total surplus from the auction to the auctioneer and the bidders is maximized because the units are allocated to the bidders with the highest valuations:

$$\sum_{n=0}^{N} u_n = \sum_{q=1}^{Q} v^{(q)}$$

The second property is a corollary of the first one because the bids correspond to the valuations, and the units are allocated to the highest Q bids.

Thus the first property is that:

$$b_n^{(q)} = v_n^{(q)}$$

Experiment 16.4

Compare the outcome of a Vickery auction with a uniform price auction when there are two items for sale by auction.

4 Price and Quantity Decisions

By definition a monopolist chooses the aggregate quantity marketed to all demanders in the industry. Analogously a monopsonist chooses the aggregate quantity bought from all suppliers in the industry. Examples of monopoly are easy to spot, from corner stores in remote areas, to professionals serving niche industries, to highly concentrated industries which collude under the guise of government regulation or evade antitrust law, whose purpose is to prevent their rent seeking behavior. Monopsony seems less frequent. Labor markets in some mining towns are monopsonistic. There is, essentially, one employer buying (almost) all the labor services offered by its residents.

This section analyzes analyze three applications of quantity choice. Initially we suppose that the demand valuations of buyers are known. This does not mean that the monopolist knows the valuations of each buyer, but only that he knows the number of buyers with with any given valuation. We look at two cases, where there are constant marginal costs, and where supply is fixed. The second application is a scenario where the seller can set a reservation price on a single item up for auction, but does not know the demand curve, only the probability distribution from which the bidders are drawn. The third application uses the principle of restricting transactions to improve the terms of trade in a model of intermediation. Specialists who have exclusive rights to trade in a pariticular stock on a stock exchanges intermediate between suppliers and demanders, driving a wedge between buy and sell prices to make a living. In this way they both monopolists and monoponists. We analyze how tehi affects the prices they quote to traders seeking to adjust their portfolios.

Price Discrimination

When analyzing monopoly, an important issue is the quantity the monopolist chooses to supply and sell. This affects the price or prices of the transaction, because the more items produced the less Regulators argue that compared to a competitively organized industry where there are many firms supplying the product, a monopolist restricts the supply of the good and charges higher prices to high valuation demanders in order to make rents out of his position of sole source.

Is this true in practice?

Designing a monopoly game with a quantity choice

What is the equilibrium quantity bought and sold?

First question: how does seller commit not to selling item and disposing of it for nothing instead. Also a problem in the ultimatum game.

Ultimatum game with each consumer perfect discrimination. Discriminating monopolist. Suppose the monopolist knows the valuations of the players, and can commit to prices. Make a take it or leave it offer to each person: multi person ultimatum game.

In the standard approach to monopoly a single seller decides how much to produce and market at the same price to each buyer. The underlying premise to this model is that the monopolist can commit to any level of production, but cannot charge different prices to different consumers.

4.1 Uniform versus discriminatory pricing

Legal issues aside, there are three necessary and sufficient conditions for full price discrimination to work. The monopolist must know how much each consumer is willing to pay for his product, he must be able to prevent resale between customers, and he must be able to commit to an all or nothing offer to each consumer. If any one these three conditions breaks down, extracting all the consumer surplus becomes much more problematic. In this case the monopolist might simply sell his good or services at a uniform price. In this case maximal rent for the monopolist market the good where marginal revenue equals marginal cost, supplying less than the amoutn that would exhaust the total gains from trade in order to maximize its rent.

To be more specific, suppose there are 12 potential buyers with valuations which decline from $\$20$ to $\$9$ in single units, and that the monopolist's production cost is $\$9$. The second column in Table 17.1 show the number of demanders who have have valuations that are greater than or equal to the corresponding price in the first column. For example there are 4 demanders with valuations of 17 or more. The third column, Total Revenue, is the product of the first two. An element in this column shows the total revenue accruing to the monopolist from charging the price in the same row to all the demanders with valuations that were at least as high as the price. Marginal Revenue measures the change in Total Revenue from increasing the quantity sold by one unit. Thus increasing the Quantity from 3 to 4 raises Total Revenue by $16,$ from 54 to $68,$ the fourth entry in the fourth column.

We first suppose that the monopolist can produce unlimited quantities of the good for a constant unit cost, and must choose a quantity to supply at a uniform price. The next two column shows the Total Cost of supplying the number of units demanded at that price. Marginal Cost, calculated using the same method as Marginal Revenue, is the increase in Total Cost from producing an extra unit. The last column, Profit, is simply the difference between Total Revenue and Total Cost.

