1 Introduction

Throughout this book we have repeatedly emphasized the importance of assessing the choices of the other players in forming a best response. For example we distinguished between those situations where the strategies of the other players do not determine a player's optimal response from cases where they do. In the former case the player has a weakly dominant strategy, while in the latter he does not, so other criteria must be developed, such as precedence or backwards induction, and mutually best responses, when formulating a plausible notion of equilibrium. The concept of a competitive equilibrium has quite a different character to the noncooperative solutions that game theory yields, and accordingly provides a sharp contrast with the definitions of equilibrium we have extensively drawn upon to explain strategic play.

The defining characteristic of the competitive equilibrium price is that it equates demand with supply when traders treat this price vector parametrically. In competitive equilibrium traders act as if they could buy or sell as little or as much as they want without affecting the prevailing price, which is set so that neither unplanned inventories nor unfilled orders grow. The most widely used approximation for market structure is the competitive equilibrium model. Many industries are viewed as a collection of firms, each of which chooses its production plan, taking as given the prices of its inputs and outputs, and no consumer thinks she can affect the terms of trade.

Models of competitive equilibrium obviate the need for strategic behavior to predict the outcomes of games, in contrast to the noncooperative game theoretic solutions we have developed. The purpose of this chapter, then, is to study competitive equilibrium with a view to evaluating its predictions when they differ from those of noncooperative game theoretic definitions, and in games where noncooperative solutions are difficult to apply. The next section of this paper defines competitive equilibrium for a single market. To accomplish this, we explain the notion of price taking behavior, show how to derive supply and demand curves of individual traders, and explain the market clearing condition.

Then in Section 3 we extend the definition of competitive equilibrium to multi-market settings, a vector of prices that equate demand with supply in each market. We investigate a general equilibrium model of an exchange economy, and in a production economy, where some commodities are used in production, others in consumption, and still others in both production and consumption.

Section 4 analyze the optimality properties of competitive equilibrium. Competitive equilibrium are efficient, exhausting all the potential gains from trade, yielding the same efficient resource outcome as several market structures we explored in the previous chapter. In an application of product differentiation we exploit this fact in deriving the competitive equilibrium. Models of competitive equilibrium can be used to measure distortions created by regulation and taxation. We investigate production subsidies and import tariffs, which drive wedges between the demand and supply price.

When there is differential information amongst traders about product quality, the competitive equilibrium price not only equilibrates supply with demand. It also aggregates the information of the more informed traders. How much trading takes place in competitive equilibrium compared to the full information analogue depends on the precise nature of the informational asymmetries. Two extreme cases are that the competitive equilibrium is not affected at all, and that there is massive market failure with no trading. Section 5 analyzes these two extremes, as well as an intermediate case, where traders form rational expectations about the product’s value to them given the competitive price that demanders pay and suppliers receive.

In contrast to most other goods and services, which are often differentiated and tailored to individual customers and customer segments, financial assets provide services that are defined so that one unit of a given stock is a perfect substitute for every other. Furthermore the differences between financial assets are in terms of the risk characteristics, which are potentially observable over time. Finally, a large fraction of an economy’s wealth is traded in financial markets. Since studying financial markets is such a fruitful area for applying the tools we have developed to trading strategies in experimental markets, many of the examples we discuss below are concerned with stocks and currency exchange.

The final section of this chapter applies competitive equilibrium to financial markets. We begin this section by assuming that traders are value maximizers. In this case competitive equilibrium predicts that asset prices follow a random walk, similar to the prediction we derived for a sequential auction, and that as the liquidation date of an asset approaches the variance in its current price increases. A value maximizer cares only about the mean return of his portfolio, not its higher order moments, whereas a risk averse investor, and also a risk loving investor, has preferences over the portfolio variance as well. The last three applications in this section assume traders are risk averse expected utility maximizers. We investigate how risky assets are discounted in competitive equilibrium, relative to bonds and also relative to each other.

2 Competitive Equilibrium in a Single Market

In models of competitive equilibrium traders are assumed to treat prices parametrically, choosing quantities to buy and sell without regard to any effect their own trading behavior might have on price. This is not an unreasonable approximation for many situations. Individual shoppers would never dream that their individual purchases could affect the price they pay because of their contribution to aggregate demand. A new car buyer knows buying a more full efficient vehicle will not reduce the cost of a gallon of fuel, only that he will consume less gallons per mile. The supply curve maps the total quantity traders would sell as a function of price. Similarly the demand curve maps the amount traders would buy at each price, if each individual trader thought that his own demand would not have any price repercussions. The competitive equilibrium price is a point where the curves cross.

The main features of competitive equilibrium can be explained within a model where there is a single market with a fixed number of traders, some of whom are endowed with units of a perfectly divisible commodity, and all of whom have cash, which as usual stands for a numeraire good. We begin our analysis of partial equilibrium by showing how to derive the supply and demand curves of the traders. To quantify these concepts, suppose there N players. Player n is characterized by her initial endowment of money m_n, her endowment of the commodity s_n, as well as her private valuation of it v_n. Thus the n^{th} player is defined by the triplet (m_n, s_n, v_n). For expositional convenience, we order the players from the lowest to the highest valuation. Thus $v_1 \leq v_2 \leq \ldots \leq v_N$.

To make matters more concrete, we also carry along two numerical examples. In particular suppose $N = 10$, with private valuations that take on the integer values from \$1 to \$10. Thus $v_n \in \{1, \ldots, 10\}$. We assume that the third player (with valuation \$3) is initially endowed with 1 unit of the stock, the fourth player has 2 units initially, the fifth player has 1 unit, and the eighth player has 3 too. In terms of our notation $s_3 = s_5 = 1$, $s_4 = 2$, and $s_8 = 3$ while $s_n = 0$ for $n \in \{1, 2, 6, 7, 9, 10\}$. In addition, everybody has \$12 to buy parts or whole units of the commodity, meaning $m_n = 12$ for all $n \in \{1, \ldots, 10\}$. The second example is similar. The only difference between the two examples is that each player places value on having at most one unit of the object. Thus in the the second example the valuation $v_n \in \{1, \ldots, 10\}$ only applies to the first unit.