Price	Quantity	Total Revenue	Marginal Revenue	Total Cost	Marginal Cost	Profit
20	1	20	20	9	9	11
19	2	38	18	18	9	20
18	3	54	16	27	9	27
17	4	68	14	36	9	32
16	5	80	12	45	9	35
15	6	90	10	54	9	36
14	7	98	8	63	9	35
13	8	104	6	72	9	32
12	9	108	4	81	9	27
11	10	110	2	90	9	20
10	11	110	0	99	9	11
9	12	108	-2	108	9	0

Table 17.1

Inspecting the last column of Table 17.1, we see that monopoly profits are maximized at $\$36$ by setting a uniform price of $\$15,$ and supplying 6 demanders. The profit maximizing quantity choice is also characterized by a local condition. At quantity $6,$ Marginal Revenue exceeds Marginal Cost, meaning that Total Revenue would rise by more than Total Cost from increasing production by one unit from 5 to $6,$ but at quantity $6,$ Marginal Cost exceeds Marginal Revenue, meaning that Total Revenue would rise by less than Total Cost from increasing production by one unit from 6 to $7.$

More generally, let $Q(p)$ denote the fraction of a large population who would buy the good if its price was $p.$ Since $q(p)$ is a declining function throughout, it has an inverse, which we denote by $P(q),$ which shows what uniform price can support selling can invert it. Thus $pQ(p)$ and $qP(q)$ both total revenue divided by the number of potential buyers. Total cost is denoted by $C(q).$ Thus the monopolist's problem is to pick choose the q that maximizes

$$qP(q) - C(q)$$

If the mapping $P(q)$ and $C(q)$ were differentiable, then a necessary condition for opitmality is that

$$P(q) + q\frac{\partial P(q)}{\partial q} = \frac{\partial C(q)}{\partial q}$$

The term on the left side of the equation is marginal revenue. When the monopolist increases quantity supplied by $\Delta q,$ there are two effects on total revenue. First the Δq extra units are sold at $P(q)$, and second the price of all q units fall by $\frac{\partial P(q)}{\partial q}\Delta q.$ The sum of these effects is Marginal Revenue. Total cost increases by $\frac{\partial C(q)}{\partial q}\Delta q.$ If

$$P(q) + q\frac{\partial P(q)}{\partial q} > \frac{\partial C(q)}{\partial q}$$

then profits increase by setting $\Delta q > 0$ and increasing production. If

$$P(q) + q\frac{\partial P(q)}{\partial q} > \frac{\partial C(q)}{\partial q}$$

then profits can be increased by contracting production and setting $\Delta q < 0.$ Thus equality holds at the profit maximizing choice.

Now compare this with a perfectly discriminating monopolist.

$$\int P(q)dq - C(q)$$

and obtain the first order condition that the inverted demand curve is set to the marginal cost, thus exhausting all potential gains from trading

$$P(q) = \frac{\partial C(q)}{\partial q}$$

Experiment 16.5

Suppose there are 12 potential buyers. There is one valuation at $\$20,$ one at $\$18,$ three at $\$15,$ one at $\$10,$ one at $\$9,$ two at $\$8,$ one at $\$6$ and two at $\$5.$ Assume the item is produced in batches of 3, total costs rising by $\$4$ for every 3 units produced. Compare the resutls from a perfect discriminating monopolist versus charging a uniform price. Construct a table and draw a diagram similar to Table 17.1 to show how much the monopolist should produce and at what price. Are the inequalities with respect to marginal revenue and marginal cost preserved? Is there equality of marginal revenue to marginal cost at the profit maximizing quantity? compare the profist of a uniform pricing monopolist with one which can price discriminate.

4.2 Financial Partners

New firms have a low probability of success, most failing within two years. Most entrepreneurs starting new firms use up their own time and wealth to no avail (apart from the experience itself). Of the remainder, many new firms reward their founders with much toil for modest wages. If founders were rational, we could infer that a relatively small proportion of new firms prove extremely lucrative for their founders. That is, entrepreneurship entails a huge gamble with the founder's time, and sometimes his or her initial wealth, for the prospect of very large rewards. The entrepreneur often knows more about the expected value of his risky project than everyone else. Our work on bargaining and contracts explains why it is hard for entrepreneurs have difficulty funding their projects. There is no self financing, efficient bargaining mechanism. Thus the entrepreneur sells the project for less than its true expected value, or owns some of the project himself, accepting its inherent risks.

By definition newly created firms are the brainchild of one individual or a very small group of coworkers. When seeking to sell their idea, or attract outside funding in return for partial ownership, they must prove to potential buyers or investors that their project is valuable (hidden information), simultaneously protect their idea or invention from theft by rivals with a lower cost of capital or some other advantage in development (adverse selection), and prove they are motivated to ensure the project's success (moral hazard). Not surprisingly reliable data are difficult to find on how likely venture capitalists are to support a typical start up, but it is a safe bet that venture capitalists are besieged with countless business plans from entrepreneurs seeking funding and only a tiny proportion of new firms incorporated annually are financed by professionally managed venture capital pools.

Because raising outside funds is very costly, entrepreneurs might exchange shares in their projects for labor and capital inputs to known acquaintances, called insiders. Marriage, kinship and friendship are examples of relationships that lead to inside contacts. There are three limitations from turning this direction. Insiders are acquaintances, so few in number. Consequently they charge a premium for taking a major share in the project if they are risk averse. They might be tempted to steal the innovation, and betray the trust placed in them. Finally they are less qualified than outsiders specially picked because of their expertise to evaluate or contribute to the project, since their position as insiders are often unrelated to the project's challenges.

How would an entrepreneur write contracts with N insiders to partially insure himself about a project that outsider investors avoid? Setting $n = 0$ to indicate the entrepreneur and $n \in \{1, \ldots, N\}$ for the insider partners we suppose the utility function of the n^{th} player is given by:

$$-\exp(-\gamma_n a_n)$$

where a_n denotes his assets if he does not participate in the project, and γ_n is the coefficient of risk aversion. We assume the payoff from the project is a random variable which denote by x, drawn from a normally distributed random variable with mean μ and variance σ^2. The n^{th} insider partner pays f_n in return for a share of s_n from the proceeds from the project, and thus receives a final payoff of $(f_n - a_n - s_n x)$. The formula for a moment generating function of the normal distribution implies that the expected utility of the partner is:

$$-\exp\left[\gamma_n(f_n - a_n - s_n\mu) + \frac{(\sigma\gamma_n s_n)^2}{2}\right] = -\exp\left[\gamma_n\left(f_n - a_n - s_n\mu + \frac{\gamma_n(\sigma s_n)^2}{2}\right)\right]$$

By inspection, the certainty equivalent to the n^{th} partner from the random payoff $s_n x$ bought with a fee of f_n is:

$$s_n\mu - \frac{\gamma_n(\sigma s_n)^2}{2} - f_n$$

The greater the variance of the random variable, and the greater the coefficient of risk aversion, the more the partner must be paid to accept a given share in the enterprise for a given certainty equivalent. With regards the entrepreneur, his expected utility from the partnership with the insiders, who pay f_1 through f_N, for shares $s_1$through s_N, is:

$$-\exp\left[-\gamma_0\left(a_0+\mu+\sum_{n=1}^{N}(f_n-\mu s_n)\right)+\frac{1}{2}\left(\sigma\gamma_0-\sigma\gamma_0\sum_{n=1}^{N}s_n\right)^2\right]$$

His certainty equivalent for undertaking the project in partnership is therefore:

$$\mu+\sum_{n=1}^{N}(f_n-\mu s_n)+\frac{\gamma_0\sigma^2}{2}\left(1-\sum_{n=1}^{N}s_n\right)^2$$

One way of solving the entrepreneur's problem is to maximize his expected utility subject to the constraints imposed upon him by his financial partners. An easier method is to maximize the entrepreneurs's certainty equivalent, subject to the same set of constraints imposed upon him by his financial partners, but again expressed in terms of certainty equivalents. We adopt the second approach.

If the entrepreneur is fully informed about the opportunities and preferences of hias partners, then he would offer contracts that were individually tailored them. Determining the fee schedule for each insider is the entrepreneur's first step towards solving the partnership offer. To construct the fee schedule that exhibits an partner's willingness to buy shares in the venture, we compare the partner's expected utility from participating in the venture with what he would obtain otherwise. The n^{th} partner accepts a nonnegotiable partnership offer (f_n, s_n) if it has a positive certainty equivalent. Since the entrepreneur's certainty equivalent is strictly increasing in the fee, and each partner's certainty equivalent is strictly decreasing in the fee, the entrepreneur can extract all the rent by making an all or nothing offer, equating the fee with the certainty equivalent value of the shares offered to each partner:

$$f_n^o=s_n\mu-\frac{\gamma_n(\sigma s_n)^2}{2}$$

In the next step the entrepreneur determines how many shares to offer each partner. Substituting the optimal fee f_n^o for each partner $n\in\{1,\ldots,N\}$ into the entrepreneur's certainty equivalent value for the contract, he chooses $(s_1,s_2,\ldots,s_N)$ to maximize:

$$\mu-\sum_{n=1}^{N}\frac{\gamma_n(\sigma s_n)^2}{2}+\frac{\gamma_0\sigma^2}{2}\left(1-\sum_{n=1}^{N}s_n\right)^2$$

Since the objective function is quadratic in each choice variable s_n, there is a unique interior stationary point, which we denote now by $(s_1^o,s_2^o,\ldots,s_N^o)$. Differentiating with respect to s_n, the first order condition reduces to

$$s_n^o=\left(1-\sum_{n=1}^{N}s_n^o\right)\frac{\gamma_0}{\gamma_n}\equiv s_0^o\frac{\gamma_0}{\gamma_n}$$

Summing over n yields

$$\sum_{n=1}^{N}s_n^o=1-s_0^o=s_0^o\sum_{n=1}^{N}\frac{\gamma_0}{\gamma_n}$$

and solving for the optimal residual stock holdings of the entrepreneur, s_0^o,and hence for his partners', we obtain for all $n \in \{0,\ldots,N\}$ the solution shares:

$$s_n^o = \left[\sum_{m=0}^{N} \frac{\gamma_n}{\gamma_m}\right]^{-1}$$

Although not imposed, this interior solution satisfies the constraints that each player is allocated a strictly positive share in the firm, that is $0 \leq s_n^o \leq 1$ for all $n \in \{0,\ldots,N\}$, and is therefore the global optimum.The fees to insiders are increasing in μ and declining in σ, projects with higher means and lower variances commanding greater fees. However the solution shares do not depend on mean or variance of the project, but only on the attitudes of each member towards risk. Furthermore the ownership share of the entrepreneur is determined exactly the same way as his partners, the inverse of a weight reflecting his risk aversion compared to everyone else. This allocation of shares maximizes the sum of the certainty equivalents, a necessary condition for the entrepreneur to extract all the potential gains from the partnership.