2.1 Individual optimization

In the model described above, given a fixed price p, the objective of player n is to pick the quantity of the commodity she trades, denoted q_n to maximize the value of her portfolio subject to constraints that prevent short sales (selling more than she owns) or bankruptcy (not having enough cash to cover her purchases). From the n^{th} player's own perspective the value of her portfolio is:

$$m_n - pq_n + v_n(s_n + q_n)$$

She chooses the quantity to trade q_n to Constraints in the optimization problem. In our example the third player's portfolio is

$$12 - pq_3 + 3 + 3q_n$$

The short sale constraint prevents her from selling more commodity units than she is endowed with:

$$s_n + q_n \geq 0$$

or in the eighth player's case

$$3 + q_8 \geq 0$$

The solvency constraint prevent her from purchasing more of the commodity than her cash endowment permits given the prevailing price:

$$m_n \geq pq_n$$

For each of the 12 players this implies

$$12 \geq pq_n$$

These constraints can be combined as:

$$\frac{m_n}{p} \geq q_n \geq -s_n$$

or, in the case of the eighth player:

$$\frac{12}{p} \geq q_8 \geq -3$$

The solution to this linear problem is to specialize the stock if v_n exceeds p, specialize in money if p exceeds v_n, and choose any feasible quantity q_n if v_n and p are equal. For example, the fourth player sells 2 units if the price is 5 to achieve a net utility of 22, and buys 6 units if the price is 2 achieving a net utility of 24. Denoting the solution to this problem by $D_n(p)$, the demand curve for the n^{th} player, then:

$$D_n(p) = \begin{cases} \frac{m_n}{p} \text{ if } v_n > p \\ -s_n \text{ if } v_n < p \end{cases}$$

and:

$$\frac{m_n}{p} \geq D_n(p) \geq -s_n \text{ if } v_n = p$$

We illustrate the individual demand and supply curves for the fourth player in Figure 20.1.

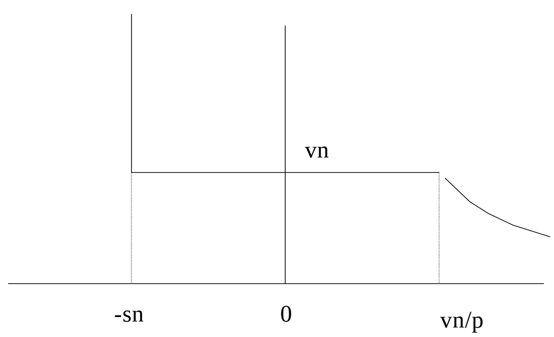


Figure 20.1
Excess demand for the fourth trader

Notice that vertical portion of this valuation is the maximum feasible units the player can sell (which occurs at $s_4 = 2$ in the figure), the horizontal portion of this excess demand curve passes through the midpoint of the horizontal axis at the height of the player's valuation (here $v_4 = 4$), the sloping portion begins at the point where all cash is spent on the commodity at a price equal to the player's valuation (in this case

$m_n/v_n = 3$) and has a slope of m_n/q (which is $12/q$ for all the players in this example).

2.2 Demand

The aggregate demand schedule for the commodity is defined as the total number of stocks that would be purchased as a function of its price. It is obtained by summing across the individual demands of players for each price. Following our usual notational practice, let $1\{v_n > p\}$ denote an indicator function that takes a value of one if $v_n > p$ and zero if $v_n \leq p$. Denoting the aggregate demand schedule by $D(p)$, the demand from those players who wish to increase their commodity holdings is:

$$D(p) = p^{-1} \sum_{n=1}^{N} 1\{v_n > p\} m_n$$

In our numerical example this formula becomes

$$D(p) = 12p^{-1} \sum_{n=1}^{10} 1\{n > p\}$$

Thus $D(p)$ declines in p for two reasons. As p falls the number of players with valuations exceeding p increases, jumps in demand of m_n/v_n occurring at each price v_n in the declining sequence $\{v_N, v_{N-1}, \ldots, v_1\}$. In between the jumps the slope of the demand schedule is its strictly negative derivative:

$$\frac{\partial D(p)}{\partial p} = -p^{-2} \sum_{n=1}^{N} 1\{v_n > p\} m_n$$

Figure 20.2 illustrates aggregate demand in the numerical example.

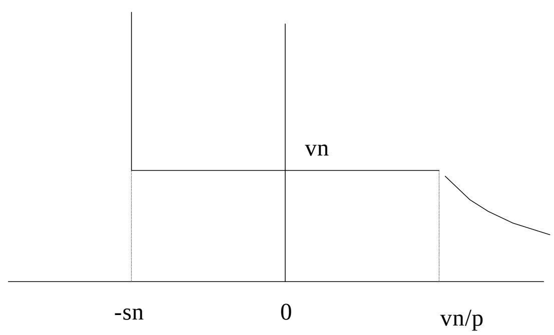


Figure 20.2
Aggregate demand

It intersects the midpoint of the horizontal axis at 10 because no one is willing to pay any more for the commodity than that. There are flats at each of the integers points on the vertical scale as new customers add their individual demands. The slope of the curve

2.3 Supply

Aggregate supply schedule is the total number of stocks that traders would like to sell as a function of its price. It is obtained in a analogous fashion to the aggregate demand schedule. Summing over the individual supply of each player we obtain the aggregate supply curve $S(p)$, the total supply of the asset from those players who want to sell their shares, as a function of price:

$$S(p) = \sum_{n=1}^{N} 1\{v_n < p\}s_n$$

The supply curve is an increasing step function with steps at $\{v_{1,} \ldots , v_N\}$ with the step at v_n having length s_n.

For the numerical example under consideration

$$S(p) = 1\{3 < p\} + 1\{4 < p\}2 + 1\{5 < p\} + 1\{8 < p\}3$$

Figure 20.3 provides a graphical depiction of the inverse supply curve.

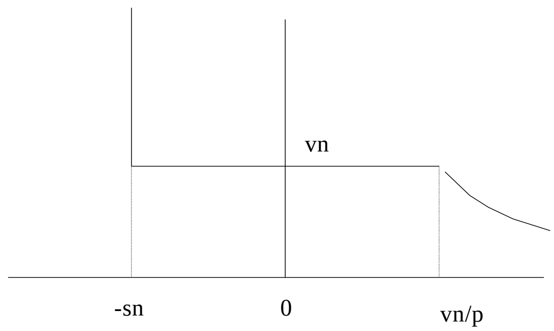


Figure 20.3
Aggregate supply

In this case the steps are unevenly spaced because not all endowments are the same, ranging from 1 to 3 units, and are of uneven height although the the population is evenly distributed across each of the integer points of the valuations, not everyone is endowed with commodity units.