Discriminatory pricing is more lucrative than uniform pricing, since it encompasses it as a special case, and offers many other options besides. However discriminatory pricing might not be feasible if, for example, insiders can trade their shares or contract dividends from their shares with each other. In this case any attempt to price discriminate would unravel. Rather than pay a higher price to join the partnership than some other insiders, an insider offered relatively unfavorable terms would simply approach a partner who was being offered a better deal, and agree on terms of trade that benefited both partners. The potential for retrading effectively constrains the entrepreneur to charge a uniform price for each share. For similar reasons the entrepreneur cannot, in this case, control how many units each shareholder should buy.

Accordingly we now suppose that the entrepreneur sets a price p for the whole venture, and each insider decides s_n, what share to purchase, costing ps_n. The solution to this problem can be solved in three steps. First we derive an insider's demand for shares in the partnership for any given price p. This yields a demand schedule for the n^{th} insider. Then we aggregate demand across insiders to find the total number of shares demanded at any given price. The last step is to choose the price which maximizes the certainty equivalent of the entrepreneur anticipating the demand response derived in the first two stages.

Deriving insider demand for shares in the venture is essentially a simple portfolio problem. Given p insider n picks to s_n to maximize the certainty equivalent

$$ps_n - a_n - s_n\mu + \frac{\gamma_n(\sigma s_n)^2}{2}$$

Differentiating with respect to the share s_n yields the first order condition:

$$(p-\mu) + \sigma^2\gamma_n s_n = 0$$

or the linear demand schedule:

$$s_n(p) = \frac{\mu - p}{\gamma_n \sigma^2}$$

Summing the individual demands we obtain the aggregate demand for shares by the insider:

$$s(p) = \left(\frac{\mu - p}{\sigma^2}\right)\left(\sum_{n=1}^{N} \frac{1}{\gamma_n}\right)$$

We are now ready to take the last step. The entrepreneur chooses p to maximize his certainty equivalent:

$$\mu + (p - \mu)s(p) - \frac{\gamma_0 \sigma^2 [1 - s(p)]^2}{2}$$

The first order condition is

$$s(p^o) + (p^o - \mu)s'(p^o) + \gamma_0 \sigma^2 [1 - s(p^o)]s'(p^o) = 0$$

The three expressions determining the optimal price have intuitive interpretations. As a first approximation, raising the price of a share increases revenue by the number of shares sold, namely $s(p^o)$. Also as the price rises, demand contracts at the rate of $s'(p^o)$ eliminating the expected losses from selling the marginal shares, which are $(\mu - p^o)$ per unit. (Recall from above that $s(p^o) > 0$ implies $\mu > p^o$.) Both expressions are positive. But retaining greater ownership in the project exposes the entrepreneur to more risk. Noting his share is $s_0 = 1 - s(p^o)$, he discounts his whole portfolio in the project by an extra $\gamma_0 \sigma^2 s_0 s'(p^o)$ for the marginal price increase due to greater exposure.

An expression for the optimal price can be derived from the first order condition. Differentiating aggregate demand equation for $s(p)$ with respect to price yields:

$$s'(p) = \frac{-1}{\sigma^2}\left(\sum_{n=1}^{N} \frac{1}{\gamma_n}\right) = \left(\frac{-1}{\mu - p}\right)s(p)$$

Substituting the expression for $s'(p)$ into the first order condition, some algebraic manipulations yield:

$$2(\mu - p^o) + s(p^o)\sigma^2 \gamma_0 = \sigma^2 \gamma_0$$

Now substituting the equation for $s(p)$ we solve for p^o to obtain:

$$p^o = \mu - \sigma^2 \gamma_0 \left(1 + \sum_{n=0}^{N} \frac{\gamma_0}{\gamma_n}\right)^{-1}$$

The optimal price varies one-for-one with the mean return from the project. The higher its variance, and the more risk averse is any player, the lower the price.

Under the perfect price discrimination, the entrepreneur achieves a higher expected utility and sells more shares than under uniform pricing arrangement. The first point is a direct consequence of the fact that it is not optimal to present the same terms to partners if they have different attitudes towards risk. As to the second claim, we now prove that the entrepreneur retains more shares when there is uniform pricing. Substituting the solution for p^o into the aggregate demand equation proves that

$$s(p^o) = \left[\sum_{n=0}^{N} \frac{\gamma_0}{\gamma_n} - 1\right]\left[1 + \sum_{n=0}^{N} \frac{\gamma_0}{\gamma_n}\right]^{-1}$$

Now note that $1 - s(p^o)$, shares held by the entrepreneur under the optimal uniform pricing scheme, exceeds s_0 because:

$$\frac{1 - s(p^o)}{s_0} = 2\left[\sum_{n=0}^{N} \frac{\gamma_0}{\gamma_n}\right]\left[1 + \sum_{n=0}^{N} \frac{\gamma_0}{\gamma_n}\right]^{-1} > 1$$

Experiment 16.7

Compare the two cases in practice for some different parameter values

5 Segmenting the Market

Breaking down perfect price discrimination. Now imagine that it cannot prevent players from buying in any market they like.

Now let monopolist condition on characteristics that are related to their valuations which he cannot observe.

5.1 Quantity discounts

One way to discriminate between customers is by the amount they wish to purchase.

Experiment 16.9

discounting
premium pricing

5.2 Packaging

Packages

Consider now another related method for segmenting market demand to extract greater economic rent. The firm exploits the idea that customers who demand several of the firm's products might exhibit more elastic demands (be more price sensitive) than customers who only wish to purchase a smaller subset of the firm's products.

Examples from the real world

Firms sell assembled goods such as cars or other durable goods for new car buyers and demand from previous buyers, plus replacement parts arising from collision damage or wear and tear.

Restaurants (furniture stores, car dealers) offer complete dinners (suites, high performance and luxury packages) with a limited (selected) range of items, and also offer portions a la carte (set pieces, individual components).

Ski resorts (amusement parks, cellular phone companies internet or cable operators) offer vacation packages for lodging and tickets (entry or connection plus service charges) as well as sell tickets (services) by themselves.

Two markets

To illustrate these ideas suppose Nokia has a monopoly on cellular phones and Worldcom has a monopoly on providing cellular phone service.

There are two groups of demanders. One group already has a phone and only wishes to buy the phone service. The other group must purchase a phone as well on order to send and receive calls by cellular phone.

Two games

We first consider the outcomes of a Nash equilibrium solution, where consumers are price takers (and can only submit market orders), and each firm maximizes its own profit, by setting price as a best reply to the other firm's, anticipating the induced consumer demand.

Then we consider a cooperative or collusive outcome, where as before consumers are price takers, but the two firms jointly set prices in both markets to maximize the sum of their profits.

We seek conditions under which the game is less profitable than the cooperative outcome, and measures of how much. These conditions explain when a merger is profitable.

Experiment 16.10

packaging

6 Durable Goods

To motivate our study of durable goods, consider the following extension of the monopooly pricing problem in Section 4. Suppose that each valuation of an infinite number of potential consumers is independently and identically uniformly distributed between α and β. As we explained in Section 4, a charging uniform price might arise when there is a resale market. In this case, a uniform price is some $p \in [\alpha, \beta]$ that induces the fraction of the population $(p - \alpha)/(\beta - \alpha)$ to buy the good Alternatively stated, to achieve market penetration of q the monopolist should set the price $P(q)$ at:

$$P(q) = \alpha - (\beta - \alpha)q$$

Suppose the unit cost of production is γ. The total profits accruing to the monopolist from producing q and selling it at a uniform price is therefore:

$$(P(q) - \gamma)q = (\alpha - \gamma)q - (\beta - \alpha)q^2$$

This is a quadratic function in quantity with first derivative:

$$(\alpha - \gamma) - 2(\beta - \alpha)q$$

Using the marginal cost equals the marginal revenue condition we obtain (q_m, p_m) the profit maximizing solution price and quantities as:

$$q_m = \frac{(\alpha - \gamma)}{2(\beta - \alpha)}$$

$$p_m = \frac{(\alpha + \gamma)}{2}$$

Figure 17.2 summarizes model and these derivations, displaying the demand parameters (α, β), the inverse demand curve $P(q)$, the marginal revenue curve $P'(q)$, the marginal cost curve γ and the solution (q_m, p_m).

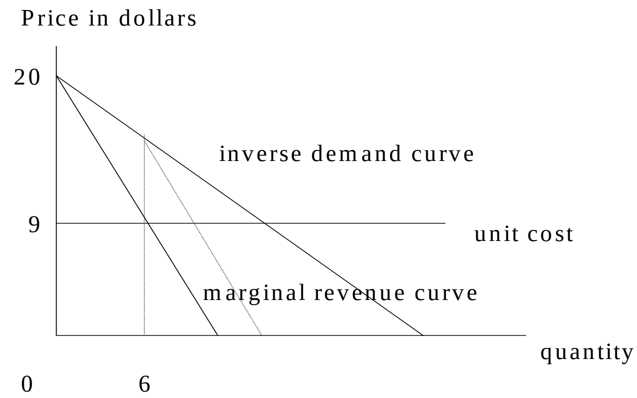


Figure 17.2
The Cost of Commitment

We now add a time dimension to the problem, by assuming that a unit of the good purchased ar date zero will be used continuously for a time period normalized to unit length. At that time a new product is introduced to supercede the current product line, rendering it useless, a simplifying assumption that does not affect the main points which follow.

Consider what would happen if the monopolist offered the good at the initlal date for p_m and sold it to the q_m consumers with the highest valuations, as indicated in the figure. The very next instant there would be a population of $(1 - q_m)$ potential consumers whose valuations are uniformly distributed between α and p_m. Following exactly the same logic used to derive (q_m, p_m) the static solution to the residual demand curve is:

$$q_m' = \frac{(\alpha - \gamma)(1 - q_m)}{2(\beta - \alpha)}$$

$$p_m' = \frac{(\alpha + \gamma)}{2}$$

If the monopolist sells to q_m for price p_m in the the first instant of the game, shouldn't it sell q_m' for p_m' in the second instant, and so on?