2.4 Trade

Subtracting the supply curve from the demand curve we obtain $D(p) - S(p)$, the net demand for the asset. At the competitive equilibrium price p^e net demand is zero so $D(p^e) = S(p^e)$, and supply exactly offsets demand. There are no unfilled orders at the competitive equilibrium price from disappointed demanders, and no unplanned inventory accumulation either. In other words, the market clears.

From the definitions given above the competitive equilibrium price p^e solves:

$$\sum_{n=1}^{N} D_n(p^e) = D(p^e) \equiv S(p^e) \equiv \sum_{n=1}^{N} S_n(p^e)$$

For completeness we define the equilibrium quantity traded as $q^e \equiv D(p^e)$.

Figure 20.4 depicts the equilibrium price and quantity, which can be found by superimposing the demand schedule on the supply schedule and labelling their crossing point.

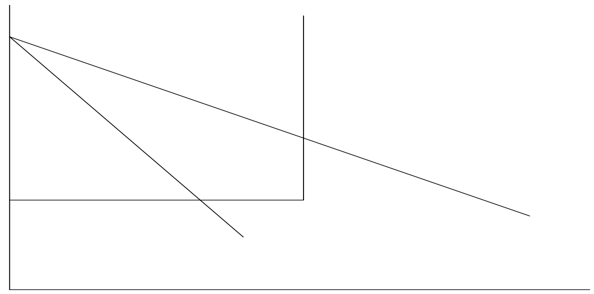
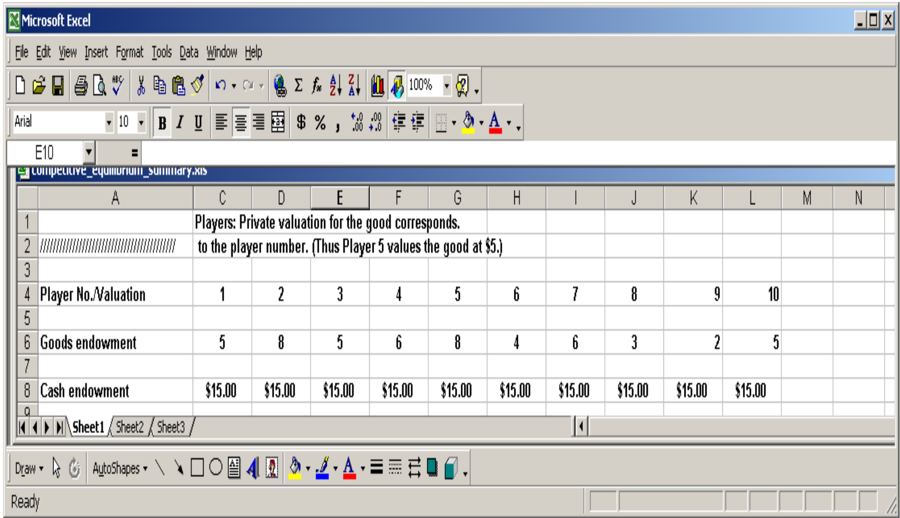


Figure 20.4
Competitive Equilibrium

In this framework the supply curve is slopes , and the demand curve slopes down (apart from the flat stretches). Consequently the competitive equilibrium for this model can be derived iteratively, by reducing the price whenever demand exceeds supply and increasing when supply exceeds demand . We illustrate this procedure with the numerical example. Substituting $p = 10$ into the supply and demand equations, we obtain $S(10) = 6$ and $D(10) \leq 1.2.$ Since $S(10) > D(10),$ there is excess supply at price $10,$ so $p^e < 10.$ Setting $p = 3,$ we see that $S(3) \leq 3$ and $D(3) > 28,$ which implies $D(3) > S(3),$ and hence $3 < p^e < 10.$ Continuing in this fashion, $D(7) > S(7),$ and $D(9) < S(9)$ implying $7 < p^e < 9.$ At $p = 8,$ the top two valuation consumers demand 1.5 units each, while suppliers are indifferent between selling any quantity between 3 and 6 units. (Note that at that price the player with valuation $8,$ is indifferent between selling her 3 units or buying up to 3 units.) Therefore $p^e = 8$ and $q^e = 3$ as Figure 20.4 shows.

Experiment 20.1

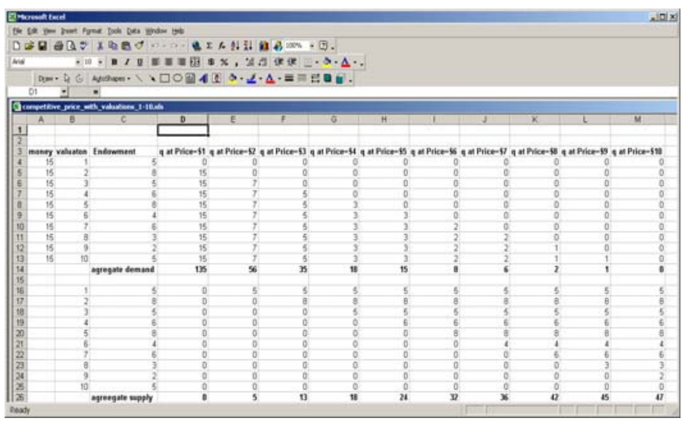
In the following experiment market game subjects participating in the experiment were assigned known valuations for a good, and also endowment of cash and the good as described in Table 20.5. The private valuation of each player corresponds to the player number. Each player was endowed with $15 cash and the good endowment between 2 units and eight units.



	A	C	D	E	F	G	H	I	J	K	L	M	N
1		Players: Private valuation for the good corresponds.											
2	//////////////////////////////////////	to the player number. (Thus Player 5 values the good at $5.)											
3													
4	Player No./Valuation	1	2	3	4	5	6	7	8	9	10		
5													
6	Goods endowment	5	8	5	6	8	4	6	3	2	5		
7													
8	Cash endowment	$15.00	$15.00	$15.00	$15.00	$15.00	$15.00	$15.00	$15.00	$15.00	$15.00		

Table 20.5: Player's valuations and endowment data

a. First we will construct a net demand curve. In order to do this all the quantities for a given price are summed up to get aggregate demand. Similarly the aggregate supply is calculated. The calculation is presented in Figure 20.6.



	A	B	C	D	E	F	G	H	I	J	K	L	M
3	money	valuaton	Endowment	q at Price=$1	q at Price=$2	q at Price=$3	q at Price=$4	q at Price=$5	q at Price=$6	q at Price=$7	q at Price=$8	q at Price=$9	q at Price=$10
4	15	1	5	0	0	0	0	0	0	0	0	0	0
5	15	2	8	15	0	0	0	0	0	0	0	0	0
6	15	3	5	15	7	0	0	0	0	0	0	0	0
7	15	4	6	15	7	5	0	0	0	0	0	0	0
8	15	5	8	15	7	5	3	0	0	0	0	0	0
9	15	6	4	15	7	5	3	3	0	0	0	0	0
10	15	7	6	15	7	5	3	3	2	0	0	0	0
11	15	8	3	15	7	5	3	3	2	2	0	0	0
12	15	9	2	15	7	5	3	3	2	2	1	0	0
13	15	10	5	15	7	5	3	3	2	2	1	1	0
14			agregate demand	135	56	35	18	15	8	6	2	1	0
15													
16		1	5	0	5	5	5	5	5	5	5	5	5
17		2	8	0	0	8	8	8	8	8	8	8	8
18		3	5	0	0	0	5	5	5	5	5	5	5
19		4	6	0	0	0	0	6	6	6	6	6	6
20		5	8	0	0	0	0	0	8	8	8	8	8
21		6	4	0	0	0	0	0	0	4	4	4	4
22		7	6	0	0	0	0	0	0	0	6	6	6
23		8	3	0	0	0	0	0	0	0	0	3	3
24		9	2	0	0	0	0	0	0	0	0	0	2
25		10	5	0	0	0	0	0	0	0	0	0	0
26			agreegate supply	0	5	13	18	24	32	36	42	45	47

Figure 20.6: Calculations of the aggregate demand and supply

The net demand is the difference between aggregate demand and aggregate supply for a given price. The graph of the aggregate demand and supply is presented in Figure 20.7.

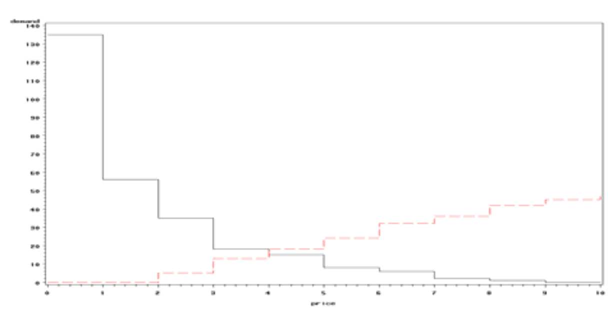
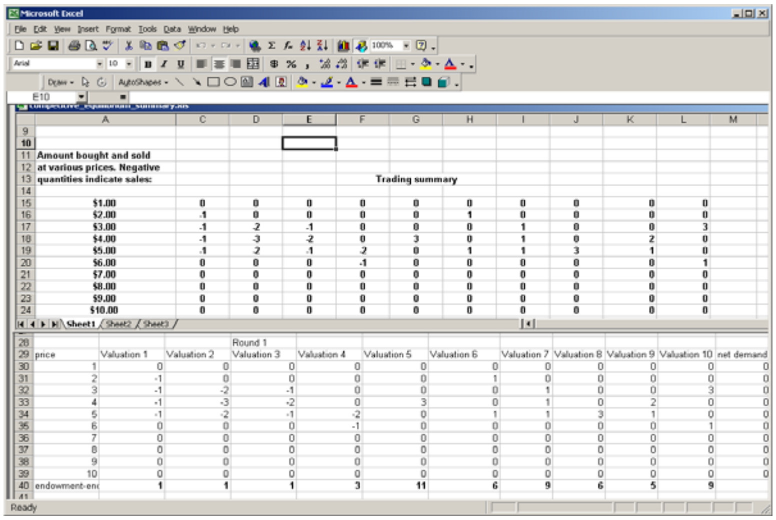


Figure 20.7:
Supply and Demand structure for the market experiment

b. From the Figure 20.7, the competitive equilibrium price is $4.

c. The trading history of the players are summarized in figure 20.8.



Amount bought and sold at various prices. Negative quantities indicate sales:

Trading summary

A	C	D	E	F	G	H	I	J	K	L
$1.00	0	0	0	0	0	0	0	0	0	0
$2.00	-1	0	0	0	0	1	0	0	0	0
$3.00	-1	-2	-1	0	0	0	1	0	0	3
$4.00	-1	-3	-2	0	3	0	1	0	2	0
$5.00	-1	-2	-1	-2	0	1	1	3	1	0
$6.00	0	0	0	-1	0	0	0	0	0	1
$7.00	0	0	0	0	0	0	0	0	0	0
$8.00	0	0	0	0	0	0	0	0	0	0
$9.00	0	0	0	0	0	0	0	0	0	0
$10.00	0	0	0	0	0	0	0	0	0	0

Round 1

price	Valuation 1	Valuation 2	Valuation 3	Valuation 4	Valuation 5	Valuation 6	Valuation 7	Valuation 8	Valuation 9	Valuation 10	net demand
1	0	0	0	0	0	0	0	0	0	0	0
2	-1	0	0	0	0	1	0	0	0	0	0
3	-1	-2	-1	0	0	0	1	0	0	3	0
4	-1	-3	-2	0	3	0	1	0	2	0	0
5	-1	-2	-1	-2	0	1	1	3	1	0	0
6	0	0	0	-1	0	0	0	0	0	1	0
7	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0
endowment-enc	1	1	1	3	11	6	9	6	5	9	

Figure 20.8: Trading history

d. First we will plot the relationship between the valuation and the price:

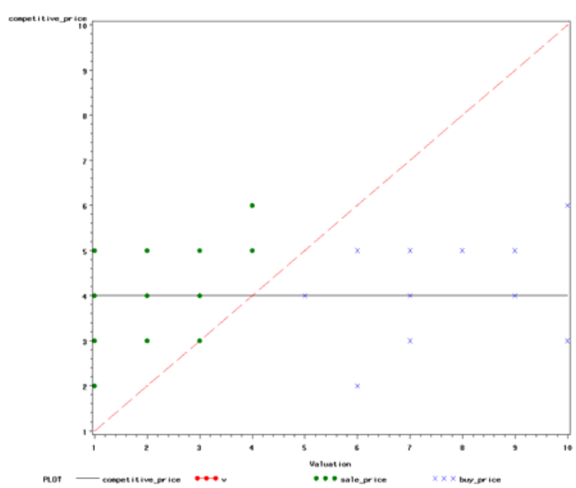


Figure 20.9: Value - Price diagram

Here we consider how closely the market matches the prediction of competitive equilibrium that the gains from trade will be exhausted.