This rhetorical question points to several new factors to consider. First, if the monopolist knows that it can charge a high price to one set of consumers, and then sell additional units at a lower price to a group of consumers with lower valuations, then it would not act as if opitmally choosing a uniform pirce initially. Presumably it should simply chrage each consumer her valuation, or reservation price, starting with the highest valuation customers and working orking its way dowm the demand curve. Having solved this problem and sold the good to all consumers with valuations greater than, suppose the monopolist is presented with an opportunity to repeat this exercise with the remaining consumers. Second, if individual customers anticipate the price will fall almost instantaneously as successive groups find the good affordable, won't the smarter ones delay their purchase a few moments until the price reaches either marginal cost γ or the minimal valuation $\alpha,$ which ever is higher? Our remarks implicitly raise the issue of of whehter the producer can in principle charge different prices for the same good over time, rentals, commitment and time consistency, commitment to high pricing policies.

Another set of issues stems from the assumption that the length of time consumers use the product is determined in part by the monopolist, the wear and tear in use by conumers, as well as unanticipated events. Recognizing these factors at play the monopolist How monopolists produce and sell durable goods introduce further complications arising from the facts that the and that the durability of teh good is another strategy tool. Do monopolists build in obsolescence? This section is devoted to studying these two issue, first pricing and then durablity.

6.1 Pricing

The basic points can be made in a simple framework where demanders have one of two valuations, v_0 or $v_1,$ and $v_1 > v_0 > 0,$ and the consuming the good over the continuous interval of time $t \in [0,1]$ provides services of value v_i for $i \in \{1,2\}.$ For convenience we assume there are no costs from producing the good.

We first consider the case in which the monopolist can commit to a given price path that depends on time alone. If the monopolist could commit to setting the price at v_1 for the entire time interval then its profits would be $v_1,$ since only the first demander would buy the product at the beginning of the game when $t = 0$, whereas if the monopolist charged v_0 for the entire game, then both demanders would buy the product realizng profits of $2v_0$.

Is it profitable to charge some price $p_1 > v_0$ for some of the game and supply the high demander, and then reduce the price to v_0 at some point $t_0 \in (0,1)$ and supply the remaining low demander? Any two price policy must respect an incenitive compatibility constraint that ensures the high demander does not wait until t_0 before buying at a discount price v_0:

$$(v_1 - v_0)(1 - t_0) \leq v_1 - p_1$$

The expression on the left of the inequaltiy is the high demanders total surplus from buying at period t_0 for $v_0,$ and the fight expression is her total surplus from paying p_1 at the beginning of the game. The monopolist maximizes total profits from this scheme, choosing (p_1, t_0) to maximize

$$p_1 + (1 - t_0)v_0$$

subject to the incenitve compatibilty constraint. Since profits are strictly increasing in $(1 - t_0)$ the incentive compatibity constraint is met with strict equality, wchih implies

$$(1 - t_0) = (v_1 - p_1)/(v_1 - v_0)$$

Substituting this expression for $(1 - t_0)$ into the expression for profits above and simplifying, we deduce the maximal profits from a two part scheme are

$$p_1 + \frac{(v_1 - p_1)}{(v_1 - v_0)}v_0 = \frac{(v_1 - v_0)p_1 + (v_1 - p_1)v_0}{(v_1 - v_0)} = \frac{(v_1 - 2v_0)p_1 + v_1 v_0}{(v_1 - v_0)}$$

If $v_1 > 2v_0$ then setting $p_1 = v_1$ is optimal in which case the profits are v_1 while if $v_1 < 2v_0$ then setting $p_1 = v_0$ is optimal and the profits are $2v_0$. In neither case are profits greater than wht could be obtained by a single price policy.

by monopolist would entertain this scheme if and only if it generated profits that were at least as high as what could be obtained by staying with one price. Therefore the participation constraint in a two price scheme with commitment is

$$\max\{v_1, 2v_0\} \leq p_1 + (1 - t_0)v_0$$

The left expression $\max\{v_1, 2v_0\}$ is the maximal profits from setting a constant price, while the right side expression are the total profist from selling to both demanders The participation constraint implies

$$v_1 - p_1 \leq (1 - t_0)v_0$$

Combining the inequality above with the incentive compatibility constraint we obtain

$$(v_1 - v_0)(1 - t_0) \leq v_1 - p_1 \leq (1 - t_0)v_0$$

which implies $v_1 \leq 2v_0$ for all $t_0 \in (0, 1)$. Therefore the two price policy

We now consider the case in which the monopolist cannot commit to following a price path, and consider subgames that occur at teh point of sale t_0 to the high demanders at any price above v_0. At t_0 the monopolist lowers its price to v_0 and sells the good to all the remaininig demanders. Anticipating the equilbrium for this subgame, high demanders would not by at any price above v_0 unless the incenitve compatibiity constraint is solved. Therefore a two price policy does not occur along the equilibrium path of teh solution to the game where commitment is impossible. But this only leaves the one price policy of charging v_0 the entire game.

A second possibility is that the firm keeps track of its sales. In this case it knows whether the high value customer has purchased or not and reduces its price only once the good has been sold. Under tehse circumstances the profits of the firm are $(v_1 + v_0)(1 - t)$. The high valuation demander

Uniform price monopolist. In this game the monopolist charges a uniform price by committing to everyone the lowest price he offers to anyone. The airlines industry tries to solve this problem by guaranteeing one low price. Uniform price monopolist In this game the monopolist charges a uniform price by committing to everyone the lowest price he offers to anyone. In durable goods case, renting or leasing might break the market up into a weekly service market

Providing consumers limited opportunities to transact might also put them in a take it or leave it situation.

Experiment 16.11

pricing durable goods

6.2 Maintenance and obsolescense

Do monopolists build in obsolescence? Teh basic result is that value maximizing monopoplists minimize costs. Not if they control how well teh good is maintained. Our analysis leads us to the question as to whether a monopolist should rent the services from the product and maintaining it, rather than sell it.

Experiment 16.12

choosing durablility

7 Demand Uncertainty

To begin the analysis of quantity choices when the valuations are unknown, we consider an auctioneer with one unit for sale, who has some alternative use for the object, and can credibly commit to keeping the object if the bidders do not meet some reservation price. Notice that it is not credible to set a reservation price that is unknown above its alternative use to the auctioneer. In that case How should this reservation price be set?

7.1 Setting a reservation price

We note first that the revenue equivalence theorem applies in this case as well, providing the valuations of the bidders are identically and independently distributed with cumulative distribution function $F(v)$. For this reason we can derive the expected revenue from all the auctions satisfying the conditions of this theorem by narrowing our analysis to the second price sealed bid auction.

Suppose the n^{th} bidder has the highest valuation of all the bidders, and all the bidders play their weakly dominant strategy of submitting their valuations as bids. There are three broad outcomes to consider. Every bid might fall below the reservation price. Only one bid might lie above the reservation price. Two or more bids are submitted above the reservation price. We consider each outcome in turn.

With probability $F(r)^N$ every bidder has a valuation less than $r,$ and the item is unsold, so the auctioneer garners utility of $v_0.$ If $r > v_0,$ then there is a chance that the auction is inefficient. In particular the probability that the highest valuation of the bidders lies between v_0 and r is

$$P_1(r) = F(r)^N - F(v_0)^N$$

Therefore conditional on all bids falling below $r,$ the probability that at least one of the valuations lies above v_0 is

$$\frac{F(r)^N - F(v_0)^N}{F(r)^N} = 1 - \left(\frac{F(v_0)}{F(r)}\right)^N$$

As N increases, the expression $(F(v_0)/F(r))^N$ converges to zero, and hence the conditional probability converges to one for any fixed $r > v_0.$ In other words, when the item does not meet the reservation price, we can be sure that it has been mis-allocated to the auctioneer with arbitrarily high probability if the number of bidders is sufficiently large. For example if $F(v)$ is uniformly distributed on the $[0,1]$ interval, $v_0 = 1/2,$ and the item is unsold, then the probability that the item was not sold to its highest value when there are 10 bidders is $1 - (1/2)^{10} = 0.999.$ Of course if $v_0 = 0,$ then failing to sell the unit invariably yields an expost inefficient outcome. Thus the fact that there is a nontrivial probability that an inefficient outcome trade off between efficiency and opitmality from the seller's objective

With probability $(1 - F(r))$ the valuation of the first bidder exceeds $r,$ and with probability $F(r)^{N-1}$ all the remaining valuations are less than $r,$ and if both events occur the first bidder pays $r.$ Since the events are independently distributed the first bidder wins the auction and pays r with probability $(1 - F(r))F(r)^{N-1}.$ the same reasoning applies to all N bidders. Therefore the probability that one of the bidders wins the auction and pays r is:

$$P_2(r) = NF(r)^{N-1}[1 - F(r)]$$

Taking logarithms we see that

$$\log P_2(r) = \log N + (N-1)\log F(r) + \log[1 - F(r)]$$

For all Thus $\log P_2$ diverges to negative infinity as N increases, or P_2 converges to zero as the number of bidders increases, for any fixed $r.$ Intuitively, as the number of bidders increases, the chance that less than two exceed any given valuation less than the highest possible shrinks.

The only remaining outcome is that which would occur for certain if there was no reservation price. From the perspective of the n^{th} bidder, the probability that v_n exceeds those of the remaining $N - 1$ bidders is $F(v_n)^{N-1}.$ The probability that all the remaining bidders have valuations less than r is $F(r)^{N-1}.$ Consequently the probability that the n^{th} bidder wins the auction with a valuation has a on the Therefore the probability that the the winning bidder pays r is $F(r)^{N-1}.$

$$P_3(r) = N\left[1 - F(r)^{N-1}\right]\int_r^{\bar{v}} F(v_n)^{N-1} dv_n$$

Since $P_3(r)$ and $P_3(r)$ both converge to zero and that

$$P_3(r) = 1 - P_1(r) - P_2(r)$$

we may immediately conclude that

With probability $\left(1 - F(r)^{N-1}\right)$ the valuation of the second highest bidder exceeds $r.$ The probability density function for the second highest valuation and bid in that case is:

$$\frac{\left(F(v)^{N-1} - F(r)^{N-1}\right)}{\left(1 - F(r)^{N-1}\right)}$$

Differentiating with respect to v we obtain its probability density function, namely

$$\frac{(N-1)F(v)^{N-2}F'(v)}{\left(1 - F(r)^{N-1}\right)}$$

Conditional on having the winning bid of v_n, the expected payment is

$$rF(r)^{N-1} + \left(1 - F(r)^{N-1}\right) \int_r^{v_n} \left[\frac{v(N-1)F(v)^{N-2}F'(v)}{\left(1 - F(r)^{N-1}\right)} \right] dv$$

The expected revenue to the auctioneer is simply N times this quantity.