3 General Equilibrium

Motivation for general equilibrium: where the demand curves for partial equilibrum come from

Competitive equilibrium is defined as the market clearing price when players optimize without recognizing their strategic positions. In competitive equilibrium the market clears when every trader ignores the effects of his own choices on the terms of trade, and maximizes his objective function falsely believing he can buy or sell unlimited units of commodity at that price, subject only to his own endowment, production and wealth constraints. This broad definition is routinely applied to price vectors in multiple markets for commodities and financial securities. This section extends our discussion to economies with multiple markets. First we analyze exchange economies, and then we introduce the production sector.

3.1 An exchange economy

In an exchange economy the players are endowed with commodities, goods and services, which they can buy or sell at unit prices that define the fixed exchange rate between the different commodities types, and in competitive equilibrium the total amount for sale equals the total amount ordered by purchasers.

Suppose there are K different types of commodities and N players. Let $\bar{x}_n \equiv (\bar{x}_{1n}, \ldots, \bar{x}_{Kn})$ denote the endowment of player $n \in \{1, \ldots, N\}$, and let $x_n \equiv (x_{1n}, \ldots, x_{Kn})$ denote the consumption allocation of the n^{th} player. In a competitive equilibrium all units of a given commodity are bought and sold at the same price. Let p_k denote the price of the k^{th} commodity, and define the price vector for all commodities as $p \equiv (p_1, \ldots, p_K)$. A player is permitted to consume any commodity bundle she likes subject only to her budget constraint, that the value of the bundle she consumes does not exceed the value of her endowment. The budget constraint of player n is thus

$$\sum_{n=1}^{N} p_k (x_{kn} - \bar{x}_{nk}) \leq 0$$

A player's preferences over consumption bundles are captured by a utility function, an index function that ranks commodity bundles by ascribing a real number to each bundle. Let $u_n(x_n)$ denote the utility function of the n^{th} agent. She maximizes $u_n(x_n)$ by choosing a vector $x_n$subject to the budget constraint defined above.

This leads us directly to the definition of a competitive equilibrium. It is any price vector $p^e \equiv (p_1^e, \ldots, p_K^e)$ that clears the market, so that all sellers and buyers can transact their supplies and demands at that price. In terms of the notation we have developed, p^e is a competitive equilibrium if it satisfies the K equalities

$$\sum_{n=1}^{N} x_{kn}^e = \sum_{n=1}^{N} \bar{x}_{nk}$$

where for each trader $n \in \{1, \ldots, N\}$, the consumption vector $x_n^e = (x_{n1}^e, \ldots, x_{nK}^e)$ solves

$$u_n(x_n^e) = \max_{x_n} \left\{ u_n(x_n) : \sum_{n=1}^{N} p_k^e (x_{nk} - \bar{x}_{nk}) \leq 0 \right\}$$

3.2 Existence of competitive equilibrium

This definition of competitive equilibrium can be readily extended to models where firms held by shareholders maximize their value, and consumers derive their wealth from shares they own plus their labor market activities. As above we let $x \equiv (x_1, \ldots, x_K)$ denote the K different commodities used as factors in production, consumption goods and services, or both. Iron ore is an example of the first, ice cream an example of the second, while a person's time can be used in production as a labor input or in consumption as leisure. For each firm $i \in \{1, \ldots, I\}$ we define a production function as the frontier of their production set Z_i, with K dimensional elements $z_i \equiv (x_1, \ldots, x_K)$, where as the image of the mapping $f(x_i)$

The budget set of consumer $j \in \{1, \ldots, J\}$ is

$$\sum_{k=1}^{K} p_k x_{jk} \leq \sum_{k=1}^{K} p_k \bar{x}_{jk} + \sum_{i=1}^{I} s_{ij} v_i$$

Given her endowment of natural resources $\bar{x}_j = (\bar{x}_{j1}, \ldots, \bar{x}_{jK})$ and her shares $s_j = (s_{j1}, \ldots, s_{jK})$, she chooses her consumption vector $x_j \equiv (x_{j1}, \ldots, x_{jK})$ to maximize $u_n(x_j)$ subject to her budget constraint for prices $p = (p_1, \ldots, p_K)$.

In a competitive equilibrium both shareholders and consumers take prices as given. Under this assumption shareholders unanimously support the goal of maximizing firm value, since increasing the value of their shares enlarges their consumption sets. Thus firm i chooses x_i to maximize

$$\sum_{k=1}^{K} p_k z_{ik}$$

subject to the constraint that $z_{ik} \in Z$.

The K market clearing conditions that determine the competitive equilibrium price vector $p^e \equiv (p_1^e, \ldots, p_K^e)$ is then

$$\sum_{i=1}^{I} z_{ik} + \sum_{j=1}^{J} x_{jk} = \sum_{j=1}^{J} \bar{x}_{jk}$$

We remark that there might be a vector of prices that solve these K equations, begin this subsection with a caveat, that a competitive equilibrium does not always exist. For example competitive equilibrium analysis cannot be applied when there are sunk costs and decreasing variable costs of production. (A sunk cost is a costly commitment made upon entry into the industry, while decreasing variable costs mean that marginal unit costs do not increase with the scale of production.) To prove this assertion, consider an industry in which firms must pay an investment cost of c_0 to enter the industry, and additional costs of c_1 to produce any units of the good. Labelling by x_i the production volume for sale by the i^{th} firm we see that its value from producing x_n is

$$\pi(x_i) = (p - c_1)x_i - c_0$$

From this expression it is easy to see that, since production must be positive, the only way for the fir to avoid losses is to stay out of the industry, or enter if the price exceeds variable costs. However unit variable profits are positive, then any level of profit can be attained by setting production levels sufficiently high. That is since $c_0 > 0$ and $x_i \geq 0,$ it follows that $\pi(x_i) \geq 0$ if and only if $p > c_1$ or $x_i = 0.$ But if $\pi(x_i) \geq 0$ and $x_i > 0,$ then $p > c_1$ and any value π can be attained by setting $x_n = (\pi + c_0)/(p - c_1).$ Since consumers of the product are only willing to buy a finite amount of it for a positive price, a competitive equilibrium does not exist in this framework.

More generally, a competitive equilibrium fails to exist in models of production and trade where there are declining marginal costs of production, and in multiproduct markets where combinations of product lines can be produced at ever falling average costs. These are called natural monopoly industries. To predict the solution outcomes of natural monopoly industries, we must revert to the tools of noncooperative game theory.