The auctioneer also receives v_0 with probability $F(r)^N$.

$$F(r)^N v_0 + NF(r)^{N-1}[1 - F(r)] + N\int_r^{\infty} v(N-1)F(v)^{N-2}F'(v)dv$$

Differentiating total expected revenue with respect to r we obtain:

$$\frac{\partial A(r)}{\partial r} = NG'(r)[1 - F(r)]r + NG(r)\{1 - F(r) - rF'(r)\}$$
$$- NG'(r)[1 - F(r)]r + NF(r)^N v_0$$

This expression can be simplified using the fact that $F(r)^N = G(r)$. Substituting for $F(r)^N$ and factoring out $G(r)$, we see that

$$\frac{\partial A(r)}{\partial r} = NG(r)[1 - F(r) + (v_0 - r)F'(r)]$$

There are two stationary points at $r = \underline{v}$ where $G(r) = 0$ and at r^o solving

$$(r^o - v_0)F'(r^o) = 1 - F(r^o)$$

In the case where

$$F(v) = (v - \underline{v})/(\bar{v} - \underline{v})$$

then

$$r^o = (\bar{v} - v_0)/2$$

(If supply is sufficiently restricted then the competitive amount is exactly what a monopolist would charge anyway.)

Experiment 16.8

Suppose v is distributed on support $[\beta, \infty]$ for some $\beta > 0$ with exponentially with cumulative distribution function:

$$F(v) = \alpha(1 - \exp(\beta - v))$$

Exercise **1.** *Conduct first price and oral auctions auction with different values of N.*

2. *Conduct first price auctions and varying the parameters (α, β)*

3. *Solve numerically the optimal reserve price as a mapping of the (α, β) values.*

4. *Plot the values of the reserve prices picked in the auctions.*

7.2 Learning about product demand

Quantity choices with a common valuation

When valuations are independent, learning more about the willingness to pay of those bidders who would buy if a designated quantity. In the case of common valuations, the monopolist wishes to produce more if the common value is high versus low. This sets up a sequential search strategic problem in which the monopolist conditions quantity produced and sold with the bidding.

We now turn to a production and marketing problem. A monopolist has just invented a new product but does not knows how much demand for the product there is. It must also build a plant and does not know what capacity to pick. During the first few periods the monopolist samples several markets to estimate demand for the new product more precisely. At the beginning of period t the monopolist decides whether to stop sampling demand and choose the capacity or not. If not, the monopolist uses a different technology to conduct trials in another market. Let m_t be the test market sampled in period $t.$ The monopolist stops sampling at time $\tau,$ and chooses the capacity c at that time. Higher capacities are more costly. During the sampling phase demanders recognize that at some point the monopolist will stop sampling, pick a capacity and price. Thus demanders who are sampled have two opportunities to buy the product, once when they are sampled and also after the new product is introduced.

New Product Pricing

A firm tries to invent a new product. Even if it fails, the product might be marketed for a time before it is withdrawn in favor of a better and more . Research is costly, so the firm faces a trade off between concentrating its efforts on marketing an inferior product versus developing an effective cure. Suppose the firm can devote its workforce to selling the product it has versus developing a new model and undertaking research. Undertaking research costs r per period. The value of a cure is c, but there is no value from taking a fake. A proportion of the population falls, say $p,$ falls ill each period and, because episodes are independently distributed across time and across the population, the probability of falling ill two periods in a row is $p^2.$ A natural question to ask is how the firm will spread its resources between marketing fakes versus undertaking research to discover a cure.

8 Summary

The second reason why is that even if preferences are known, people controlling .this is the problem of monopoly. If the monopolist can seaprately enter games with each of its clients, then we would expect the gains from trade to be exhausted. How the gains from trade are split would depend on the nature of the bargaining games played with each trading partner. But if the market cannot be fully segmented then this result breaks down. In the extreme case of a uniform price monoply too little trading takes place.

This chapter extended our study of auctions to multiunit auctions, and moved from there into an analysis of monopoly, by analyzing strategic considerations of determining the quantity to sell.

If the monopolist can vary its output, then both its cost structure and its marketing strategy helps to determine the scale of production. In each case the marginal cost of production is set to the marginal revenue from an additional sale, taking account of the fact that selling more units can only be achieved with price reductions. Different mechanisms for sales are associated with different schedules for marginal revenue, and thus lead to different solutions. We found that if the monopolist can play an ultimatum game with each of its potential customers separately, then the monopolist should produce where its marginal cost of production is equated with the lowest valuation customer that is serves. This is also the solution outcome if the monopolist is unable to deal with its clients independently and cannot commit to limiting production. In the first case, the monopolist extracts all the consumer surplus; in the second the buyers do. An intermediate case occurs if the monopolist sells each unit at the same price, and commit to production limits. In this case the marginal revenue curve lies below the inverse demand curve, so the price of all units is above marginal cost, in contrast to the other two cases.