3.3 A production economy

Consider a production economy with I consumer workers, J manufacturing firms and an unspecified number of suppliers of $K - 1$ raw materials, in which there are only two factors affecting the i^{th} consumer's well being, his leisure and his allocation of goods and services, or material wealth. He is endowed with 365 (annual days) of leisure but can sell his labor by working for one of the manufacturers. Aside from labor, the $(K - 1)$ other inputs in the production of goods and services are raw materials of various types available in fixed aggregate supply.

Let y_j denote the production of the j^{th} firm, and suppose it purchases the input vector $(x_{1j}, x_{2j}, \ldots, x_{Kj}),$ where the first input is labor, and the other inputs are raw materials. We assume the production function of the firm is:

$$y_j = \Pi_{k=1}^{K}(x_{kj})^{\alpha_k}$$

where:

$$\sum_{k=1}^{K} \alpha_k \equiv \alpha < 1$$

This condition is necessary and sufficient for the technology to exhibit decreasing returns to scale. We specify the utility of the i^{th} consumer worker as

$$w_i + \log(365 - l_i)$$

where w_i represents wealth derived from labor income and firm dividends spent on goods and services.

Aggregate resource constraints in the factor markets require the total labor supply to exactly offset the total demand for labor by manufacturers, and for the endowment of raw materials to be greater than the total quantity used in manufacturing. Symbolically:

$$\sum_{i=1}^{I} l_i = \sum_{j=1}^{J} x_{1j}$$

and:

$$\sum_{j=1}^{J} x_{kj} \leq \bar{x}_k$$

for $k \in \{2,\ldots,K\}$ where $\bar{x}_k$ is the total stock of the k^{th} exhaustible resource. In the product market aggregate consumption expenditure equals aggregate production, that is:

$$\sum_{i=1}^{I} w_i = \sum_{j=1}^{J} y_j$$

A competitive equilibrium for this economy is a price vector $(p_1,p_2,\ldots,p_K)$, specified relative to the numeraire price of goods and services, which satisfies the aggregate resource constriants defined above when consumer workers maximize their utility by choosing their labor supply given the wage rate p_1, the owners of the manufacturing firms choose factor inputs to maximize the value of their firms given the input prices $(p_1,p_2,\ldots,p_K)$, and those owning stocks of the k^{th} exhaustible resource sell them to manufacturing firms if p_k is strictly positive.

In competitive equilibrium the j^{th} manufacturer chooses $(x_{1j},x_{2j},\ldots,x_{Kj})$ to maximize:

$$\Pi_{k=1}^{K}(x_{kj})^{\alpha_k} - \sum_{k=1}^{K} p_k x_{kj}$$

The first order condition for this problem is:

$$\alpha_k y_j = p_k x_{kj}$$

Making y_j the subject of the equation Comparing the first order condtions for the k^{th} and l^{th} inputs, we see that:

$$\frac{p_k x_{kj}}{\alpha_k} = \frac{p_l x_{lj}}{\alpha_l}$$

Combining this condition with the definition of production we can solve the optimal input level as a function of prices. Note that

$$y_j = \Pi_{k=1}^{K}(x_{kj})^{\alpha_k} = x_{lj}^{\alpha}\Pi_{k=1,k\neq l}^{K}\left(\frac{\alpha_k p_l}{p_k \alpha_l}\right)^{\alpha_k}$$

Substituting this equation for y_j into the first order condtion for the input x_{lj} yields the factor demand for that input:

$$\begin{aligned} x_{lj} &= \frac{\alpha_l}{p_l} y_j \\ &= \frac{\alpha_l}{p_l} x_{lj}^{\alpha}\Pi_{k=1,k\neq l}^{K}\left(\frac{\alpha_k p_l}{p_k \alpha_l}\right)^{\alpha_k} \\ &= \left[\frac{\alpha_l}{p_l}\Pi_{k=1,k\neq l}^{K}\left(\frac{\alpha_k p_l}{p_k \alpha_l}\right)^{\alpha_k}\right]^{1/(1-\alpha)} \end{aligned}$$

Because the production technology is convex and firms are identical, they make the same input selections. Furthermore the marginal product of each resource is strictly prositive, so all the the non labor resources command a strictly positive price in equilibrium and hence are exhausted. Therefore:

$$x_{kj} = \frac{\bar{x}_k}{J}$$

for each $k \in \{2,\ldots,K\}$ and $j \in \{1,\ldots,J\}$ and thus:

$$y_j = \Pi_{k=1}^{K}\left(\frac{\bar{x}_k}{J}\right)^{\alpha_k} = J^{-\alpha}\Pi_{k=1}^{K}\bar{x}_k^{\alpha_k}$$

where total labor supply is deonted by $\bar{x}_1$. Substituting these two equations back into the first order condion we now obtain the competitive equilibrium price of the l^{th} exhaustible resource as a function of the total labor supply:

$$\alpha_k y_j = p_k x_{kj}$$

$$p_l = \frac{\alpha_l y_j}{x_{lj}} = \alpha_l J^{(1-\alpha)}[\Pi_{k=1,k\neq l}^{K}\bar{x}_k^{\alpha_k}]\bar{x}_l^{\alpha_l-1}$$

Turning to the labor supply decision, the i^{th} worker maximizes

$$p_1 l_i + d_i + \log(365 - l_i)$$

with respect to l_i given the dividends he receives from his shares in the ownship of the raw materail extraction manufacturing companies. For values of $p_1 > 1/365$, the first order condition for this problem yields the optimal choice, an interior solution with the closed form

$$l_i = 365 - \frac{1}{p_1}$$

Otherwise the worker does not participate in labor force.

Since all consumer workers have the same preferences and leisure is additively separable from consumption, differnces in dividend payments do not affect labor supply choices in this framework. Therefore everyone makes the same labor supply choice in this model, and the aggregate labor supply is

$$\bar{x}_1 = \sum_{i=1}^{I} l_i = I\left(365 - \frac{1}{p_1}\right)$$

Upon equating this equation with the demand for labor we obtian

$$I\left(365 - \frac{1}{p_1}\right) = \left[\frac{\alpha_1}{p_1}\Pi_{k=2}^{K}\left(\frac{\alpha_k p_l}{p_k \alpha_l}\right)^{\alpha_k}\right]^{1/\left(1-\sum_{k=1}^{K}\alpha_k\right)}$$

Similarly substituting for $\bar{x}_1$ in the price equations for the raw materials yileds $K-1$ equations that depend on the raw material stocks $(\bar{x}_2,\ldots\bar{x}_K)$ and p_1. This gives us K equations in K unknowns that an be solved numerically.

3.4 Product differentiation

The applications we have considered presuppose all the product or service units bought and sold within a market are more or less identical. However the concept of competitive equilibrium can also be applied to markets where each supplier sells a uniquely differentiated product, and every demander value the products differently. For example consider a market setting where there are I demanders and J suppliers. The product or service supplied by producer $j \in \{1,\ldots,J\}$ is indexed by a characteristic p_j, which we assume is a positive real number. Similarly demanders $i \in \{1,\ldots,I\}$ are distinguished by another characteristic denoted $a_i,$ also a positive real number. For convenience producers are consumers are ranked by their respective characteristics from the lowest to the highest. Thus $v_i \leq v_{i+1}$ for all $i \in \{1,\ldots,I-1\}$ and $w_j \leq w_{j+1}$ for all $j \in \{1,\ldots,J-1\}.$ The is a function of their characteristics, which we denote by $f(v_i,w_j)$. Thus if the i^{th} demander buys a service unit from the j^{th} supplier at price p_{ij}, then the demander attains a utility of $f(v_i,w_j)-p_{ij}$ and the supplier receives p_{ij}.

The residential housing market is really a product spectrum in which no two houses are alike, and practically all demanders have special or idiosyncratic desires. Teams within a professional league put different values on any given player joining the league, depending on their current stock of players and their fan base, while each player brings a unique combination of skills to the league and the team he joins. Tourist attractions that compete for the holilday maker's time, and specialized food and beverage products, often fall into this category. The following application analyzes the market for wine, an industry where production units are imperfect substitutes for each other in consumption. In deriving the competitive equilibrium price, we exploit the fact that it supports an efficient allocation of resources.

Suppose the amount a person is willing to pay for a bottle of wine depends his tastes $v,$ and the quality of the wine $w.$ By quality we mean a one dimension characterstic that is desirable. Denote the reservation price of a consumer by the function $u(v,w).$ We assume $u(v,w)$ is increasing in both arguments; wine connoisseurs have higher values of v than beer drinkers and higher quality wines are associated with higher values of $w.$ We also assume that all $v_2 > v_1$ and $w_2 > w_1$ this assumption

$$u(v_2,w_2)+u(v_1,w_1) > u(v_2,w_1)+u(v_1,w_2)$$

This assumption means that those enjoy drinking wine are willing to pay higher prices for higher quality wine, than those who do not like drinking it as much. It is satisfied if and only if the cross partial derivative of $u(v,w)$ is positive:

$$\begin{aligned} 0 &< u(v_2,w_2)-u(v_2,w_1)+u(v_1,w_1)-u(v_1,w_2) \\ &= \int_{w_1}^{w_2} u_2(v_2,w)dw - \int_{w_1}^{w_2} u_2(v_1,w)dw \\ &= \int_{v_1}^{v_2}\int_{w_1}^{w_2} u_{12}(v_2,w)dwdv \end{aligned}$$

Under this assumption Pareto optimal allocations exhibit positive assortive matching. Those who like wine the most drink the highest quality wine. Let $F(v)$ denote a probability distribution function that characterizes wines tastes across the population, which we assume is strictly increasing (meaning that everyone has different tastes), and let $G(w)$ denote the probability distribution function characterizing wine quality supplied by wineries. Positive assortive matching means that when people are ordered by the quality of wine they drink, the same ranking emerges as when they are ordered by their taste for wine. Denoting by v_n the taste of the n^{th} consumer and by w_n the quality of the wine he would consumes. Then positive assortive mating is formally defined by the equation $F(v_n) = G(w_n)$. Since $F(v_n)$ is strictly monotone increasing it has an inverse $F^{-1}(x)$ for all $x \in (0,1)$. Therefore wine of quality w' is bought by a consumer with taste $v' = F^{-1}[G(w')]$ in competitive equilibrium if the supplier can make a profit. Otherwise the consumer We can also use this fact to derive the competitive equlibrium price of wine, and how much wine, or equivalently how much low quality wine, is produced.

Suppose the cost of growing vines and making wine does not depend on its quality, and is constant at c. Competitive presssure implies that the lowest quality wine offered to the market will be priced at cost c, and sold to the consumer who has the lowest valuation of all wine drinkers. Let $\underline{v}$ denote the threshold and the highest valuation amongst the the lowest by the cheapest wine will and that the quality of the wine. Similarly let $\underline{w}$ denote the lowest quality wine sold. Then $u(\underline{v}, \underline{w}) = c$. Since $F(\underline{v}) = G(\underline{w})$ we can solve for $\underline{w}$ from the equation

$$u(F^{-1}[G(\underline{w})], \underline{w}) = c$$

Now consider the n^{th} consumer with valuation $v_n > \underline{v}$ reviewing the wine list $p(w)$, which indicates bottle price as a function of quality. He chooses w_n to maximize

$$u(v_n, w) - p(w)$$

The first order condition for this optimization problem is

$$p'(w_n) = u_2(v_n, w_n)$$

Substituting for $v_n = F^{-1}[G(w_n)]$ we obtain an equation for the price quality wine gradient

$$p'(w) = u_2(F^{-1}[G(w_n)], w)$$

Integrating up from $\underline{w}$ we obtain

$$p(w) = c + \int_{\underline{w}}^{w} p'(x)dx = c + \int_{\underline{w}}^{w} u_2(F^{-1}[G(x)], x)dx$$

The effects of new wine growing region can be easily ananlyzed in this framework. We need to scale the size of the demand and supply after we compute the new distirubtion of wineries

$$G(x) = \alpha G_1(x) + (1 - \alpha)G_2(x)$$

4 Welfare

The prisoner's dilemma illustrates why games reach outcomes in which all players are worse off than they would be in one of the other outcomes.

Notice that a competitive equilibrium , then it uses up all the surplus. It is impossible to make one or more players better off without making someone else worse off. This is obvious if it is a single price. This important result explains why many economists recommend markets as a way of allocating resources.

To prove that a competitive equilibrium is Pareto optimal.

The outcomes generated by competitive equilibrium exhaust all the gains from trade. Then in a competitive equilibrium it is impossible to make somebody better off without making one of the other players strictly worse off. This can be formalized as follows.

4.1 Pareto optimality

Competitve equilibrium is Pareto optimal. Pareto optimality

Let the symbol $\succsim$ denote "at least as good as" and the symbol $\succ$ denote. Let $\{x_1^e, \ldots, x_N^e\}$ denote the consumption outcome generated by the competitive equilibrium price p^e.

Competitive Equilibrium Allocations are Optimal

Let x_n^e denote the bundle of goods and services consumed by the n^{th} trader when prices p^e are in competitive equilibrium. If the allocation $\{x_1^e, \ldots, x_N^e\}$ was not a Pareto optimal allocation, then there exists some other bundle called $\{x_1', \ldots, x_N'\}$ with the property that x_n' is preferred to x_n^e by every trader $n,$ and at least one trader, say the m^{th} strictly prefers it. It therefore must be the case that

$$p^e \cdot x_m' > p^e \cdot x_m^e$$

otherwise trader m would consume x_m' instead of x_m^e when prices are $p^e.$ If $\{x_1', \ldots, x_N'\}$ was feasible (market clearance) requires

$$\sum_{n=1}^{N} x_n^e = \sum_{n=1}^{N} x_n'$$

Hence

$$\sum_{n=1}^{N} p^e \cdot x_n^e = \sum_{n=1}^{N} p^e \cdot x_n'$$

From above it follows that:

$$\sum_{\substack{n=1 \\ m\neq n}}^{N} p^e \cdot x_n < \sum_{\substack{n=1 \\ m\neq n}}^{N} p^e \cdot x_n'$$

It now follows that for at least one other trader, say the l^{th}

$$p^e \cdot x_l' < p^e \cdot x_l$$

By continuity of preference we can increase x_l' a little bit and still fall within the budget constraint. Therefore x_n' could not be utility maximizing for each trader, and so neither could be x_n^e.

4.2 Tax incidence

Distortions to competitive equilibrium allocations

We concluded in the previous section that if there are a goodly number of both buyers and sellers trading in a market, then the competitive equilibrium outcome approximates well the extent of trading and the prices observed. We draw upon this conclusion in this section to use competitive equilibrium concepts to predict how the market reacts to taxes and quotas of various types.

Sales and Production Taxes

A sales tax is paid by the buyer at the point of purchase; a production tax is paid by the firm when the item is manufactured. In competitive equilibrium the incidence of equal taxes and the reallocation of resources induced by the taxes are exactly the same. Assuming that all the production units are sold is then can put this argument to reallocation and the prices.

The diagram also shows why taxes and subsidies are called distortions to competitive equilibrium. In competitive equilibrium depicted here it is impossible to reallocate resources in such a way so that at least one player is made better off without making anyone else worse off, a property called Pareto optimality. This is the sense in which a competitive equilibrium exhausts all the potential gains from trade. To prove the claim, note that every trader who buys units values the commodity at least as highly as p^e but that everyone who sells units values it at most p^e. Therefore taking units between buyers and sellers cannot compensate the buyers. Furthermore every buyer who has units specializes in the commodity; since there is We shall shows later in the chapter that the property of Pareto optimaltiy applies more generally in competitive equilibrium.

Experiment 20.5

This exercise is designed to investigate the incidence and the distortions effects of production and sales taxation within an experimental setting.

Exercise **1.** *Run a production tax*

2. *Investigate a sales tax*

3. *Compare the outcomes by player type under these two regimes. Test for significant differences between them.*

Experiment 20.6

Tariffs on imported goods create a differential between imports sold and domestically produced goods. A subsidy to locally produced goods (perhaps in the form of tax breaks for establishing the firm) coupled with an equivalent sales tax creates a similar distortion. In considering the car industry we imagine a small country which cannot affect the world price of automobiles, but can affect the. the world trade authority severely limits the ability of countries to imposes tariffs at a national level, but the situation within countries is different especially when countries do not have jurisdiction over state and regional tax law.

Exercise *This exercise is designed to investigate the incidence and the distortions effects of tariffs and production and sales taxation within an experimental setting.*

1. *Investigate a tariff*

2. *Investigate the combination of a sales tax and a domestic production subsidy*

3. *Compare the outcomes by player type under these two regimes. Test for significant differences between them.*

4.3 Money

The government has a monopoly on the supply of fiat money.

8 Summary

This final chapter analyzed competitive equilibrium, which we defined as a price that clears the market when all traders maximize their objective functions without accounting for the effect of their own behavior on the demand and supply of the product. By stripping the equilibrium concept of its connections to the market microstructure, we showed that the role for strategic play is greatly diminished. In a competitive equilibrium the only connection firms have to other firms is through price, and that price is independent of the firms own choices. There is therefore no role for strategic behavior, and it important for strategists to be able to identify these situations.

There are three main advantages from using the competitive equilibrium as a tool for predicting questions about production and trade. As we showed in the previous chapter, the more players in the market, the harder it is to maintain collusive arrangements that enforce a monopoly outcome. Thus the smaller the share of a firm in industry production, the smaller the effect of changes in its own production on aggregate industry output, so the effect on price, determined by total demand, is small. Therefore little is lost by a firm that ignores the small downward effect on prices induced by increasing its own quantity. Hence in many industries the assumption that individual traders cannot affect the prices through their own actions has a solid empirical foundation. In bargaining and auction games with only small numbers of players, the predictions from competitive equilibrium do not correspond well to the noncooperative solutions developed in previous chapters, but as the number of bidders in an auction grows, we also observed that the competitive equilibrium set converged to the singleton predicted by a wide range of auction mechanisms. This naturally raises the question about which equilibrium predictions are more accurate.

Second, under some fairly general conditions, all the gains from trade are exploited in competitive equilibrium. This property is called Pareto optimality, and means that after all trade has been conducted, it is impossible to make one player better off without making another worse off. This rationale for the competive equilibrium is that if the trading mechanism does not attain Pareto opitmality, then there are potential gains to all the players from changing it.

Finally, compared to models which analyze strategic behavior explicitly analyzing models of competitive equilibrium for predicting how firms behave and also the distribution and size of the gains from trade is straightforward. This simplifies the analysis, which might otherwise be intractable if modeled as a noncooperative game. We have illustrated the power of competitive equilibrium predictions in both real and financial asset markets, reviewing the predictions that arise when players maximize present value, how those predictions are affected by inducing risk aversion, and investigating the effects of distortions. The results of experiments help us to evaluate how well competitive equilibrium predicts outcomes in games that are difficult to solve using noncooperative game theoretic methods.